# UNIVERSITY OF MARYLAND
## DEPARTMENT OF PHYSICS
COLLEGE PARK , MARYLAND 20742

**PHYSICS 732** **FINAL Homework assignment**
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1. The cuprate high T c superconductors are highly anisotropic materials containing stacks of 2-dimensional of $CuO_2$ planes in a square lattice separated by charge reservoir planes. The $CuO_2$ planes are thought to be responsible for the anisotropic conduction and the superconductivity in these materials. The Cu states are highly localized so that a tight binding treatment is thought to be appropriate. The resulting band structure displays several features that are important for the properties of these materials: unusual van Hove singularities and Fermi surface nesting near half filling (i.e., large areas of the Fermi surface that are separated by a common wave vector). Perform a 2-dimensional tight binding calculation of the band structure of these planes. The Cu atoms form a nearly simple cubic lattice. Take a model with s-states on the simple cubic Bravais lattice.

a). Find E(k) and sketch the constant energy surfaces in the first Brillouin zone.

b). Find the density of states. If you cannot obtain an analytic result
find the limiting behaviors at the bottom, top and middle of the band and sketch the result.

c). If the $CuO_2$ plane contains one electron per unit cell what is the geometry of the corresponding Fermi Surface? Is it a metal or insulator?

d). What are the nesting wavevectors?

2. Consider a quantum well of GaAs in a GaAs/AlGaAs heterostructure. If the width of the GaAs quantum well varies from N to N+1 unit cells in some region of the well an electron can be bound to the wider fluctuation relative to the narrower quantum well. Under these conditions the fluctuation produces a quantum dot.

a. Calculate the electron energies in a uniform width well relative to the bottom of the conduction band for 30 nm and 26 nm widths in the square well approximation and the effective mass approximation. Take the electron mass $m^*/m_0$=0.07 and a band gap of 1.5 eV.

b. Take the 30 nm wide region to be an island in the plane of size 100 nm by 100 nm surrounded by a 26 nm wide region of the well. Calculate the binding energy of an electron to the quantum dot in the square well approximation and the effective mass approximation.

c. Take the 30 nm fluctuation region to be a circular island of diameter D. Calculate the energy of the lowest bound state and the ionization energy. Determine if there is a minimum diameter D that supports a state localized to this island.

3. Find the superfluid velocity $v_s$ near an isolated vortex in an extreme type II superconductor by examining the fluxoid quantization more carefully. Obtain analytic results in the two limits: $\xi < r < \lambda$ and $r > \lambda$. Use the result to estimate the radius of the core of the vortex.

4. Consider a single Josephson junction in the resistively shunted Josephson junction (RSJ) model in zero magnetic field. Take the $\beta_c \ll l$ limit (i.e., the capacitance $C \approx 0$).
a. Find the I-V characteristic of the junction for the case of a voltage biased junction (source impedance « R). Sketch the result.
b. Now consider the case of current biasing (the more realistic case). Hint: the integrals are in tables. Sketch the variation of $\delta = \theta_1 - \theta_2$ with t and show that $\langle d\delta/dt \rangle = 2e(I^2 - I_c^2)R/h$ where $\langle \cdots \rangle$ denotes the time average. Find and sketch the I-V characteristic.

5. Consider a linear chain of Heisenberg spins with the Hamiltonian )
$\mathcal{H} = J(S_1 \bullet S_2 + S_2 \bullet S_3 + S_3 \bullet S_4 + S_4 \bullet S_1)$
a. Show that this can be rewritten as
$\mathcal{H} = ½\, J[(S_1 + S_2 + S_3 + S_4)^2 - (S_1 + S_3)^2 - (S_2 + S_4)^2]$
b. Find the ground state energy of the system.
c. Compare with the classical result. Discuss.
d. What is the total spin of the ground state?

6. A thin layer of metal transmits electromagnetic radiation according to the approximation

$$T = \frac{1}{\left(1 + \frac{dZ\sigma_1}{2}\right)^2 + \left(\frac{dZ\sigma_2}{2}\right)^2}$$

where T is the transmitted fraction of the energy, $\sigma_1$ and $\sigma_2$ are the real and imaginary part of the conductivity, d is the thickness of the metal, and $Z = 377\ \Omega$ [MKSA units] or $4\pi/c$ [CGS units].
(a) Assume that a superconductor is described by $\sigma(\omega) = \Omega_p^2\, \delta(\omega)$; the Cooper pair condensate results in a delta function at zero frequency. Calculate the transmission coefficient T for low, but nonzero frequencies, and show that the frequency dependence of the transmission can be used to evaluate $\Omega_p$
(b) Based on the Drude model (with $1/\tau = 0$), estimate the (super) conduction electron density n, if T = 0.02 for d = 0.01 -cm-wavelength IR radiation. The thickness of the film is d = 1000 A.