UNIVERSITY OF MARYLAND
DEPARTMENT OF PHYSICS
COLLEGE PARK , MARYLAND 20742

PHYSICS 732 **HOMEWORK ASSIGNMENT #6**
DR. H. D. DREW **DUE TUESDAY MAY 8, 2007**

Read Chapter 34 of Ashcroft & Mermin and chapter 27 of Marder

1. Ashcroft and Mermin, Problem 34.1.

2. Electrodynamics of dirty superconductors:
Consider the two fluid model of a superconductor in which we assume that for $0 \le T \le T_c$ the current density may be written as the sum of two contributions $j = j_n + j_s$, where j_n comes from thermally excited quasiparticles and the conductivity is of the Drude form σ_D with carrier density n_n and relaxation time τ and j_s derives from the London equation with a carrier density $n_s = n - n_n$.
a. Show from Maxwell's equations that the dispersion relation for electromagnetic waves in the superconductor of wavevector q and frequency ω is
$$q^2c^2 = 4\pi\sigma_D i - c^2\lambda^{-2} + \omega^2$$
and compare the penetration dept with the London penetration depth, λ_L.
b. Show that for $\omega\tau << 1 \quad and \quad \omega << \omega_P$ (the plasma frequency) that the normal electrons do not participate significantly in the dispersion relation and find the penetration depth of the wave to lowest order.
c. Examine the case of $\omega\tau >> 1 \quad and \quad \omega << \omega_P$.
d. Consider the magnetic penetration depth for superconductors as a function of the mean free path of the electrons by using the conductivity sum rule. That is, assume that in the normal state the conductivity is the Drude form σ_D. Model the conductivity in the superconducting state by $\sigma_1 = \Theta(\omega - 2\Delta)\sigma_D$, where Θ is the step function (this is not equivalent to the two fluid model). Find the penetration length in the limits $\omega\tau << 1 \quad and \quad \omega\tau >> 1$. Express your answers in terms of λ_L and the coherence length ξ.

3. Estimate the parameter $g(E_F)/V_0$ for simple metals in the "jellium" approximation. From this and the phonon parameters estimate the energy gap and T_c for Pb.

4. From BCS theory: calculate the free energy difference between the superconducting and normal states at T = 0. From the Cooper pair problem, calculate the radius of the Cooper pair and compare with the BCS coherence length.

5. Find the following for the quasiparticle excitations of BCS superconductors:
a. Show that the quasi particle operators $\alpha_{k\uparrow} = u_k a_{k\uparrow} + v_k a^+_{-k\downarrow}$, $\alpha_{k\downarrow} = u_k a_{k\downarrow} - v_k a^+_{-k\uparrow}$ obey Fermion anticommutation relations.

b. Prove that BCS ground state is the α vacuum.
c. Find the density of states. Compare with the normal state.
d. Find the velocity as a function of momentum k. Compare with the normal state.
e. Find the quasiparticle effective mass, $m^* = \hbar^2 k_F \partial k / \partial E_k$, as a function of momentum k.
Compare with the normal state.
f. Find the electrical current of a quasi particle as a function of its momentum k.