UNIVERSITY OF MARYLAND

DEPARTMENT OF PHYSICS

COLLEGE PARK, MARYLAND 20742

PHYSICS 732 **HOMEWORK ASSIGNMENT #4**
DR. H. D. DREW Due Tuesday, April 10, 2007

Read Ashcroft and Mermin, chapter 31, 32, 33
Read Marder, chapter 24, 25, 26.

1. Conducting systems with half filled bands are unstable to formation of density waves which gaps the Fermi surface and produces an insulator. Consider a one dimensional system with lattice constant a in the tight binding approximation. Take a hopping nearest neighbor matrix element t and determine the E(k) relation (ignore the overlap matrix elements). Now suppose there is a weak periodic potential V_Q where $Q = \pi/a$ is half of the reciprocal lattice vector of the original lattice.
a. Calculate the resulting energy dispersion near $k = \pm\pi/2a$ and sketch the result.
b. Calculate the matrix elements for optical absorption near $k = \pm\pi/2a$. Is the transition "allowed" or "forbidden"?

2. Work out the edge state energy levels as a function of k_x for a Hall bar sample in which the edge is abrupt, i.e., V(y) rises to infinity at $y = \pm a$.
This is related to the 1-d double pendulum problem. Plot the first three energy levels as a function of orbit center.

These two problems are related to the Laughlin's argument for the Quantum Hall effect, Aharanov-Bohm effect and phenomenon of persistent currents in normal metals.

3. Show that the magnetic Hamiltonian is invariant for the gauge transformation $A \rightarrow A - \nabla\omega$ if the wave function is multiplied by the phase factor $\exp[-i(\frac{e}{\hbar c})\omega] = \exp[-i\frac{e}{\hbar c}\int \delta A \cdot dr]$

4. Consider non interacting spinless electrons confined to a ring of radius r_0. The cross section of the ring is sufficiently small that you can assume that the electrons are in the lowest quantized states. Solve for the energy levels associated with the θ motion in zero magnetic field. Now consider the addition of a magnetic flux ϕ going through the center of the ring while the magnetic field at the ring is zero. Calculate the energy levels as a function of ϕ. Suppose that there are 4 electrons on the ring. What is the magnetic field dependence of the magnetic moment of the ring.