Department of Physics, University of Maryland, College Park

Dec. 17, 1999 **Physics 731: FINAL EXAM** ***Name:*** ____________

I. Consider a *D*-dimensional crystal with *N* primitive cells, each of volume *v*, *p* atoms per cell, *Z* valence electrons per atom.

1. How many distinct, independent values of **k** are there (assuming periodic boundary conditions)?

2. a) How many phonon branches are there?

b) How many of these branches are acoustic?

c) In a high-symmetry direction, how many are transverse?

3. a) What is the volume (in reciprocal space) of the second Brillouin zone?

b) What is the ratio of the volume of the third Brillouin zone to the second?

c) What is the "volume" (in k-space) of the free-electron "sphere" (or circle or line)?

d) What is the "volume" (in k-space, in an extended zone scheme) enclosed by the Fermi surface?

e) What is the *minimum* number of energy bands that are fully or partially occupied?

f) What is the plasma frequency (at $\mathbf{k} = 0$)?

5. a) For a fixed magnetic field **H** and p = 1, how many distinct magnon modes are there?

b) Assuming a free-electron distribution, what is the minimum value of H such that only 10 Landau levels are occupied? (Show your work.)

c) i) If the material is an insulator or semiconductor, circle which of these must be even.

Z *p* *D* *pZ* *pZD*

ii) Give an example of a single-element metal for which the circled quantity is even.

II. 1. Give an example of a (single-element) metal that has optical phonons. (Lattice structure is okay.)

2. Give an example of a metal that becomes a type I superconductor.

III. Give a) the temperature (T) dependence of the following specific heats <u>at low T</u>. (Prefactors not needed.) For each item, also give b) the name of the characteristic temperature that T is much less than when it is "low" and the order of magnitude (10^n K) of this characteristic temperature, and c) the model or law (if any) that produces this temperature dependence (e.g., Debye, Bloch, Drude, Einstein).

1. Specific heat due to acoustic phonons.

2. Specific heat due to optical phonons.

3. Specific heat due to electrons in a normal metal.

4. Specific heat due to electrons in a superconductor.

5. Specific heat due to electrons in a semiconductor.

6. Specific heat due to spin waves in an insulator at weak H.

IV. Consider the band structure for Ge at the right.

1. Is the gap direct or indirect? (Justify.)

2. Mark clearly the features due to spin-orbit coupling.

3. For Si, is the gap a) bigger by an order of magnitude,
b) somewhat bigger, c) about the same size (±10%),
d) somewhat smaller, e) smaller by an order of magnitude?

4. With a **dotted** line, draw in the Fermi energy ε_F.

5. With a **solid** line, draw the chemical potential μ at a moderate temperature (high enough to be distinguishable from the dotted line!). Describe briefly your reasoning in positioning μ relative to ε_F.

V. For **4** of these 5 experimental techniques, state a) what precisely is measured and b) what physical information one learns.

1. de Haas - van Alphen   2. Hall effect (classical, not quantum)   3. Optical absorption

4. Angle-resolved photoemission   5. Scanning tunneling microscopy

VI. Consider a Bloch function $\psi_{\mathbf{k}}(\mathbf{r})$ for a **bcc lattice** with conventional lattice constant a. At the site at the origin suppose this wavefunction has the value $\psi_{\mathbf{k}}(0) = A$. Let $\mathbf{k} = (\pi/a)(1, 0, 0)$.

1. Find the value of $\psi_{\mathbf{k}}$ at the nearest neighbor site with x, y, z > 0.

2. Find the value of $\psi_{\mathbf{k}}(\mathbf{r})$ at the second-neighbor site $\mathbf{r} = \mathbf{R} = a\,(1, 0, 0)$.

3. Find the value of $\psi_{\mathbf{k}}(\mathbf{r})$ at the second-neighbor site with y>0.

4. Is there any lattice site at which $\psi_{\mathbf{k}}(\mathbf{R}) = 0$? If yes, find one explicitly. If no, explain why not.

5. If $\psi_{\mathbf{k}}(a(0.1, 0, 0)) = 0.8A$, what is the value of $\psi_{\mathbf{k}}(a(1.1, 0, 0))$? (Justify your answer.)

VII. Consider a one-dimensional lattice with lattice periodicity a and a weak periodic potential (so that the nearly-free electron model applies) $U(x) = -2V \cos(2\pi x/a) + V \cos(4\pi x/a)$.

1. Write explicitly (by inspection after converting cos to exponentials) the non-zero Fourier components U_K.

2. In the reduced zone scheme, sketch the three lowest bands. (You can do this even if you didn't do part a.)

3. What is the size of the gap between the first and the second bands?

4. What is the size of the gap between the second and the third bands?

5. Circle on your sketch anywhere the effective mass is negative.

6. *Near the zone boundary*, how do the effective masses in the first and the second bands compare?

7. For what value of V is the effective mass in the third band the same as in the first band, *both evaluated at the center of the zone* (the "Γ" point)?

8. Sketch the density of states (vs. energy) for this system. Label clearly the energies on the horizontal axis.

VIII. Consider an inversion layer at the surface of a semiconductor, described by the sketches at the right.

1. On the left sketch, draw the bound states with ε_0 and ε_1.
2. On the right sketch, mark the energies ε_0, ε_1, ε_4 on the vertical axis.

3. Recall (hopefully) that the density of states $g(\varepsilon)$ has the form of an ascending staircase. Explain its form, specifically:

a) Label the value of ε associated with each jump and draw ε_F on the sketch of $g(\varepsilon)$.

b) Why is the DOS flat between jumps?

c) What determines the height of the jumps in $g(\varepsilon)$?

IX. For each of the following modes that can be described as simple harmonic oscillators, indicate what is oscillating. (In one case, this may be problematic.) If it cannot, describe it briefly.

Phonon

Plasmon

Surface plasmon

Photon

Polariton

Cooper pair

Magnon

X. In a pn junction

1. On which side of the junction is the positive "end" of the interface electric dipole? Briefly, why?

2. What produces the drift or generation current?

3. What produces the diffusion or recombination current?

4. Which is sensitive to the bias voltage V, and what is its dependence on V?

5. In which direction does the rectification occur (which is the preferred direction of electric current)?

XI. 1. Is a semiconductor with many donor impurities (and few or no acceptors) intrinsic, n-type, or p-type?

2. Give an example of a typical donor impurity, or indicate the column of the periodic table from which the donor atoms typically come.

3. According to the hydrogen-atom analogy, what is the (ground-state) binding energy of the donor impurity (and relative to what energy in the semiconductor)?

4. An exciton is a bound state of a donor electron and an acceptor hole. Following up on the H-atom analogy, it is similar to positronium (bound pair of electron and positron). How, then should its ground-state binding energy compare to that of the donor level?

XII. At the transition of a superconductor to a normal metal, circle any of the following which vanish and indicate what happens to the others:

1. Superconducting electron density
2. London penetration depth
3. Gap at the Fermi level
4. Difference between normal and superconducting electronic specific heat
5. Density of vortices (in type II)