Department of Physics, University of Maryland, College Park

Dec. 19, 2005 **Physics 731: FINAL EXAM** ***Name:*** ______________________________(Print)

*"I pledge on my honor that I have not given or received any unauthorized assistance on this examination, and that I personally prepared my 8-1/2x1l inch sheet."*______________________________**(Sign)**

Easy and hard problems are intermixed; don't get bogged down. You are allowed a personally-prepared sheet of formulas, front side from midterm, back side new.

1. Consider a D-dimensional crystal with volume V, N_c primitive cells, N_a identical atoms, and N_e valence electrons, zero magnetization.

a) How many distinct, independent values of **k** are there (assuming periodic boundary conditions)?

b) i) What is the volume (in reciprocal space) of the 8th Brillouin zone?

ii) With which other Brillouin zones does it share a bounding plane?

c) What is the number of longitudinal acoustic modes?

d) What is the density of spin-up electrons?

e) If the material is an insulator or semiconductor, circle the one ratio of these that must be even.

N_e / N_c $\quad$ N_a / N_c $\quad$ N_e / N_a $\quad$ N_c / N_a $\quad$ N_a / N_e $\quad$ N_c / N_e

2. What is the order of magnitude of the ratio the radius of a hydrogenic orbital about a donor impurity in Si to that of an H atom? (Circle the best choice.)

100 $\quad$ 10 $\quad$ 2 $\quad$ 1 $\quad$ 1/2 $\quad$ 0.1 $\quad$ 0.01

b) What factors are responsible for this ratio not being 1?

c) Are any of these factors different for a similar hole orbital around an acceptor impurity? (If yes, which?)

d) Should one expect to find bound-states between 2 donor impurities comparable to H_2 molecules? Explain.

3. For each of the following statements, respond with the appropriate letters. Many statements are true for more than one model; include all the letters! If unsure of a choice, write a brief explanation why.

A) True for the Drude model of metals

B) True for the Sommerfeld (free electron) model of metals

C) True for the nearly free electron model

D) True for the (isotropic) Debye model of lattice vibrations

E) True for the Einstein model of lattice vibrations

F) True for spin waves/magnons

a) All electrons/phonons/magnons (i.e., whichever one is relevant for the choice) have the same velocity.

b) The energy of each mode is linearly proportional to $|\mathbf{k}|$.

c) In the limit of small excitation energy, this energy is proportional to $|\mathbf{k}|^2$.

d) The largest existing or occupied **k**-mode is proportional to the 1/3 power of the density of atoms.

e) 1000 K is a low temperature.

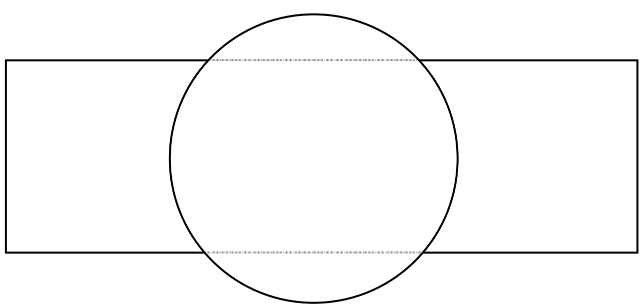
f) Can explain a Hall coefficient that is positive.

g) Conserves the number of [energy] carriers in its description of thermal conductivity.

h) Gives a specific heat that goes like $\exp(-T_0/T)$ at low temperature T.

i) Gives a specific heat that goes like a power-law of T at low T.

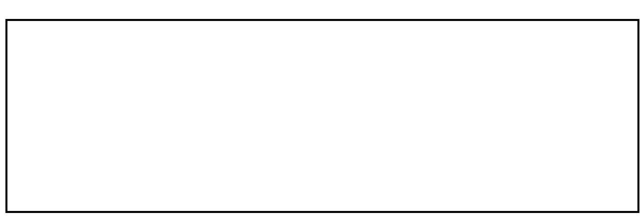
j) Gives a specific heat that goes like a constant at high T (relative to the characteristic temperature).



4. Consider a rectangular 2D lattice with the 1st Brillouin zone illustrated to the left and a free-electron circle.

a) Redraw on the rectangle below the circle in the nearly free electron approximation. Include a dashed line showing the boundary of the 2nd Brillouin zone (BZ).

b) Redraw below (separately) the orbits in the first and in the second BZ and indicate for each whether it is electron-like, hole-like, or open.



5. Define (using formulas) the following types of electron effective masses

a) specific heat

b) band (due to dispersion of $\varepsilon(\mathbf{k})$, effective mass tensor)

c) cyclotron

d) Fermi liquid

e) Are they all the same for the free electron/Sommerfeld model?

6. Consider a simple (single-band, nearest-neighbor, no overlap) tight-binding band describing a 2-dimensional square lattice: $\epsilon(\mathbf{k}) = -2\gamma\,[\cos(k_x a) + \cos(k_y a)]$.

a) Determine the inverse effective mass tensor for arbitrary **k**.

b) For what values of **k** (if any) is the effective mass essentially a scalar (so proportional to the unit matrix)?

c) For arbitrary **k**, is this tensor diagonal? Do you expect it to be diagonal for other 2D lattices, such as hexagonal?

7. An electron with $\mathbf{k}_e$, moving to the right, enters a region with a magnetic field **H** pointing out of this sheet, as depicted:

$$\longrightarrow \qquad\qquad\qquad\qquad \odot$$

a) Indicate the path of the electron as it enters the region

b) Suppose this electron state has negative effective mass. If this electron state is missing from an otherwise full band, draw an arrow to describe the appropriate length and direction of $\mathbf{k}_h$ and indicate how it evolves as it enters the region of the magnetic field.

8. For each of the following, indicate by writing the appropriate letter before each property if it is

A) proportional to the electronic density n
B) proportional to $n^{1/2}$
C) independent of n,
D) inversely proportional to n ($\propto 1/n$),
E) proportional to $n^{-1/2}$
F) other (specify the n-dependence).

a) plasma frequency ω_p

b) London penetration depth (λ_L) [n_s for n in the possible answers]

c) Drude conductivity σ

d) Mobility μ

e) Hall coefficient R_H

f) Cyclotron frequency ω_H

g).Specific heat of free electrons

h) ε_F [for free electrons in 3D]

i) $g(\varepsilon_F)$ [for free electrons in 3D]

j) Area of an extremal [Fermi-level] electron orbit in a [strong] H field

9. In Hartree-Fock theory for free electrons, $\epsilon_0(k) = \hbar^2 k_F^2/2m \rightarrow \epsilon\,(k)$

a) Is $\varepsilon(k)$ greater than, less than, or equal to $\epsilon_0(k)$ for $k < k_F$? What is the physical reason for this?

b) Is $\epsilon\,(k)$ greater than, less than, or equal to $\epsilon_0(k)$ for $k > k_F$?

c) Show that in Hartree-Fock, the Fermi velocity is infinite.

d) What would be the consequences of this? I.e., how do you know from experimental evidence that other corrections cancel this divergence?

10. a) Draw a sketch of the energy levels vs. position for a p-n junction. i) Indicate the net charge distribution near the junction. ii) Indicate the direction of the *hole* generation and recombination currents.

b) Which of these currents is sensitive to the height of the barrier?

c) What is depleted in the depletion zone?

11. What is the ratio of the number of states in the highest filled Landau level to the lowest filled Landau level in a layer (in **k**-space) of a metal perpendicular to a strong magnetic field? (If the problem does not give enough information, indicate what further you need to know.)

12. In the spin wave picture, show that $\langle \mathbf{k} | S_\perp^2(\mathbf{R}) | \mathbf{k} \rangle = S + (2S-1)/N$.
Hint: start by finding $S_\perp^2(\mathbf{R}) | \mathbf{R} \rangle$ and . $S_\perp^2(\mathbf{R}) | \mathbf{R}' \neq \mathbf{R} \rangle$ (in the notation of A&M).

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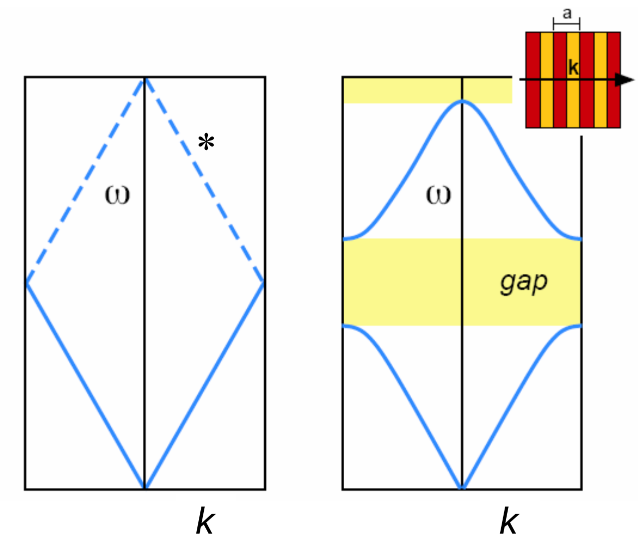


13. According to an online tutorial by Johnson and Joannopoulos, "Photonic crystals are periodically structured electromagnetic media, generally possessing *photonic band gaps*: ranges of frequency in which ***light*** cannot propagate through the structure." Based on what you know (or should know) about Bloch's theorem, you can understand the key results. In the simplest case, consider a 1D modulation as illustrated in the sketch, where the dielectric constant $\varepsilon(x)$ varies periodically in the x direction, taking on 2 different values ε_{high} and ε_{low} in alternating vertical slabs.

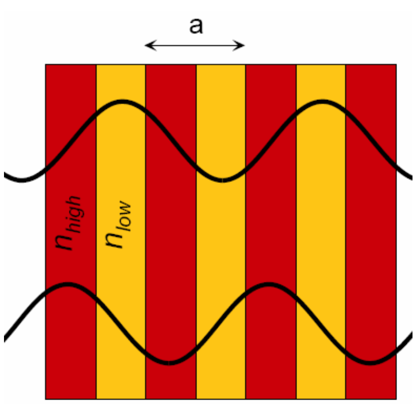
The electric field $E(x,t)$ satisfies the equation: $-\nabla^2 E = \varepsilon(x)\,(\omega/c)^2 E$, with traveling-wave solutions for $E(x,t)$.

a) In the left panel above is the empty-lattice approximation. i) Draw on the panel where the line marked by the * was before it was mapped to the position in the panel.

ii) Indicate on the horizontal axis in the panel the position of the values of k at the right and the left edge of it.

iii) Why is the dispersion linear rather than quadratic (like for electrons) for small ω?

b) Writing the average dielectric constant as $\langle\varepsilon\rangle$, show that ω at the center of the gap is $c\pi/(a\sqrt{\langle\varepsilon\rangle})$



c) The figure on the left shows 2 standing-wave solutions of the k-states bordering the gap. Recalling that $\varepsilon = n^2$, which solution corresponds to the lower edge of the gap, i.e. the situation with the lower frequency? Why?

d) If we model the dielectric by $\varepsilon(x) = 1 + \Delta\cos(2\pi x/a)$, where $0 < \Delta << 1$, so that the perturbation can be treated as perturbational, the size of the gap is $\Delta c\pi/a$. How does this compare with what you would expect from the nearly free electron approximation?

e) For a photonic crystal with variations in 3D rather than 1D (so $\varepsilon(\mathbf{r})$), to get a gap in all directions do you expect Δ_{3D}, compared to Δ_{1D}, to be:

Much smaller | Slightly smaller | Same size | Slightly larger | Much larger

Why?

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