Department of Physics, University of Maryland, College Park

Dec. 15, 2000 **Physics 731: FINAL EXAM** ***Name:*** **______________**

1. Consider a D-dimensional crystal with N primitive cells, each of volume v, p atoms per cell, Z valence electrons per atom.

a. How many distinct, independent values of **k** are there (assuming periodic boundary conditions)?

b. i) How many acoustic phonon branches are there?

ii) How many longitudinal optical branches are there (in a high-symmetry direction)?

c. i) What is the volume (in reciprocal space) of the n^{th} Brillouin zone?

ii) With which Brillouin zone[s] does the third Brillouin zone share a border that is more than a line? (Take D=3 for this question.)

iii) What is the "volume" (in k-space, in an extended zone scheme) enclosed by the Fermi surface?

iv) What is the density of electrons?

v) What is the London penetration depth (at T=0)?

2. State the smallest spatial dimension (1, 2, or 3) needed to have the following: (Explain your reasoning briefly!)

Transverse phonons

Optical phonons

Landau levels

Spin waves (magnons)

Closed electron orbits

Energy gaps in band structure

3. Suppose the effective mass tensor has the three eigenvalues $m_1^* = 3m_e$, $m_2^* = 4m_e$, $m_3^* = 5m_e$, where m_e is the bare electron mass.

a) What is the specific heat effective mass?

b) What is the minimum value of the cyclotron effective mass? In which direction does **H** point in this case?

c) What is the maximum value of the cyclotron effective mass? In which direction does **H** point in this case?

4. Consider a [2D] square lattice, lattice constant a, with a free electron gas.

a) If each atom has one electron (Z=1), what is the area of the Fermi "sphere" (i.e. circle)?

b) If the Fermi circle just touches the edge of the first Brillouin zone, how many electrons on average are there per atom? (Note that the answer is not an integer!)

c) If each atom has two electrons (Z=2), sketch the Fermi circle (superimposed on the first Brillouin zone).

d) i) Redraw your sketch in part c) in the nearly free electron approximation. ii) Show clearly, e.g. with heavy lines, which part becomes the hole orbit.

5. Answer with the following letters. Each statement may have several letters. If uncertain, write out your thinking.

A True for metals
B True for superconductors
C True for intrinsic semiconductors
D True for doped semiconductors
E Not true for any of the above

The Fermi level at T=0 lies within an energy gap.

The size of this gap decreases with increasing temperature.

Is well described by the Sommerfeld model.

Has acceptor levels

At finite temperature always has a non-vanishing density of electron and hole carriers

The fundamental unit has charge -2e.

Is never (as far as the course is concerned) a Bravais lattice (with a single-atom basis).

6. Consider a dispersion relation of the form $\omega \propto |\mathbf{k}|^n$ for a D-dimensional system.

a) Show that the associated density of states can be written as $g(\omega) \propto \omega^\gamma$ and find γ in terms of n and D.

b) If the dispersion relation describes bosons, show that the low-temperature specific heat $\propto T^\zeta$, and find ζ in terms of γ (or n and D).

c) If the dispersion relation describes electrons, does the T-dependence of the low-temperature specific heat stay unchanged (with the same value of ζ)? If yes, provide a brief justification; if no, provide an illustrative counterexample.

7. Consider thermal conductivity by electrons or phonons. Respond with
A. True for electrons only
B. True for phonons only
C. True for both electrons and phonons
D. False for both electrons and phonons, but true for atoms in a classical gas.
(In case of doubt, explain your reasoning!)

The number of carriers is conserved.

Anharmonic effects prevent infinite conductivity.

The speed of the carriers is greater at the hotter end than at the cooler end.

The density of the carriers is greater at the hotter end than at the cooler end.

There is a voltage difference between the hotter end than at the cooler end.

The thermal conductivity is proportional to the specific heat.

8. Consider an electron at k=0 in a 1D tight-binding model, with $\varepsilon(k) = \varepsilon_0 - 2V\cos(ka)$.
A constant electric field -E exerts a force on the electron in the positive direction. There are no collisions.

a) Plot k(t) vs. time until the electron reaches the zone boundary. How long a time t_0 does this take?

b) Sketch the effective mass as a function of k (not t) for positive values of k in the first Brillouin zone.

c) Sketch the acceleration of the electron as a function of time from 0 to t_0.

d) Sketch the velocity of the electron as a function of time from 0 to t_0.

9. a) What is the minimum field H such that at most 5 Landau levels of a free electron gas (with Fermi energy ε_F) are occupied at T=0?

b) If the electron density of this electron gas is doubled, does the minimum H increase or decrease, and by how much? Answer for both i) a 3D electron gas and ii) a 2D electron gas such as in an inversion layer.

10. If the chemical potential of an intrinsic semiconductor stays in the center of the band gap as temperature increases, what can be concluded about the value of m_c/m_v ?

11. For Si, $m^*/m = 1/4$ and $\varepsilon = 12$; for Ge, $m^*/m = 1/8$ and $\varepsilon = 16$.

a) If the energy $\epsilon_c - \epsilon_d$ is 1/40 eV for Si, estimate its size for Ge.

b) Do you expect Si or Ge with comparable doping to have a higher carrier density at room temperature? (Explain briefly.)

c) If the hydrogenic radius of an electron around the donor is 70 Å, estimate its size for Ge.

12. A pn junction is subjected to a forward bias. Compared to zero bias, indicate whether each of the following A) increases exponentially, B) increases modestly, C) stays nearly unchanged, D) decreases.

Drift current

Diffusion current

Total net current

Interface dipole layer

Difference between the conduction band minima in the two materials

Band gap of the p material

13. Do two of the following 3 parts:

a) In class and the book sketches illustrate the magnetization M as a function of H for type I and type II superconductors. Draw instead a sketch of the B field inside the superconductor as a function of H for these two types. Mark clearly the relevant critical fields H_{c1} etc. along the horizontal axis.

b) In homework you considered a superconducting slab with a magnetic field along a planar direction, i.e. perpendicular to a normal to the slab.

i) How thick would the slab have to be so that the field at the middle would be 1/3 that outside the slab?

ii) Estimate the minimum thickness of the slab so that the field might induce vortices. (The answer should be given in terms of the relevant characteristic length of the superconductor.)

c) Explain why metals with small densities of states at ε_F are poor superconductors. Use both formal results from homework and physical reasoning.