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Physics 731 HOMEWORK ASSIGNMENT #8 Due: Nov. 15, 2007
Deadline: Nov. 20

Reading: See syllabus

Problems to turn in (read the rest):

1. 12–2. To make things easier, you may assume that **H** = H**ž** and **M** is diagonal (but with $\mathbf{M}_{xx} \neq \mathbf{M}_{yy} \neq \mathbf{M}_{zz}$). (The solutions will give the general case, which is an unpleasant mess.)

2. 12–6 and 12–7a. (The pair count as a single problem. Use T_R for 12-6.)

3. 13–2 is worked out on p. 407 of Grosso and Paravicini, as discussed in class (and also in the solutions). If we define kinetic coefficients, for n = 0, 1, 2:

$$e^2 K_n \equiv \frac{e^2}{4\pi^3}\int \tau(\hat{E}\bullet\vec{v})\varepsilon^n\left(-\frac{\partial f_0}{\partial \varepsilon}\right)d^3k = \int\left(-\frac{\partial f_0}{\partial \varepsilon}\right)\varepsilon^n \sigma(\varepsilon)\,d\varepsilon$$

a) Show that the second equality above is true.

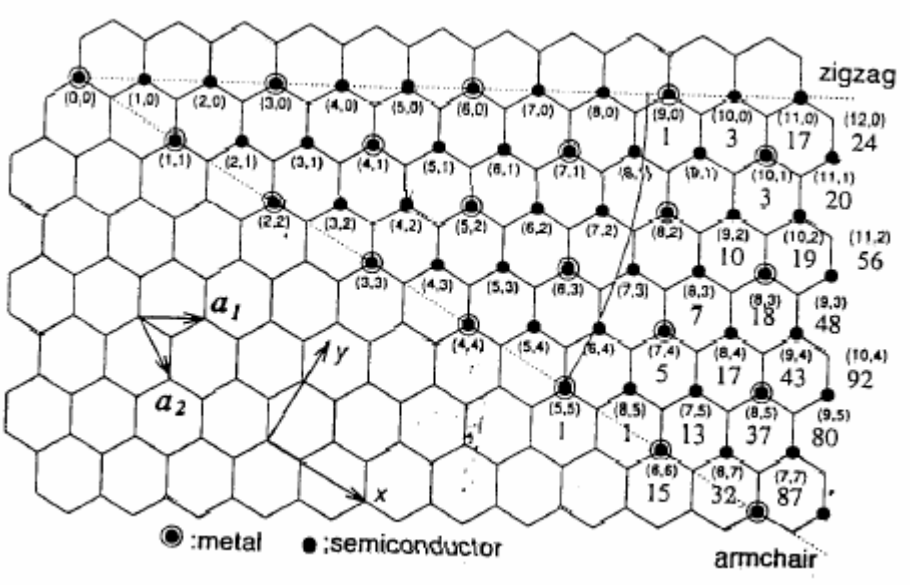
b) Show explicitly that $e^2K_n = \mu^{n-1}\sigma(\mu) + ((\pi k_B T)^2/6)\, h_n(\mu)$, to order T^2, and find $h_n(\mu)$.

c) Again to order T^2 write e^2K_n in terms of ε_F rather than μ (which, curiously, Grosso and Paravicini do not do).

4. 15–1 and 15–4 (The pair count as a single problem. Note that this is an easy extension to 3D of the 2D problem 5′of Set #7.)

Read 15–3. Since I derive this result in undergraduate modern physics courses, I presume you have already seen it. Read also 15–5, which I might have assigned had time permitted. To help visualize why γ is negative, you should look at Fig. 9.13.

5 (20 points). Consider the band structure of carbon nanotubes. The figure from a Dresselhaus[2] review, pasted below, will be helpful



Recall that in the nearest-neighbor, single-band, tight-binding model, the energy of a honeycomb is given by

$$\varepsilon(k_x, k_y) = \pm\gamma\left[1 + 4\cos\left(\frac{\sqrt{3}k_x a}{2}\right)\cos\left(\frac{k_y a}{2}\right) + 4\cos^2\left(\frac{k_y a}{2}\right)\right]^{1/2}$$

in Schönenberger notation, with a the lattice constant of the hexagonal Bravais net, so √3 times the nearest-neighbor (NN) spacing a_0. This notation assumes that the points of the hexagons are in the x direction.

a) In the armchair orientation, the wrapping is by (n,n) in terms of the primitive vectors, with the unit-cell spacing (along the circumference of the tube) being the NN spacing plus the distance across a hexagon tip-to-tip. (Cf. Fig. 19.2.) i) Show then that this unit distance $|\boldsymbol{a}_1 + \boldsymbol{a}_2|$ is 3 times the NN spacing, or $a\sqrt{3}$. By analogy with finding the allowed values of k for periodic boundary conditions, find the n allowed values of k_x here and iii) write the dispersion relation for each as a function of $k_y \rightarrow$- k, the 1D wavevector in the "infinite" direction. (Only one equation need be written, with k_x replaced by its discrete values.)

b) Draw the 2D Brillouin zone for graphene (making use of your results in earlier homework sets) and on it draw the lines corresponding to these solutions for n = 5 (solid lines) and n=6 (dashed lines).

c) Now consider the zigzag orientation, with periodicity(n,0). i) Show that the unit distance is a (so that the circumference of the tube is na) and that the wrapping can be viewed as being in the y direction. As above ii) find the allowed values of k_y and iii) write the dispersion relation.

d) Again draw the 2D Brillouin zone for graphene with solid and dashed lines for n= 5 and 6, respectively.

e) Comment on the difference between the two and why the figure says armchair tubes are always metallic but zig-zag ones are metallic only when n is a multiple of 3.