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Physics 731 **HOMEWORK ASSIGNMENT #7** **Due: Thursday, Nov. 1, 2007**
Deadline: Tuesday, Nov. 6, 2007

Read about Bloch's theorem and the nearly-free electron gas: A&M chapters 8, 9, 10; F&J pp. 60-61, 125–127

Problems to turn in (read the rest):

1. NOT ASSIGNED (but you will get the solution; a dated problem and not worth it, unless you are interested) 8–1, parts a, e, f, g, h; cf. pp. 168–170 [and figs. 5 & 6] of Kittel's 8^{th} (pp. 180–2 in 7^{th} pp. 164–6 in 6^{th}, 191–2 in 5^{th})]. Note also the Kronig-Penney model; java applets are listed on the class web-site reference list.

2. NOT ASSIGNED (but you will get the solution) 8–2, parts b & c. [Hint: change k-space variables to ones that have spherical constant-energy contours. Note also part a; this is a more rigorous version of the sloppy sketch in class.] Also, sketch $g(\varepsilon)$ near a minimum (as in part b) and near a saddle point (as in part c).

3. 9–1

4. 9–3 Feel free to use Mathematica or equivalent to find the roots of the 4x4 determinants.

5. a) 9–5a

5' (From Kittel 7–1, counts as part of 5 above) *Square lattice, free electron energies.*

a) Show for a simple square lattice (so in two dimensions) that the kinetic energy of a free electron at a corner of the first zone is higher than that of an electron at the midpoint of a side face of the zone by a factor of 2.

b) What is the corresponding factor for a simple cubic lattice (so in three dimensions)?

c) What bearing might the result of (b) have on the conductivity of divalent metals?

6. 10–1, parts a-i, a-iv, and c.

7. Consider a single-band tight-binding model for a square lattice of side a. Neglect overlap. Then $\varepsilon(\mathbf{k}) = \varepsilon_s - \beta - 2\gamma\,[\cos(k_x a) + \cos(k_y a)]$.

a) i) Find the equivalent of Eq. (10.24) for small $|\mathbf{k}|a$.

ii) If one writes this as a free-electron dispersion relation, what is the "mass"?

b) Note that the constant-energy "surfaces" (i.e. curves in 2D) are not circular for larger values of $|\mathbf{k}|a$. In which directions is the curve elongated? Specifically, find the ratio of $|\mathbf{k}|$ in the {11} directions to $|\mathbf{k}|$ in the {10} directions on a curve with

i) $\varepsilon(\mathbf{k}) - \varepsilon_s + \beta = -3\gamma/2$ ii) $\varepsilon(\mathbf{k}) - \varepsilon_s + \beta = -\gamma$ iii) $\varepsilon(\mathbf{k}) - \varepsilon_s + \beta = -\gamma/2$

c) Find the shape of the constant-energy curve for $\varepsilon(\mathbf{k}) - \varepsilon_s + \beta = 0$.

d) For general (not small) values of $|\mathbf{k}|a$, show that the electron velocity $\mathbf{v}(\mathbf{k}) \propto \nabla_{\mathbf{k}}\varepsilon$ parallel to $\mathbf{k}$ only in high-symmetry — $\langle 10\rangle$ and $\langle 11\rangle$ — directions