Department of Physics, University of Maryland, College Park

Dec. 15, 1998 **Physics 731: FINAL EXAM** ***Name:*** _____________

<u>I.</u> Consider a D-dimensional crystal with N primitive cells, each of volume v, p atoms per cell, Z valence electrons per atom.

1. How many distinct, independent values of $\mathbf{k}$ are there (assuming periodic boundary conditions)?

2. a) How many phonon branches are there?

b) How many of these branches are optical?

c) In a high-symmetry direction, how many are longitudinal?

3. a) What is the volume (in reciprocal space) of the first Brillouin zone?

b) What is the value of the structure factor at $\mathbf{K} = 0$ (assuming the form factor is 1)?

c) What is the ratio of the volume of the fourth Brillouin zone to the first?

4. What is the density of electrons n ?

5. a) How many energy bands are there?

b) What is the *minimum* number that are fully or partially occupied?

c) If the material is an insulator or semiconductor, circle which of these must be even. (Justify your choice!)

Z $\quad p$ $\quad D$ $\quad pZ$ $\quad pZD$

<u>II.</u> For each experimental technique, state what it used [primarily] to measure.

1. Neutron scattering

2. X-ray scattering

3. Raman scattering

4. de Haas - van Alphen

5. EXAFS

6. Optical absorption

III. For each of the following, indicate by writing the appropriate letter before each property if it is a) proportional to the electronic density n, b) proportional to $n^{1/2}$ c) independent of n, d) inversely proportional to n ($\propto 1/n$), e) proportional to $n^{-1/2}$ f) other (specify the n-dependence). (For superconductors, use n_s for n.)

1. Plasma frequency ω_p

2. London penetration depth (Λ or λ_L)

3. Conductivity σ

4. Mobility μ

5. Hall coefficient R_H

6. Flux quantum Φ in superconductivity

7. Cyclotron frequency ω_H

8. Specific heat of free electrons

9. ε_F [for free electrons in 3D]

10. $g(\varepsilon_F)$ [for free electrons in 3D]

IV. For each experimental technique, state what it used [primarily] to measure.

1. Neutron scattering

2. X-ray scattering

3. Raman scattering

4. de Haas - van Alphen

5. EXAFS

6. Optical (infrared) absorption

V. For each of the following statements, respond with the appropriate letters. Many statements are true for more than one model; include all the letters! If unsure of a choice, write a brief explanation why.

a) True for the Drude model of metals

b) True for the Sommerfeld (free electron) model of metals

c) True for the nearly free electron model

d) True for the (isotropic) Debye model of lattice vibrations

e) True for the Einstein model of lattice vibrations

1. All electrons/phonons have the same velocity.

2. The energy of each mode is linearly proportional to $|\mathbf{k}|$.

3. The largest existing or occupied **k**-mode is proportional to the 1/3 power of the density of atoms.

4. 1000 K is a low temperature.

5. Can explain a Hall coefficient that is positive.

6. Conserves the number of [energy] carriers in its description of thermal conductivity.

7. Gives a specific heat that goes like $\exp(-T_0/T)$ at low temperature T.

8. Gives a specific heat that goes like a power-law of T at low T.

9. Gives a specific heat that goes like a constant at high T (relative to the characteristic temperature).

VI. Suppose a 1-D band can be described by the tight-binding dispersion relation $\varepsilon(k) = \varepsilon_0 - 2\gamma \cos(ka)$, $\gamma > 0$.

1. For what values of k, $0 \le k \le \pi/a$, are there Van Hove singularities?

2. If the effective mass m^* at $k = 0$ is *twice* the bare electron mass m_e, express γ in terms of m_e and other fundamental constants.

3. Draw a sketch of m^* vs. k for $0 \le k \le \pi/a$.

4. Circle all of the follow which change sign when the subscript e becomes h:

k_e ε_e v_e m_e q_e

VII. Consider a wire strip with an electric field pointing to the left and a magnetic field pointing out of the sheet (toward you). On the strip on the left, indicate the **k** and **v** of electrons. On the right one, do the same for holes.

VIII. For each statement, indicate whether it is true or false. If true, provide a brief justification. If false, provide a counter-example.

1. There are no hole-like [partially filled] bands in alkali metals.

2. There are no hole-like [partially filled] bands in transition metals.

3. There are no hole-like bands in semiconductors.

4. There is a unique definition of the effective mass of an electron in a particular partially-filled band.

5. In an undoped, instrinsic semiconductor, the chemical potential lies half way between the conduction and the valence band in the limit of $T \to 0$.

IX. 1. In the BCS model of a superconductor, suppose the Debye frequency were doubled. What change would be needed in the density of states at ε_F to leave the gap unchanged?

2. What is paired in a Cooper pair in the BCS model of superconductivity?

X. 1. In the [bcc] first Brillouin zone of an fcc crystal with conventional lattice constant *a*, find the distance k_L from the zone center (Γ) to the middle (L) of one of the 8 hexagons (which lie in {111} planes). [Hint: use the fact that $\mathbf{k}_L$ is simply related to a reciprocal lattice vector!]

2. Show that this is the closest point on the zone boundary to Γ.

3. In the *nearly-free-electron model*, a) write an expression for the energy ε_- as $\mathbf{k} \rightarrow \mathbf{k}_L$ from inside the zone. [Your answer should be in terms of k_L and a $V_{\mathbf{K}}$, with $\mathbf{K}$ specified.]

b) If $\varepsilon_F = \varepsilon_- + \Delta$, Δ small, show that the Fermi surface intersects the hexagon in a circle of radius $\rho \propto \sqrt{\Delta}$.

XI. Do TWO of the following THREE:

1. Draw a sketch of the energy levels vs. position for a p-n junction, as in class. Indicate the net charge distribution near the junction. Indicate the direction of the *hole* generation and recombination currents. Which is sensitive to the height of the barrier?

2. Consider an electron orbiting about a magnetic field H in the z direction. Prove that in the semiclassical picture, the xy component of the orbit of the electron in real space has the same shape as that of the $k_x k_y$ plane in k-space, rotated by 90°. How do the evolutions in the z direction in the two spaces compare?

3. In homework you showed that for a thin superconducting film thickness 2d, the magnetic field inside the slab could be written as $\vec{B}(y) = H_o \hat{z} \frac{\cosh(y/L)}{\cosh(d/L)}$. a) For what equation was this the solution?

b). Does the critical magnetic field increase, decrease, or stay the same relative to that of a bulk sample of the same metal? (In your answer, indicate what determines the critical magnetic field, at which the superconductor is driven normal.) How would you proceed to make a quantitative comparison? (Write down needed integrals, but do not carry them out.)