Department of Physics, University of Maryland, College Park

Dec. 14, 2001 **Physics 731: FINAL EXAM** ***Name:*** ______________

Easy and hard problems are intermixed; don't get bogged down. You are allowed a personally-prepared sheet of formulas.

1. Consider a *D*-dimensional crystal with *N* primitive cells, each of volume *v*, *p* atoms per cell, *Z* valence electrons per atom.

a. How many distinct, independent values of **k** are there (assuming periodic boundary conditions)?

b. How many transverse optical branches are there (in a high-symmetry direction)?

c. i) What is the volume (in reciprocal space) of the n^{th} Brillouin zone?

ii) With which Brillouin zone[s] does the fourth Brillouin zone share a border that is a plane?

iii) What is the "volume" (in k-space, in an extended zone scheme) enclosed by the Fermi surface?

iv) What is the "volume" (in k-space, in an extended zone scheme) enclosed by the Fermi surface if all electron spins were alligned in one direction (e.g. up)?

v) What is the density of electrons?

vi) What is the Hall resistance?

d) If the material is an insulator or semiconductor, circle which of these must be even. (Justify your choice!)

Z *p* *D* *pZ* *pZD*

2. Give a physical example that is well described by

a two-dimensional free-electron gas

a one-dimensional free-electron gas

a hydrogenic state with a binding energy that is a small fraction of a Rydberg a simple harmonic oscillator with dispersion relation $\omega(k) = \omega_0 + \text{const.}\bullet k^2$ for small k (that is not a phonon)

3. Consider a (2D) rectangular lattice with lattice constants a and b, $a < b$. Suppose the free-electron "sphere" (i.e. circle) is large enough so that it extends into the second Brillouin zone (BZ) in one direction but not in the perpendicular direction.

a) Illustrate this situation at the top of this page, using a dotted line to represent the free-electron circle.

b) With a solid line, show how this sphere is altered in the nearly free electron approximation.

c) Indicate (separately) for the part in each BZ whether it is closed and electron-like, closed and hole-like, or open.

d) Is this a metal or an insulator/semiconductor?

e) If a magnetic field were applied perpendicular to the 2D plane, describe how you would find the time for a carrier to complete an orbit around a closed orbit, indicating what information you would need to know.

4. a) For a free-electron gas, find the strength of the magnetic H such that the third Landau level is occupied just near the belly/equator of the Fermi sphere.

b) At this value of H, what is the ratio of the occupation of the first Landau level to that of the second level? (Explain your reasoning in case you make a mistake in your numerics.)

c) Draw a sketch of the density of states.

5. a) What is the dispersion relation $\varepsilon(\mathbf{k})$ for a (single-band, as studied in class and homework) tight-binding model of an fcc crystal, assuming just nearest-neighbor interactions $-\gamma$ and no atomic overlap?

b) What is the bandwidth (difference between minimum and maximum $\varepsilon(\mathbf{k})$ for this band?

c) What is the **x** component of the electron velocity $\mathbf{v}(\mathbf{k})$?

6. Draw a sketch for a magnon on a chain of atoms (from above and perpendicular to the spins, as done in class illustrations) that has $k=\pi/a$. Use this sketch to argue that magnons are a much better approximation for small k than for large k.

7. In a one-dimensional crystal (with lattice spacing *a*), a real periodic potential produces the following gaps: 2 eV between the first and the second band, and 1 eV between the second and the third band. Assuming the nearly free electron approximation is valid, write the form of this potential.

8. a) Find the temperature dependence (the exponent of T) at low T of the energy $\langle E \rangle$ for a system in D dimensions with boson excitations having energy or frequency proportional to $|\mathbf{k}|^{\alpha}$ (at small $|\mathbf{k}|$).

b) Circle the cases in the following list for which this is relevant:

Acoustic phonons

Optical phonons

Conduction electrons in a metal

Electrons in a semiconductor

Electrons in a superconductor

Magnons in a ferromagnet

Magnons in an antiferromagnet

c) For each case above, write down (to its right) the lowest-energy excitation from the ground state (in 3D) at T=0. For systems where this goes to zero as $|\mathbf{k}|^{\alpha}$, write down this power α.

9. a) Show that if the effective mass tensor is proportional to the (3×3) unit matrix, the specific heat effective mass and the cyclotron effective mass are the same.

b) Where in the Brillouin zone is this condition most likely to occur, and for what kinds of Bravais lattices?

c) Give an example of a mass tensor for which the cyclotron effective mass is twice the specific heat mass for the magnetic field **H** in one direction and half of it when **H** is in a perpendicular direction.

10. Answer with the following letters. A statement may have more than one letter. If uncertain, write out your thinking.

A True for Type I conventional superconductors
B True for Type II conventional superconductors
C True for high-T_c superconductors
D True for metals that do not become superconductors
E Not true for any of the above

Are often just a single element (e.g. Pb, Al, Sn)

Typically have a layer structure, with great anisotropy

Have very weak electron-phonon coupling

Have a positive surface energy between normal and superconducting regions

Have a coherence length that is notably less than the penetration depth

Can have a gap function that vanishes in some directions

Invariably involves triplet rather than singlet pairing of electrons

Might form electron pairs by spin-flip rather than phonon mediation

11. Circle each of the following which are *parallel* (NOT antiparallel) to the hole drift (generation) mass current at a pn junction, and place a star before those that are *increased* by a forward bias.

hole diffusion (recombination) mass current

electron drift (generation) mass current

electron diffusion (recombination) mass current

electron diffusion (recombination) electrical current

12. Consider the characteristic features of the band structure of different kinds of materials. How would you distinguish a simple metal like Al, a d-band or noble metal like Cu, an insulator like diamond, and a highly anisotropic material like graphite? In particular, discuss distinctive features you would expect for each. Aspects you ought to consider include: location of Fermi level, similarity to empty-lattice band structure, nature of gaps, dispersion in different directions, flat bands crossing rising bands. (You can also refer to graphs of phonon dispersion relations, particularly if you are unsure about the band structure.)

Bonus questions (do only one, and do it last, after finishing the rest of the test):

In quantum dots, doped semiconductors with sizes of order 10s of nm., what sort of problems would arise with the picture presented in the text for bulk crystals?

Why is it claimed that magnetic impurities in superconductors strongly depress the transition temperature, even in materials that easily accommodate fluxoid penetration (vortices)?

Consider a 3D metal in the free electron (Sommerfeld) model. Thermal expansion says that the fractional increase of the volume of the crystal is proportional to T.

a) What is the physical origin of this expansion?
b) How does this affect the Fermi energy and the total energy of the gas?
c) How does the change in the Fermi distribution affect the energy of the gas?
d) Estimate the relative effects.