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Physics 731 **HOMEWORK ASSIGNMENT #9** **Due: Nov. 17, 2006**
Deadline: Nov. 21

Reading: See syllabus

Reminder: No class on Tuesday Nov. 14, Thursdays Nov. 16 & 23.
Make-up classes: Fridays Nov. 10 & 17, 2:30-3:45.

Problems to turn in (read the rest):

1. 12–2. To make things easier, you may assume that **H** = H $\check{\mathbf{z}}$ and **M** is diagonal (but with $\mathbf{M}_{xx} \neq \mathbf{M}_{yy} \neq \mathbf{M}_{zz}$). (The solutions will give the general case.)
2. 12–6 and 12–7a. (The pair count as a single problem. Use T_R for 12-6.)
3. 13–2.
4. 15–1 and 15–4 (The pair count as a single problem. Note that this is an easy extension to 3D of the 2D problem 5′of Set #7.)
5. 31–11.

6. Refer back to pp. 661-663; the spin susceptibility of a conduction electron gas at T = 0 K may be discussed by another method. Let $n_{\pm} \equiv (n/2)(1 \mp \zeta)$ be the concentration of spin-up (-down) electrons, i.e. parallel (antiparallel) to a magnetic field H.

a) Show that, in H, the total energy per volume in the spin-up band in a free-electron gas is

$E^+ = E_0\,(1 - \zeta)^{5/3} + (n/2)\,\mu_B H\,(1 - \zeta)$,

where $E_0 = (3/10)n\varepsilon_F$ in terms of the Fermi energy ε_F in zero magnetic field. Find a similar expression for E^-.

b) Minimize the total energy per volume $E^+ + E^-$ with respect to ζ and solve for the equilibrium value of ζ in the approximation $\zeta \ll 1$. Show then that the magnetization $M = (3/2)\, n\mu_B^2 H/\varepsilon_F$, as in the class discussion of Pauli paramagnetism.

c) We now consider the effect of exchange interactions among the conduction electrons. As a viable first approximation, we assume that electrons with parallel spins interact with each other with energy -V (with V > 0), while electrons with antiparallel spins do not interact with each other. Show that the additional term $-(1/8)\, V\, n^2\, (1 - \zeta)^2$ is added to E^+ and find a similar expression for E^-.

d) Minimize the total energy and solve for ζ again in the limit $\zeta \ll 1$. Show that the magnetization is

$$M = \frac{3n\mu_B^2}{2\varepsilon_F - \frac{3}{2}Vn} H$$

Notice that there is peculiar behavior for $V > 4\varepsilon_F/3n$. One can easily show that at H=0 the total energy for the paramagnetic state with $\zeta = 0$ is unstable relative to a ferromagnetic state with finite ζ. This is called the Stoner criterion for ferromagnetism. (adapted from Kittel, ISSP) This kind of phase transition at T=0 is now glamorized with the label "quantum phase transition."

Read 15–3. Since I derive this result in undergraduate modern physics courses, I presume you have already seen it. Read also 15–5, which I might have assigned had time permitted. To help visualize why γ is negative, you should look at Fig. 9.13.