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Physics 731 HOMEWORK ASSIGNMENT # 2 Due: Sept. 19, 2006 (Deadline: Sept. 21)
Read Ashcroft & Mermin (A&M), chaps. 5-6; Duan and Guojan, 1.4, 2.2.4, 2.2.5, 2.3.2

1. A&M 5-3
2. A&M 6-2

3a) Show as a corollary to problem 2 [A&M 6-2] that the {111} planes of a simple cubic crystal are triangular lattices. (So are the {111} planes of the bcc crystal.) What is the interplanar spacing? [Hint: problem 1, A&M 5-3, may be helpful.]
b) For an fcc crystal, viewed from the [111] direction as a sequence of stacked close-packed planes, write down a third primitive vector $\mathbf{a_3}$, given that the first two are in a close-packed plane [e.g. $\mathbf{a_1} = a_{NN}\, \mathbf{x}$; $\mathbf{a_2} = (a_{NN}/2)\,(\mathbf{x} + \mathbf{y}\sqrt{3})$]. (I.e., find the components of $\mathbf{a_3}$ along **x**, **y**, and **z**. Note that a_{NN} is the nearest-neighbor distance, so (1/2) times the conventional lattice constant. I often write a_{NN} as a_1, but in this case that might invite confusion.) Then show explicitly how the ABCABC stacking sequence is realized, i.e. that after 3 translations by $\mathbf{a_3}$ the lattice points coincide with those in the original plane, translated perpendicular to this plane by 3 times the interplanar spacing *d*.

4. A&M 6-3
5. A&M 6-5
6. Consider the reciprocal lattice of a two-dimensional (2D) or planar lattice (in a 3D space).

a) Write $\mathbf{k} = \mathbf{k}_{\shortparallel} + k_z\mathbf{e}_z$. Show that $\mathbf{K}_{3D} = \mathbf{K}_{2D} + K_z\mathbf{e}_z$, where K_z is arbitrary, so that the reciprocal lattice can be represented by a net of rods. For elastic scattering, $|\mathbf{k}| = |\mathbf{k}|'$, write the relation between $\mathbf{k}_{\shortparallel}$ and $\mathbf{k}_{\shortparallel}'$. (Hint: Consider a 2D lattice as the limit of a (3D) family of planes with interplanar spacing *d* going to infinity. Cf. class websites, category "Reciprocal Lattices," item: Ewald construction for a surface or 2D lattice.) What added constraint comes from energy conservation?
b) Generalize Fig. 6.7 to show the Ewald construction for diffraction from a 2D lattice. Note that one observes a diffraction pattern of electrons from a surface for all values and orientations of the incident wavevector **k** above a critical value.
c) Show that for electrons incident perpendicularly on a {100} surface of a copper crystal, the critical (i.e. minimum) *energy* at which the first diffracted beam appears (as incident energy is raised) is about 22 eV. (Use the handy relationship $\lambda(\text{Å}) \approx 12/[E(\text{eV})]^{1/2}$. Note that the periodic table inside the front cover of A&M provides lattice constants of the elements, as well as lots of other information; alternatively, if you have Marder's text, use his table 2.1.)

7. a) What is the reciprocal lattice of a sheet of graphene (i.e. a planar honeycomb)?
b) What is the geometric structure factor of each rod?

Read and think about (but do not turn in) A&M 5-4, A&M 6-4, as well as what happens in problem 7 if part of the graphene sheet is rolled into a carbon nanotube.