Physics 731     HOMEWORK ASSIGNMENT # 2     Due: Sept. 19, 2006  (Deadline: Sept. 21)
Read Ashcroft & Mermin (A&M), chaps. 5-6; Duan and Guojan, 1.4, 2.2.4, 2.2.5, 2.3.2

1. A&M 5-3
2. A&M 6-2

3a) Show as a corollary to problem 2 [A&M 6-2] that the \{111\} planes of a simple cubic crystal are triangular lattices.  (So are the \{111\} planes of the bcc crystal.)  What is the interplanar spacing?  [Hint: problem 1, A&M 5-3, may be helpful.]
b) For an fcc crystal, viewed from the [111] direction as a sequence of stacked close-packed planes, write down a third primitive vector \(a_3\), given that the first two are in a close-packed plane [e.g. \(a_1 = a_{NN} \hat{x}\); \(a_2 = (a_{NN}/2)(\hat{x} + \sqrt{3}\hat{y})\)].  (I.e., find the components of \(a_3\) along \(x\), \(y\), and \(z\). Note that \(a_{NN}\) is the nearest-neighbor distance, so \((1/2)\) times the conventional lattice constant.  I often write \(a_{NN}\) as \(a_\nu\), but in this case that might invite confusion.)  Then show explicitly how the ABCABC stacking sequence is realized, i.e. that after 3 translations by \(a_3\) the lattice points coincide with those in the original plane, translated perpendicular to this plane by 3 times the interplanar spacing \(d\).
4. A&M 6-3
5. A&M 6-5

6. Consider the reciprocal lattice of a two-dimensional (2D) or planar lattice (in a 3D space).
a) Write \(k = k_\| + k_z \hat{e}_z\).  Show that \(K_{3D} = K_{2D} + K_z \hat{e}_z\), where \(K_z\) is arbitrary, so that the reciprocal lattice can be represented by a net of rods.  For elastic scattering, \(|k| = |k'|\), write the relation between \(k_\|\) and \(k_{\|}'\).  (Hint: Consider a 2D lattice as the limit of a (3D) family of planes with interplanar spacing \(d\) going to infinity.  Cf. class websites, category “Reciprocal Lattices,” item: Ewald construction for a surface or 2D lattice.)  What added constraint comes from energy conservation?
b) Generalize Fig. 6.7 to show the Ewald construction for diffraction from a 2D lattice.  Note that one observes a diffraction pattern of electrons from a surface for all values and orientations of the incident wavevector \(k\) above a critical value.
c) Show that for electrons incident perpendicularly on a \{100\} surface of a copper crystal, the critical (i.e. minimum) energy at which the first diffracted beam appears (as incident energy is raised) is about 22 eV.  (Use the handy relationship \(\lambda(\text{Å}) \approx 12/[\text{E(eV)}]^{1/2}\).  Note that the periodic table inside the front cover of A&M provides lattice constants of the elements, as well as lots of other information; alternatively, if you have Marder's text, use his table 2.1.)
7. a) What is the reciprocal lattice of a sheet of graphene (i.e. a planar honeycomb)?
b) What is the geometric structure factor of each rod?

Read and think about (but do not turn in) A&M 5-4, A&M 6-4, as well as what happens in problem 7 if part of the graphene sheet is rolled into a carbon nanotube.