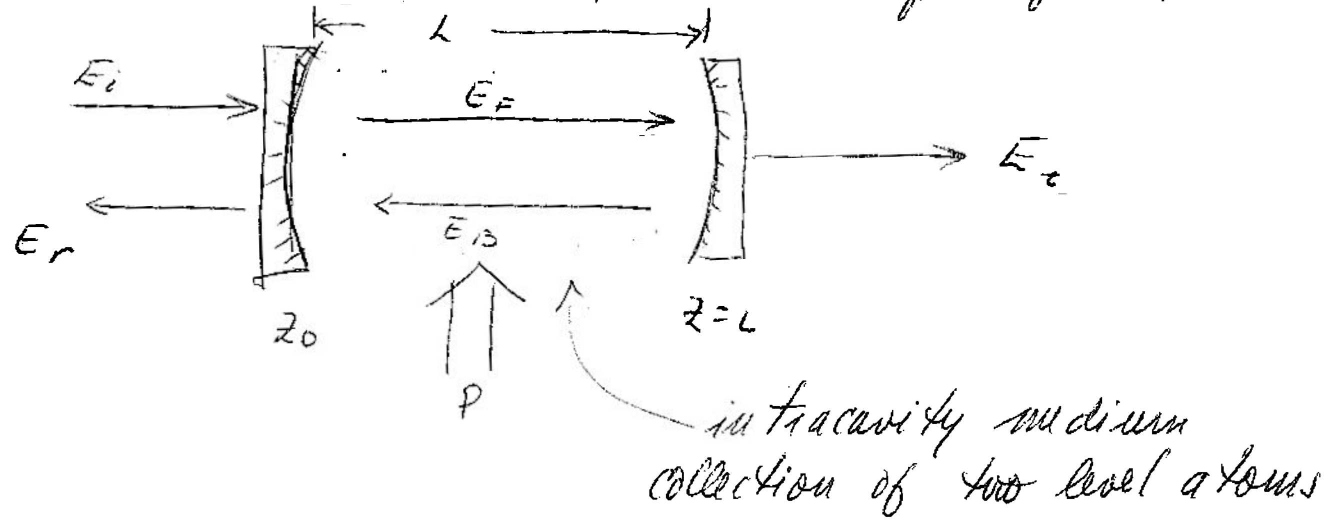
The case for atoms in cavities:

State equations for the laser and optical bistability

The purpose is to introduce a very simple model which shows the steady state characteristics of OB and the laser in some unified form



The two level atoms can absorb and saturate or can give gain, but also saturate if they are inverted by a pump on the side P. The effect of the pump simply changes the sign of the absorption
On resonance the atoms do not produce any phase shift, they simply attenuate.

What we need to do is add the waves to see what is transmitted

$$\vec{E}_i(z,t) = \mathcal{E}_i\, e^{-i(\omega t - kz)}\,\vec{x} + c.c.$$

$$\vec{E}_f(z,t) = \mathcal{E}_f\, e^{-i(\omega t - kz)}\,\vec{x} + c.c.$$

$$\vec{E}_t(z,t) = \mathcal{E}_t\, e^{-i(\omega t - kz)}\,\vec{x} + c.c.$$

and

$$\vec{E}_B(z,t) = \mathcal{E}_B\, e^{-i(\omega t + kz)}\,\vec{x} + c.c.$$

$$\vec{E}_r(z,t) = \mathcal{E}_r\, e^{-i(\omega t + kz)}\,\vec{x} + c.c.$$

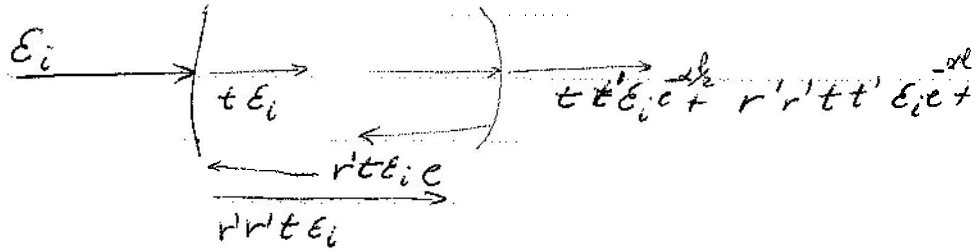
The mirrors have $\left.\begin{matrix} r\,e^{i\phi_r} \\ t\,e^{i\phi_t} \end{matrix}\right\}$ reflectivities for the field for waves <u>into</u> the cavity (may be complex).

while $r'\,e^{i\phi_{r'}}$

for waves <u>out</u> of the cavity $t'\,e^{i\phi_{t'}}$

$k = k_0 + \frac{i\alpha}{2}$ ← absorption of the atoms

We have to add the waves.



but careful with the phases.

let us make $r = r'$; $t = t'$ $\quad r^2 = R$, $t^2 = T$

$R + T = 1$

$$\mathcal{E}_T = T \mathcal{E}_i e^{-\alpha l/2} e^{-i\delta} + R T \mathcal{E}_i e^{-(\alpha l/2 - i\delta)2} + R^2 T \ldots$$

$$= T \mathcal{E}_i \frac{1}{1 - R e^{-(\alpha l/2 - i\delta)}}$$

the intensity.

$$I_t = I_i \frac{T^2}{(1+R^2) - 2R e^{-\alpha l} \cos\delta}$$

$\cos\delta = 1 - 2\sin^2 \delta/2$

$$= I_i \frac{1}{1 + \left[\frac{2\sqrt{R}}{1-R^2}\right]^2 e^{-\alpha l} \sin^2 \delta/2}$$

i

if there are no absorbing atoms.

$$\frac{|\mathcal{E}_t|^2}{|\mathcal{E}_i|^2} = \frac{T^2}{(1-R)^2 + 4R\sin^2\frac{\delta}{2}}$$

Airy function. which in the limit gives a Lorenzian.

$$\frac{|\mathcal{E}_t|^2}{|\mathcal{E}_i|^2} = \frac{T^2}{(1-R)^2}\,\frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}$$

if $R \sim 1$ and $R+T=1$ $\qquad \sin^2\frac{\delta}{2} = \frac{\delta^2}{4}$

$$\frac{|\mathcal{E}_t|^2}{|\mathcal{E}_i|^2} = \frac{1}{1 + \frac{\delta^2}{(1-R)^2}}$$

Empty cavity case

← sharpness of the fringe.

but the case we are interested in is when $\delta = 0$ and $\alpha \neq 0$ (resonant case).

$$\frac{|\mathcal{E}_t|^2}{|\mathcal{E}_i|^2} = \frac{T^2 e^{-\alpha L}}{(1 - Re^{-\alpha L})^2}$$

→ The Finesse of such a cavity is the ratio of the separation $\delta = 2\pi$ to the FWHM of transmission. $\delta_{HM} = \frac{(1-R)}{\sqrt{R}}$ $\quad$ FWHM $= 2\delta$

$$\mathcal{F} = \frac{\sqrt{R}\,\pi}{1-R} \sim \frac{\pi}{1-R}.$$

but let us make another assumption $|\alpha L| \ll 1$ and on resonance.

$$\frac{|\mathcal{E}_t|^2}{|\mathcal{E}_i|^2} \approx \frac{T^2(1-\alpha L)}{[1-R(1-\alpha L)]^2} \sim \frac{T^2}{[\underbrace{1-R}_{\text{small}}+\alpha L]^2}$$

of same order

The field inside the cavity is enhanced by the Quality factor $\sim \frac{1}{(1-R)}$ $\frac{1}{T}$ one can think of it just as the probability of escape of a photon enhancing the field inside

$$\frac{|\mathcal{E}_t|^2}{|\mathcal{E}_i|^2} = \frac{T^2}{[1-R+\alpha L]^2}$$

For an atomic medium (we shall see later from M.B.E.)

$$\alpha = \frac{\alpha_0}{1+I/I_s}$$

where I_s is the saturation power.

Homogeneously broadened only radiative, aligned dipoles.

$\alpha_0 = \sigma\rho$ ← density   where $\sigma = \frac{3}{2\pi}\lambda^2 \sim \frac{\lambda^2}{2}$

provide a photon.   polarization.

$$I_{sat} = \frac{\hbar\omega_a}{2\tau\ \sigma}$$ li

$$\frac{1}{2\tau} = \gamma_\perp$$ (transverse relaxation rate)

every two lifetimes. (2τ)

cross section for absorption (σ)

It is more convenient to normalize everything in the problem to the saturation intensity.

also instead of looking at the transmitted and incident.

$$Y = \frac{1}{T}\frac{I_i}{I_s}$$ the field inside in the absence of medium.

$$X = \frac{1}{T}\frac{I_t}{I_s}$$ the field inside <u>with</u> the medium

$$C = \frac{\alpha L}{2T}$$ ratio of ~~atomic~~ loss or gain to cavity loss.

because of travelling or standing waves the definitions may vary by factors of <u>2</u> in some places.

$$\frac{|\mathcal{E}_t|^2}{|\mathcal{E}_i|^2} = \frac{T^2}{(1-R+\alpha L)^2} \qquad 1-R=T$$

$$\frac{\dfrac{\varepsilon_0 c}{T}\dfrac{|\mathcal{E}_t|^2}{I_{sat}}}{\dfrac{\varepsilon_0 c\,|\mathcal{E}_i|^2}{T\ \ I_{sat}}} = \left(\frac{T}{T+\dfrac{\alpha_0 L}{1+\dfrac{I_{inside}}{I_S}}}\right)^2$$

$$\frac{X}{Y} = \frac{1}{\left(1+\dfrac{\alpha_0 L/T}{1+X}\right)^2}$$

$$Y = X\left(1+\frac{2C}{1+X}\right)^2$$

This is an equation of state for O, B and the laser.
let us look first at the laser $Y=0$
and C is negative:
one has the following equation.

let $2C = -\sigma$ positive

$$X\left(1+\frac{2C}{1+X}\right)^2 = 0$$

$$X\left(1-\frac{\sigma}{1+X}\right)^2 = 0$$

two solutions are possible $\underline{X=0}$

or $\underline{1+X=\sigma}$ $\quad X=\sigma-1 \quad \underline{X\geq 0}$ Intensity