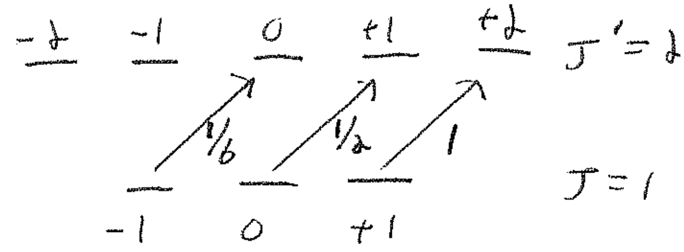
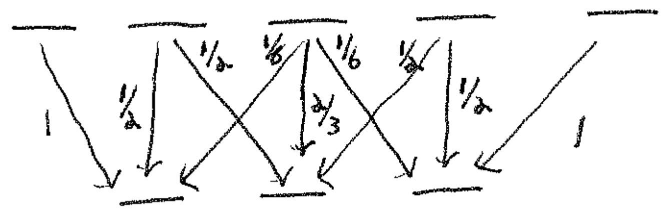
9) Using Wigner-Eckart:

a) Excitations (relative rates)



Decays (relative ratios, $\times \Gamma$ for absolute)



b) Population is being pumped toward the $|1,+1\rangle \to |2,+2\rangle$ transition. Add to this the assumption that excited state populations are small, then in equilibrium all the population is in $J=1$, $m_J = 1$ state.

c) Neglecting excited states by considering the probabilities of movement from one ground to the next:

$$\dot{\rho}_{|1,1\rangle} = \rho_{|1,-1\rangle} \cdot \left(\tfrac{1}{6} R_e\right) \cdot \left(\tfrac{1}{6}\Gamma\right) + \rho_{|1,0\rangle} \cdot \left(\tfrac{1}{2} R_e\right) \cdot \left(\tfrac{1}{2}\Gamma\right)$$

$$\dot{\rho}_{|1,0\rangle} = \rho_{|1,-1\rangle} \cdot \left(\tfrac{1}{6} R_e\right) \cdot \left(\tfrac{2}{3}\Gamma\right) - \rho_{|1,0\rangle} \left(\tfrac{1}{2} R_e\right) \cdot \left(\tfrac{1}{2}\Gamma\right)$$

$$\dot{\rho}_{|1,-1\rangle} = -\rho_{|1,-1\rangle} \left(\tfrac{1}{6} R_e\right) \cdot \left(\tfrac{5}{6}\Gamma\right)$$

where R_e = rate of excitation. $\left(\frac{I}{I_0}\right)$

9)

d) $0 = \frac{1}{36} P_{|1,-1\rangle} + \frac{1}{4} P_{|1,0\rangle} = \dot{P}_{|1,1\rangle} (R_e \Gamma)^{-1}$

$0 = \frac{1}{9} P_{|1,-1\rangle} - \frac{1}{4} P_{|1,0\rangle} = \dot{P}_{|1,0\rangle} (R_e \Gamma)^{-1}$

$0 = -\frac{5}{36} P_{|1,-1\rangle} = \dot{P}_{|1,-1\rangle} (R_e \Gamma)^{-1}$

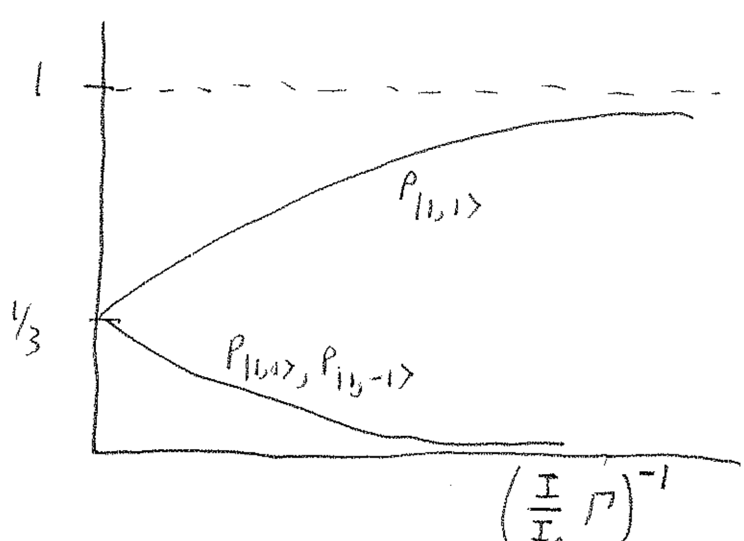
So $P_{|1,-1\rangle} = P_{|1,0\rangle} = 0$ with no restrictions on $P_{|1,1\rangle}$.

Note we must conserve number $\rightarrow P_{|1,1\rangle} + P_{|1,0\rangle} + P_{|1,-1\rangle} = 1$

so steady-state gives $P_{|1,1\rangle} = 1$, matching b)

e) solution looks like

$$\boxed{\begin{aligned} P_{|1,1\rangle} &= 1 - \frac{2}{3} e^{-\frac{5}{36}\frac{I}{I_0}\Gamma t} \\ P_{|1,0\rangle} &= \frac{1}{3} e^{-\frac{5}{36}\frac{I}{I_0}\Gamma t} \\ P_{|1,-1\rangle} &= \frac{1}{3} e^{-\frac{5}{36}\frac{I}{I_0}\Gamma t} \end{aligned}}$$



f) $\frac{1}{e}$ reached @ $t = \frac{36}{5}\left(\frac{I}{I_0}\Gamma\right)^{-1} \rightarrow \sim 7$ cycles $\rightarrow \sim 7$ photons

"More" steady-state $\rightarrow e^{-10} \rightarrow \sim 70$ photons