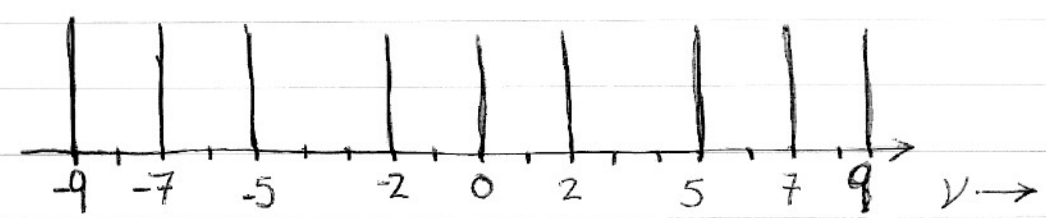
1)

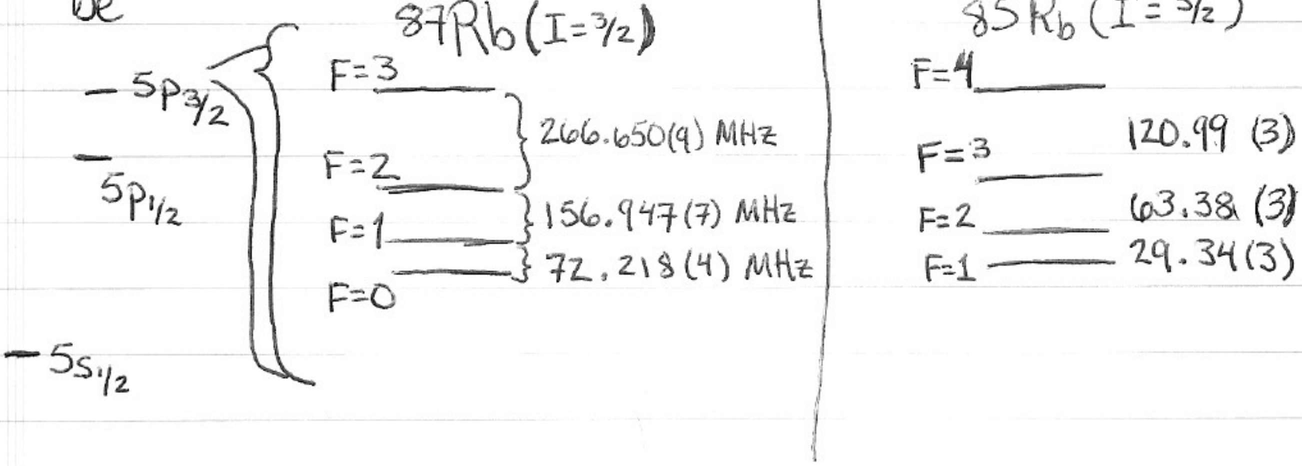
Anomalous Zeeman effect and selection rules:

The strong flourescence spectrum of a 2-electron atom in a magnetic field is observed, and has relative spacings as shown below. (In the absence of a magnetic field, there is only one line.) The flourescence is observed along a direction ⊥ to the magnetic field, so that all polarizations can be observed. Assume LS coupling is good, and that the excited level from which the spectrum originates is a ^3P, but J is unknown. Determine J, L, S for both the initial + final levels. Assume that all states of the excited level are excited to produce flourescence.



2)

The Hyperfine spacings for the $5P_{3/2}$ excited state of Rubidium have been measured to be



Find the Hyperfine constants $A_{J=3/2}$, $B_{J=3/2}$ for the two isotopes. The system is overconstrained (2 unknown and 4 energies (3 energy differences)), so either a) find the best global-fit values or b) pick 3 energies (2 intervals) and calculate $A_{J=3/2}$ + $B_{J=3/2}$. In either case, give an estimate of the error.

What do these measurements say about the difference between the isotopes.

3)

Circularly polarized light is incident on an atom with ground level $J=1$. (Ignore nuclear moments/angular momentum.) Assume the light is sufficiently detuned from any resonant transition that spontaneous scattering is negligible. The scalar and vector polarizabilities α_i at the laser frequency ω are defined by

$$\Delta U = -\frac{1}{2}\alpha_0(\omega)\left|\vec{E}_{AC}\right|^2 + \frac{i}{2}\alpha_1(\omega)\left(\vec{E}_{AC}\times\vec{E}^*_{AC}\right)\cdot\vec{J}$$

ΔU is the light shift, $\vec{E}_{AC}$ is the magnitude of the E-field of incident light, and assume for this problem that the second rank tensor polarizability is negligible at this laser wavelength, $\alpha_2(\omega)\simeq 0$.

a) If in addition a DC electric field $\vec{E}_{DC}$ is applied along the direction of propagation, what are the energies of the 3 ground states in the $J=1$ level? (Use DC polarizabilities $\alpha_i(\omega=0)$, $i=0,2$)

b) If the DC electric field is applied perpendicular to the direction of propagation, what are the energies of the 3 ground states? For fixed E_{AC}, sketch as a function of $\alpha_2(0)E_{DC}^2/\alpha_1(\omega)E_{AC}^2$ and discuss the limits $E_{AC}\gg E_{DC}$, $E_{DC}\gg E_{AC}$.

c) How does this compare to the situation with a DC magnetic field and a DC electric field in the absence of light?

4)

Optical Dipole Trap (optical tweezers for atoms)

In class we studied the "reactive" or "dipole" force using the optical Bloch equations.

Here we will see how this is used to make a simple atom trap.

A Gaussian laser beam has a transverse intensity profile

$$I(r) = I(0)\, e^{-2r^2/w^2}$$

where w, the waist, is the beam radius at $1/e^2$. The beam propagates in the z direction. It can be shown that the waist at an arbitrary point z is:

$$w(z) = w_0 \left[1 + (z/z_0)^2\right]^{1/2}$$

with $z_0 = \pi w_0^2/\lambda$. The waist is minimum $w(0) = w_0$ at the focus, $z = 0$.

a) Find $I(0)$ at $z = 0$ for a given total power P.

b) Calculate the depth of a trap formed by this focussed laser beam, in terms of the power P, the detuning δ, and the characteristic intensity I_0 where $I/I_0 = 2\Omega^2/\Gamma^2$. Consider only the dipole force - ignore the scattering force.

4) continued.

c.) For ^{23}Na, with $\Gamma/2\pi = 10\,MHz$, $J_0 = 6\,mW/cm^2$, take a total power $P = 1W$, focus to $W_0 = 10\,\mu m$, detune $|\delta/2\pi| = 100\,GHz$, calculate the depth in MHz and in Kelvin.

What is the sign of the detuning $\delta = \omega - \omega_0$?

d.) What is the oscillation frequency of an atom along the z and r directions. Write a general expression, then evaluate it for the parameters of part c).

e) Now consider the scattering of photons. What is the scattering rate for an atom at the focus. Evaluate this for the parameters give

What is the heating rate for this scattering rate? evaluate it in K/s for the parameters given.

What will happen to the ratio of trap depth to scattering rate as δ is increased. Discuss this qualitatively.

5)

$\pi/2$ pulse and free-induction decay

a) use the optical Bloch equations to calculate the response of an atom, initially in the ground state, ($w = -1/2$, $v = u = 0$) to an on-resonance pulse lasting a time T.

Take a general envelope $E_0(t)$ where for $t < 0$ $E_0(t) = 0$ and for $t > T$ $E_0(t) = 0$.
$E(t) = E_0(t) \cos\omega_0 t$.

Assume that you can ignore the decay Γ during the pulse

i. what is the condition on T such that Γ may be ignored?

ii. what is the condition on $E_0(t)$ such that $\rho_{ee}(T) = 1/2$?

iii. For an atom like Na with $\Gamma/2\pi = 10$ MHz, $I_0 = 6\ mW/cm^2$, what is the intensity of a $\pi/2$ pulse that has constant intensity from 0 to T? Choose a value of T appropriate for ignoring Γ and give a numerical value for the intensity

5) continued.

b) After the $\pi/2$ pulse, describe the evolution of the Bloch vector by calculating and sketching the evolution of:

$v(t-T)$
$w(t-T)$
$u(t-T)$
$\rho_{ee}(t-T)$

The atomic dipole moment of this decaying atom radiates. At some distant point in the radiation field plot a sketch of $E(t)$, qualitatively.

What is the spectrum of the radiated field?

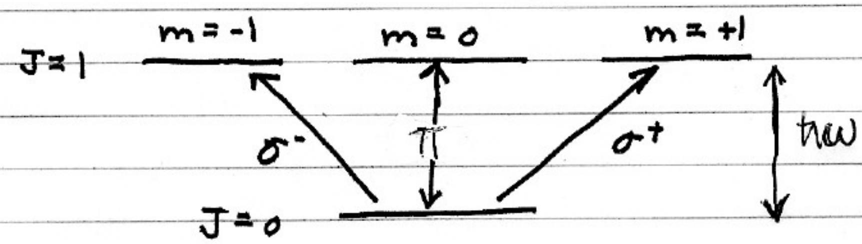
6)

A radiation-pressure trap for atoms

In class we considered 1-D laser cooling of a 2-level atom, taking into account radiation pressure and the Doppler shift.

Here we include the Zeeman shift for a 4-level atom and find a restoring force – a trap.

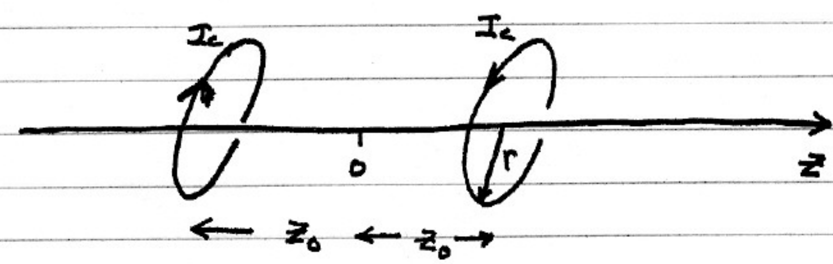
Consider a $J=0 \rightarrow J=1$ transition:



take the $\vec{z}$ axis to be the quantization axis, with laser beams incident along $\pm z$, with circular polarization.

a. show that a set of coils wound as shown, coaxial with z, with current I_c, produce a field of the form (near $z=0$) along the z axis.

$$\vec{B} = b z \hat{z}$$ and find b. Use SI units.



Express b in terms of I_c, z_0, and r.

6) continued

b. The $J=1$ state is assumed to have a magnetic moment $|\mu| = \mu_B$, a Bohr magneton for $m_J = \pm 1$. Draw an energy level diagram showing the energy levels of all 4 states as a function of z. Quantise along the $+z$ direction and label the states. A qualitative drawing is sufficient, but be sure the labels are unambiguous.

c. the cooling / trapping beams are tuned below ($\omega < \omega_0$) the zero-field resonance and one is polarized σ^+ (positive angular momentum), the other is polarized σ^-. Which beam (propagating along $+z$ or $-z$) should be polarized σ^+ and which σ^- to produce a restoring force, that forces the particle to $z=0$.

d. Express the Zeeman shift frequency as $\frac{E_Z}{\hbar} = \beta z m_J$ what is β?

e. To account for saturation, but to avoid the complication of standing-wave effects, we assume the counterpropagating beams to be alternating in time so only one is on at a time, each with 50% duty factor. What are reasonable conditions to put on the pulse duration T (i.e. $x \ll T \ll y$) to make the problem simple? Give a qualitative discussion of what x and y are.

(6) continued.

f. The intensity of each laser beam is I on average ($2I$ when it is on). The normalized intensity I/I_0 is $I/I_0 = 2\Omega^2/\Gamma^2$ where Ω is the Rabi frequency.

For small velocity v and small displacement z write an expression for the force on an atom as a function of $k = 2\pi/\lambda$; $\delta = \omega - \omega_0$; β, Γ, I, I_0, v, and z.

For what v and z is the expression valid?

g. Show that the force on an atom is the same as for a damped harmonic oscillator. What is the condition for critical damping?

h. For what values of δ and I is the damping maximum? For what values is the trap stiffness maximum? (i.e., the undamped oscillation frequency is maximum)

i. Consider ^{23}Na for which $\Gamma/2\pi = 10$ MHz, $\lambda = 589$ nm and take $b = 10^{-3}$ T/cm or $\beta = 14$ MHz/cm, and $I_0 = 6$ mW/cm^2. For the parameters that maximize the damping, what is the damping time for the velocity? Is the motion underdamped or overdamped? If you wanted to change ~~from underdamped to overdamped~~ this character what could you change?