Bose-Einstein Condensation and Coherent Atom Optics with Laser-like Atom Waves

Lecture # 30
Thursday 15 December 2005

Final lecture of PHY 721

What are laser-like atom waves?

All matter is wave-like: $\lambda_{dB} = h/p$

Hot atoms have short λ_{dB} ; 1000K Na (~10^3 m/s)

$\longrightarrow \lambda \approx 2 \times 10^{-11}$ m

Cold atoms have long λ ; 1 cm/s (< 1 μK) $\longrightarrow \lambda = 2 \times 10^{-6}$ m

But, just as ordinary (thermal) light is a jumble of different wavelengths and directions, a cold, thermal gas is a jumble of atom waves.

Laser light, or Bose-Einstein condensed atoms, are different: orderly, organized waves.

In 1924 Einstein predicted that if a gas of bosons, for example:

- Na atoms
- Rb atoms

were cold enough and dense enough, something weird and wonderful would happen –

Bose-Einstein Condensation:

A phase transition where a large fraction of the atoms stop moving ! (or, at least as much as the Heisenberg uncertainty principle allows)

What are Bosons?

There are two kinds of particles in the world:
fermions and bosons.

Fermions: half-integral spin (1/2, 3/2, etc.) electrons, protons, neutrons, ^{2}H, ^{6}Li, … are forbidden by the Pauli exclusion principle to have more than two of the same type in the same state. They are the “loners” of the quantum world. If electrons were not fermions, we would not have *chemistry*. Fermions obey the rules of Fermi-Dirac statisitics.

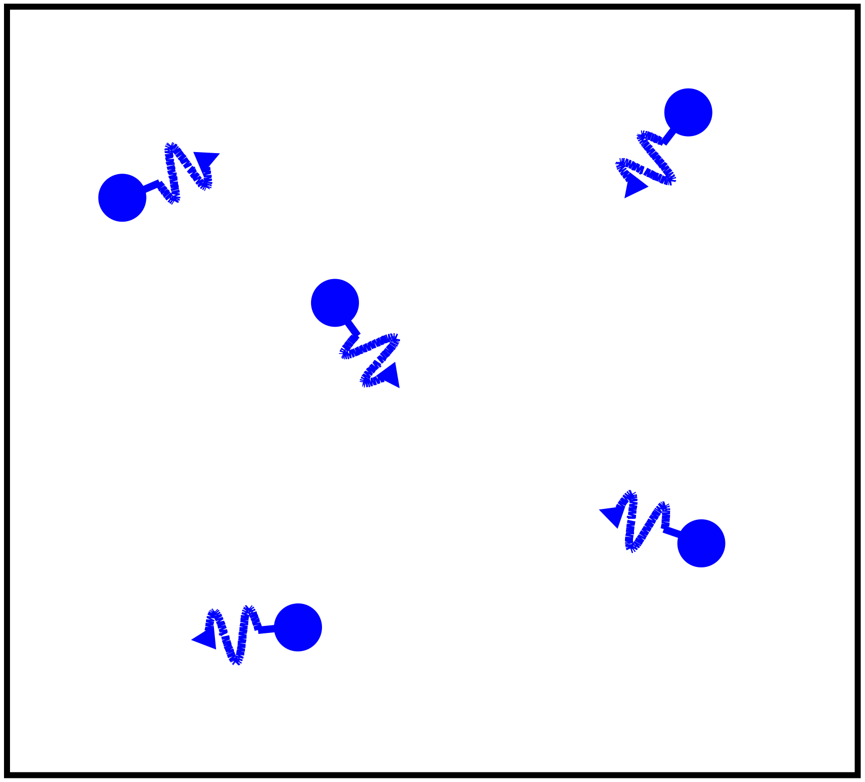
Bosons: integral spin (0, 1, etc.) photons, ^{1}H, ^{7}Li ^{23}Na, ^{87}Rb, ^{133}Cs, … love to be in the same state. They are the “joiners” of the quantum world. If photons were not bosons, we would not have lasers. Bosons obey the rules of Bose-Einstein statistics.

How cold and how dense?

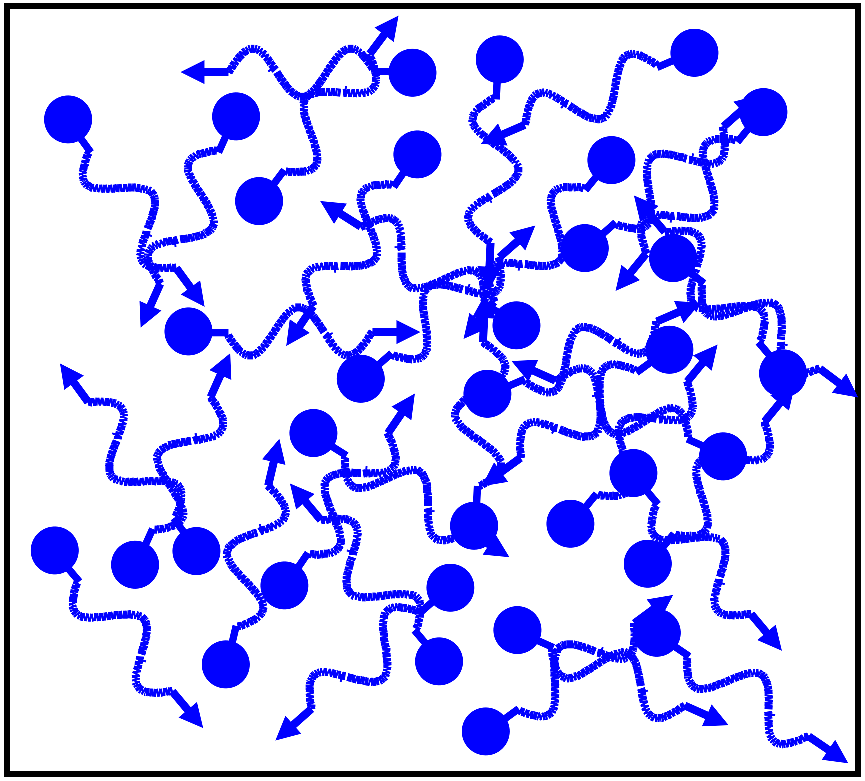
(N.B.: This was not the way Einstein thought about it)

Thermal deBrogile wavelength $\lambda_{\mathbf{dB}} = h/mv_{thermal}$

(coherence length is $\sim\lambda_{\mathbf{dB}}$)



In a hot, dilute gas, $\lambda_{\mathbf{dB}}$ is so short that it makes no difference.



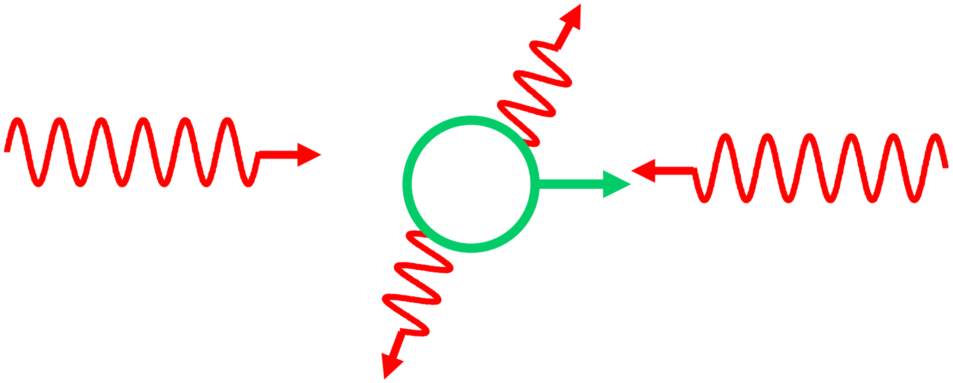
In a cold, dense gas, the wavelength can become comparable to the average interatomic spacing

Alternate views of BEC:

As the gas cools, usual thermal statistics puts more atoms in lower energy states. Once the occupation probability of the lowest state is on the order of unity, the Bose-enhancement of whatever process produces thermal equilibrium will favor more atoms going into the ground state, creating the condensate.

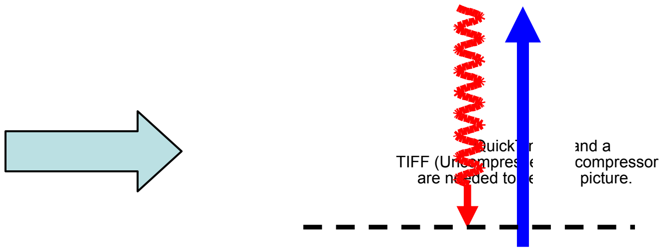
As the phase space density increases, the Bose distribution function cannot support more atoms in states of higher energy, so the extra atoms have to go into the ground state, producing the condensate. (See, e.g. Huang's book on Statistical Mechanics for the classic treatment of BEC.

Laser cooling is a good start, but doesn't get cold or dense enough



Random absorption and emission heats the atoms, creating a limit to how cold one can get.

Molecule-forming collisions in the presence of the laser light limit the density.



(Internal radiation pressure also limits density)

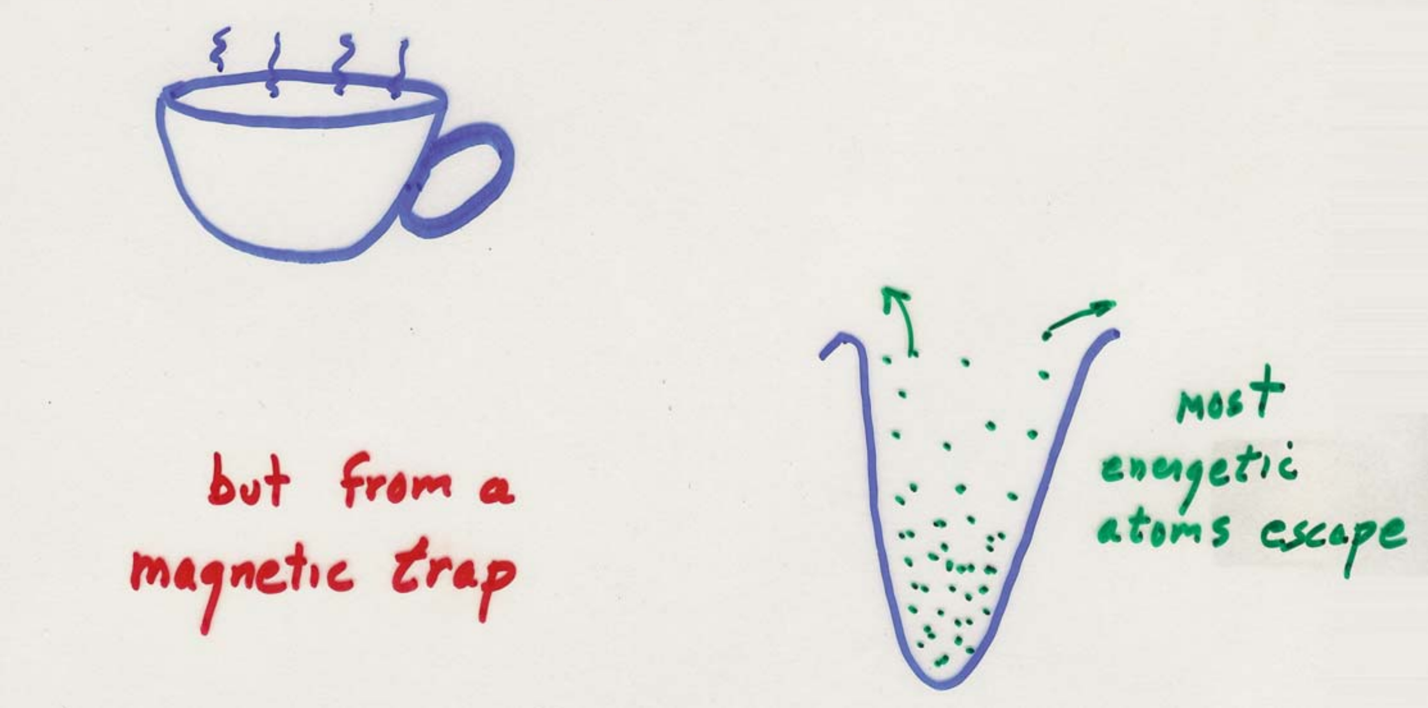
Laser cooling (and trapping) fails by about several orders of magnitude in phase-space density !

$$\phi sd = \frac{\text{atoms}}{(\text{volume} \times \Delta p_x \Delta p_y \Delta p_z)}$$

$\phi sd \approx h^{-3}$ for BEC

How do we get colder and denser?

Evaporative Cooling!



BEC

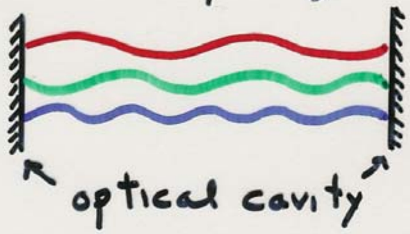
In 1995 (70 years after Einstein’s prediction) teams in Boulder, Colorado and Cambridge, Massachusetts achieved Bose-Einstein Condensation in super-cold gas.

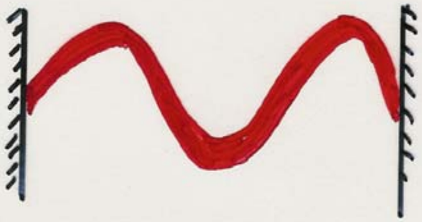
This feat earned those scientists the 2001 Nobel Prize for physics.

They and others have reached temperatures lower than one nanokelvin !

BEC provides coherent matter waves - different from "ordinary" atoms as laser light from ordinary light.

ordinary light



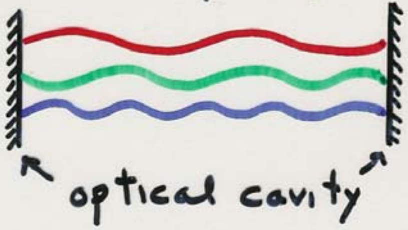


laser light:

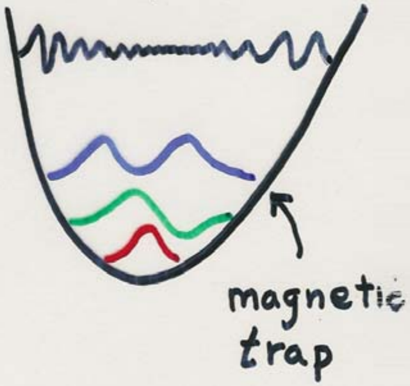
many photons in the same mode

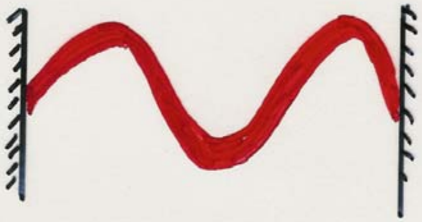
BEC provides <u>coherent</u> matter waves - different from "ordinary" atoms as laser light from ordinary light.

ordinary light



ordinary gas



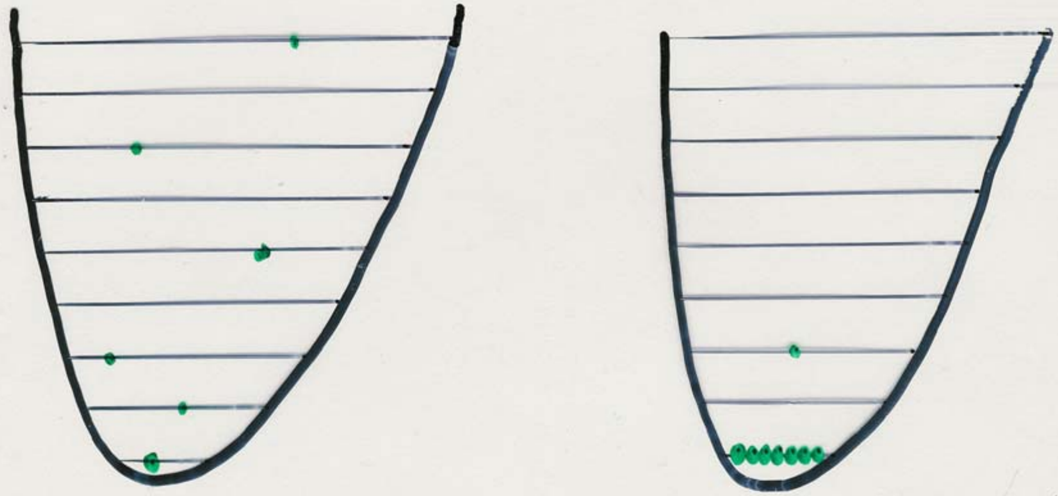


laser light:

many photons in
the same mode

BEC: many
atoms in the same
quantum state

BEC represents many atoms in a single quantum state of motion.



This is like a laser field – many photons in a single mode of the E-M field.

The intensity and coherence of a BEC are laser-like.

Making a BEC?

- Start with, e.g., Na atoms, at T ≈ 1000 K
  (or Rb-87 atoms at T ≈ 500 K, or other things)

- Laser cool the Na to ~50 μK

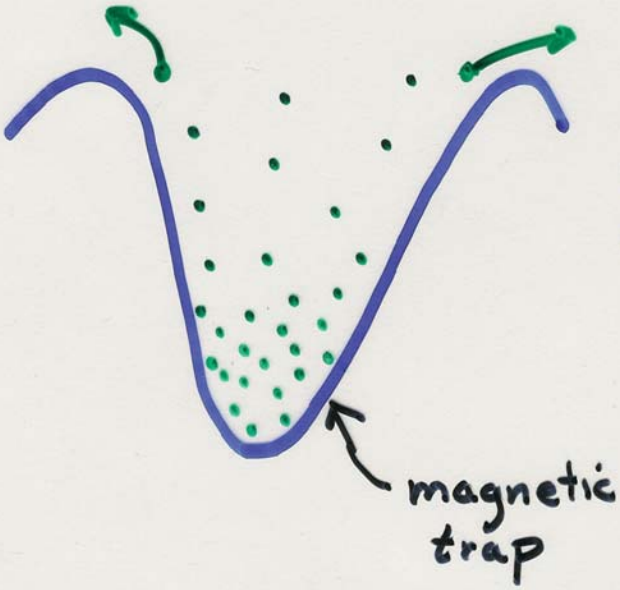
- Trap the atoms in a magnetic bottle

- Evaporatively cool

- Bose condense at T ≈ 1 μK; n ≈ 10^{13} atoms/cm^3
  (approximate parameters for Na in our lab)

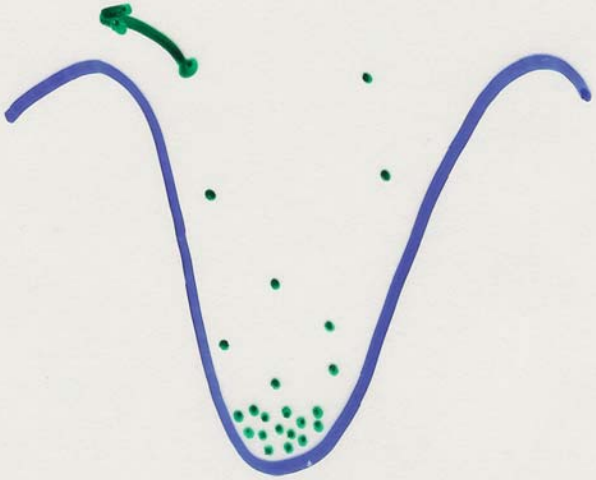
- Continue to evaporate and adiabatically expand
nearly pure condensate ~100 μm diameter, ~10^6 atoms

Evaporative Cooling

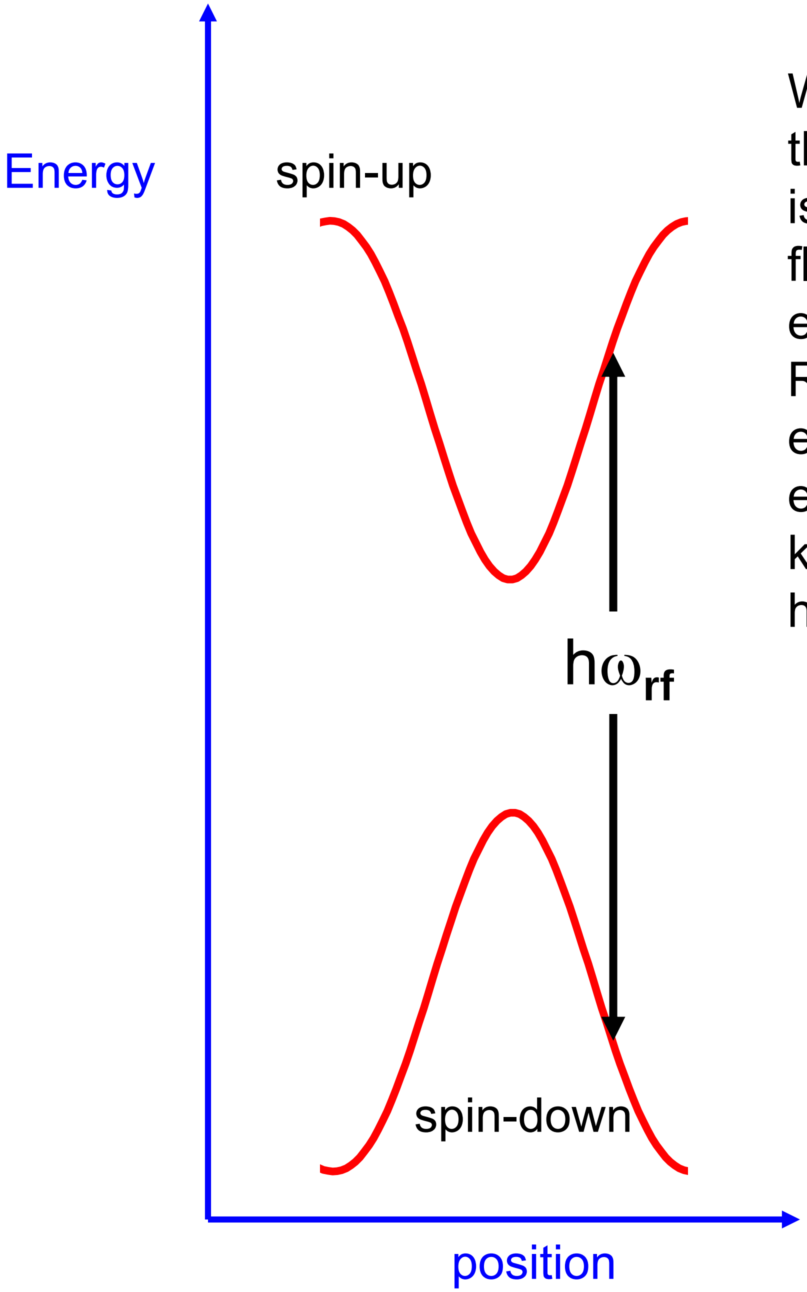


Evaporation lets the most energetic atoms escape.

The remaining ones re-equilibrate, are left colder.



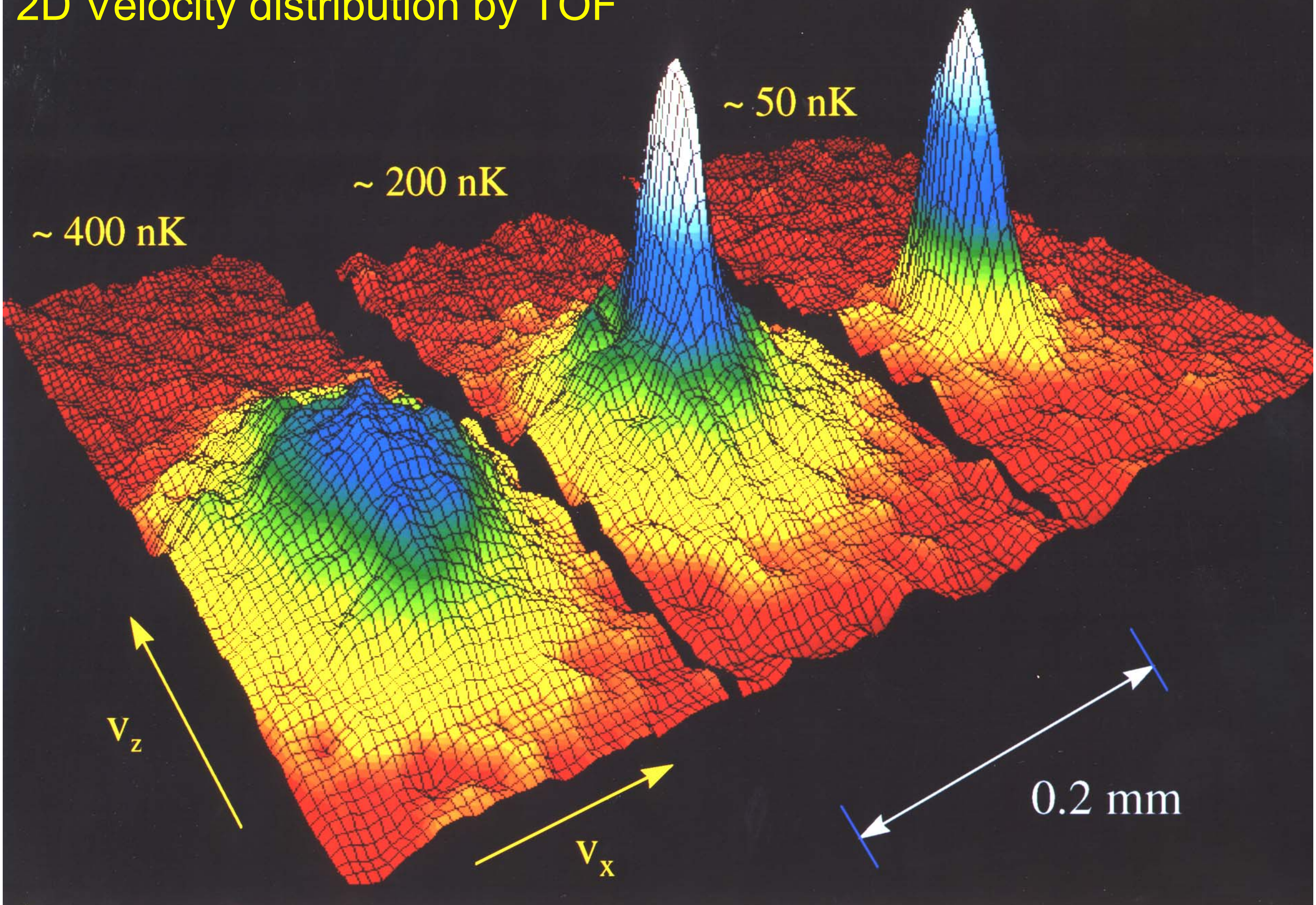
Radio-frequency forced evaporative cooling



When atoms reach the field where the rf is resonant for a spin-flip, those atoms are ejected (evaporated). Reducing rf frequency evaporates lower energy atoms, keeping trap stiffness high.

Atoms re-equillibrate through elastic collisions, producing more high velocity atoms.

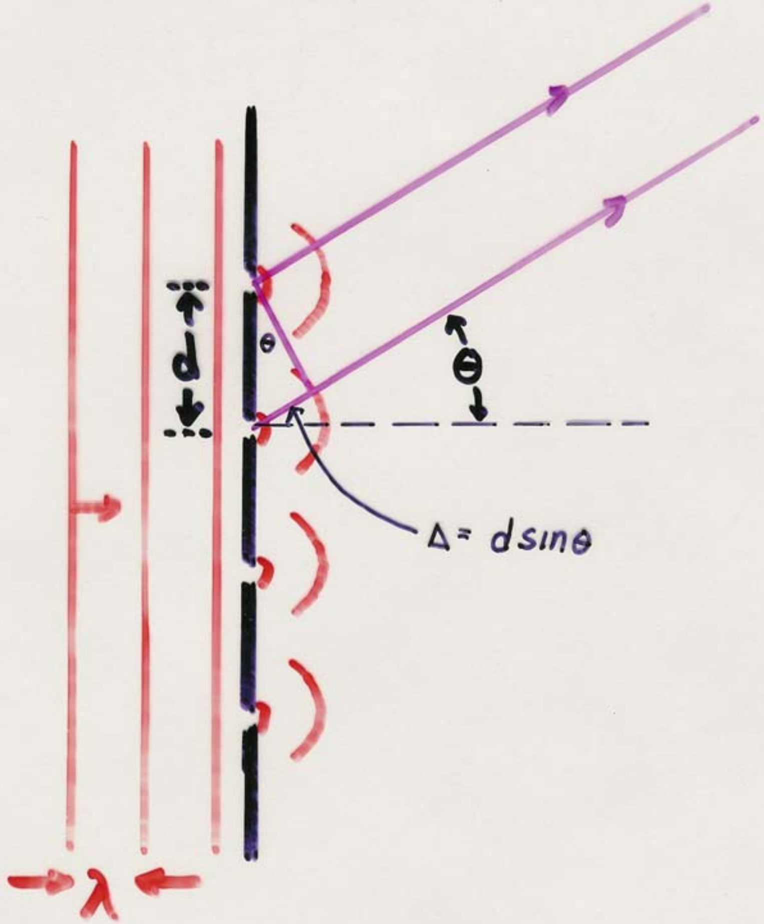
2D Velocity distribution by TOF



Coherent Atom Optics

Using coherent atoms, let’s do the sorts of things we do with coherent light--like diffraction, interference, non-linear optics, ….

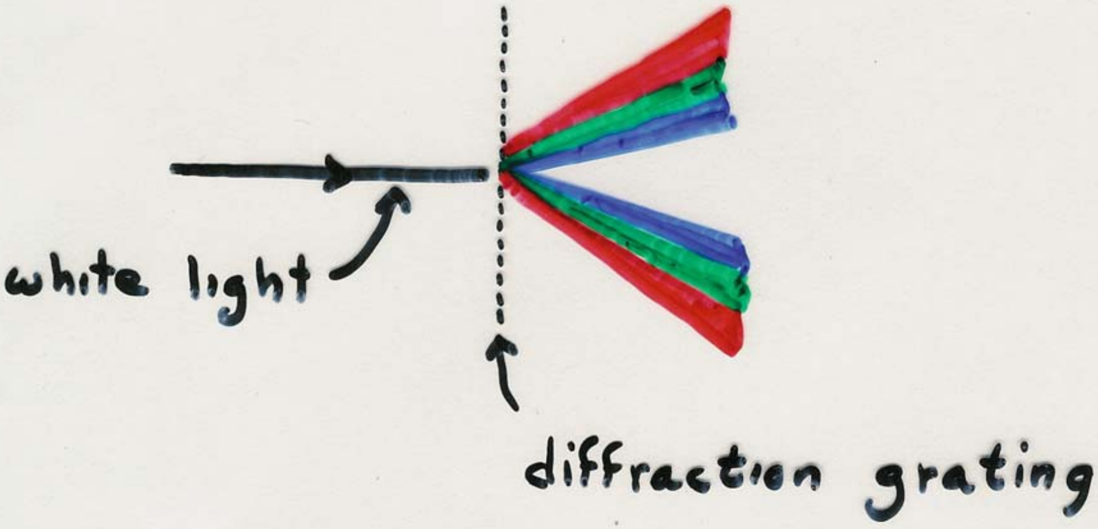
Diffraction Grating

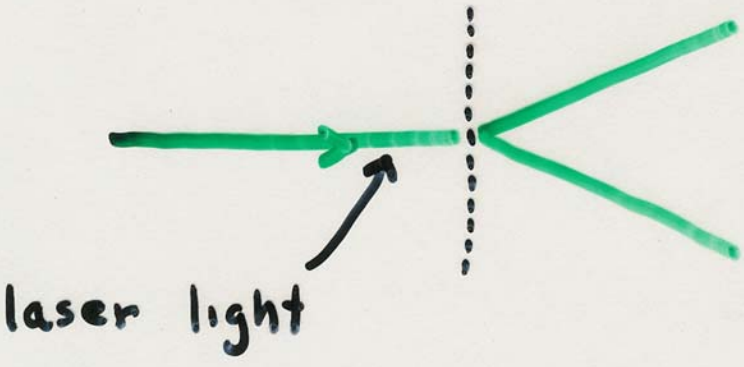


For constructive interference at θ.

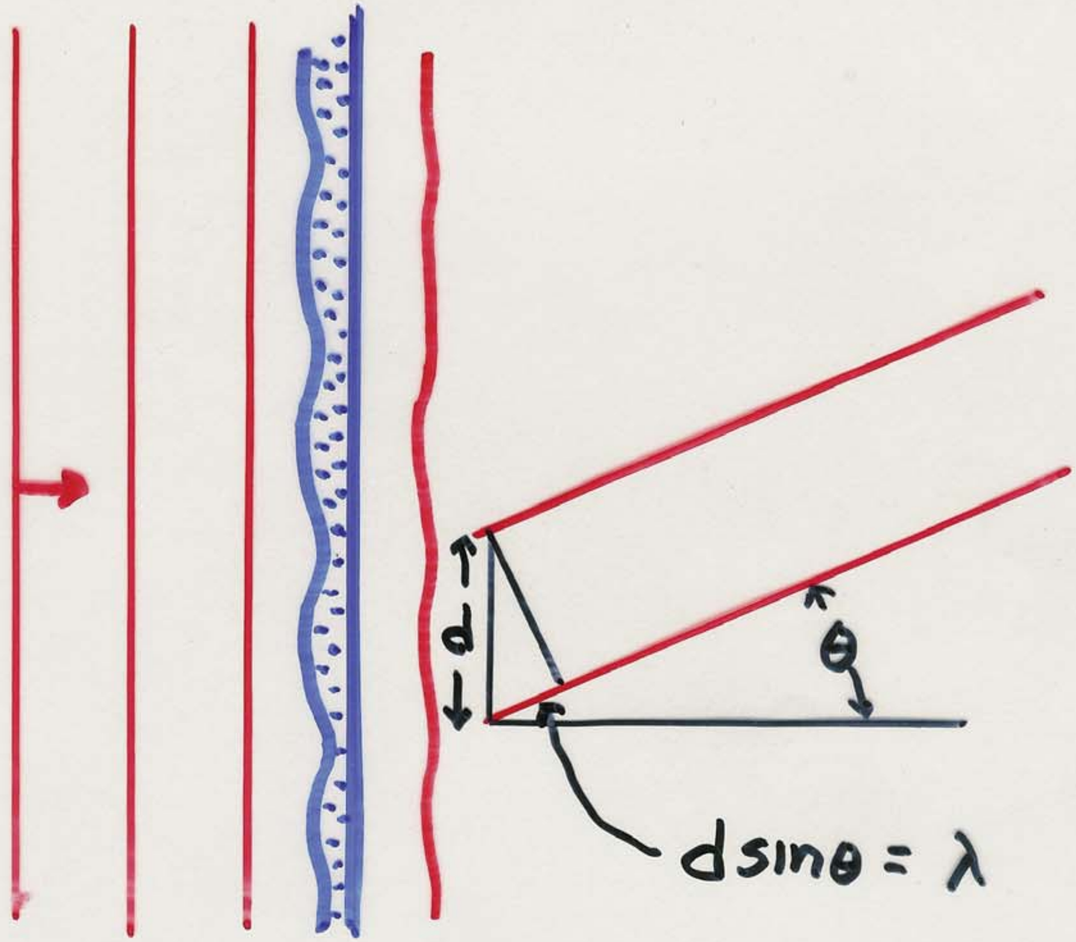
$$\Delta = n\lambda = d\sin\theta$$

optical diffraction

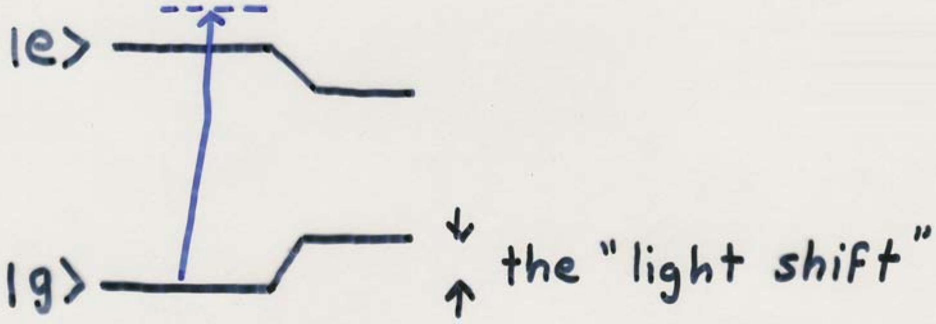




A <u>phase</u> grating diffracts light just like an amplitude grating — a periodic array of slits (at least in the far-field).

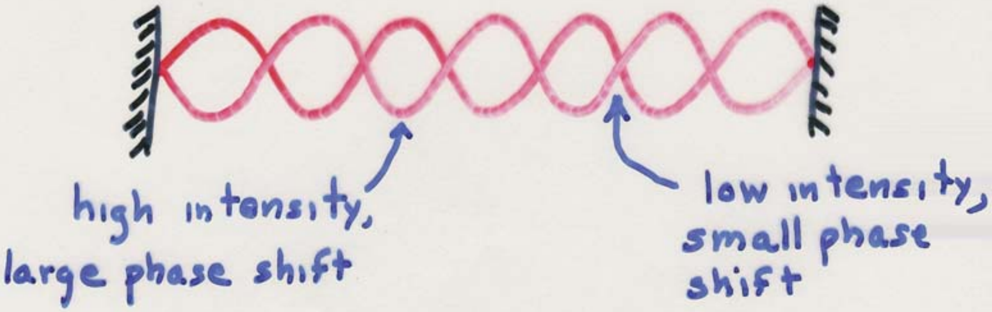


Light tuned far from an atomic resonance shifts the energy of the atoms :



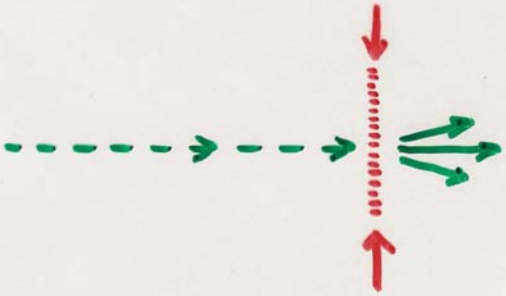
An energy shift applied for a time interval produces a phase shift in the atomic wave function.

A standing wave of light acts like a phase grating for atom waves :



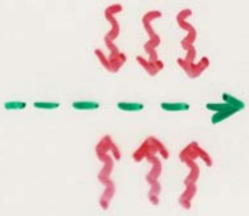
diffraction of an atomic beam by a standing wave of light

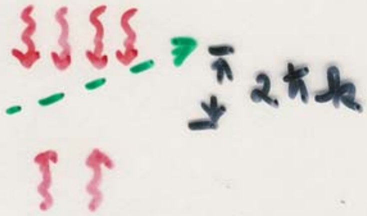
(MIT - Pritchard - 1980's)



like diffraction of light by a *thin* phase grating.

Also, interpreted as redistribution of photon momentum by absorption / stimulated emission





P. Moskowitz, P. Gould, S. Atlas, & D. Pritchard (1983)

(Incoherent atomic beam)

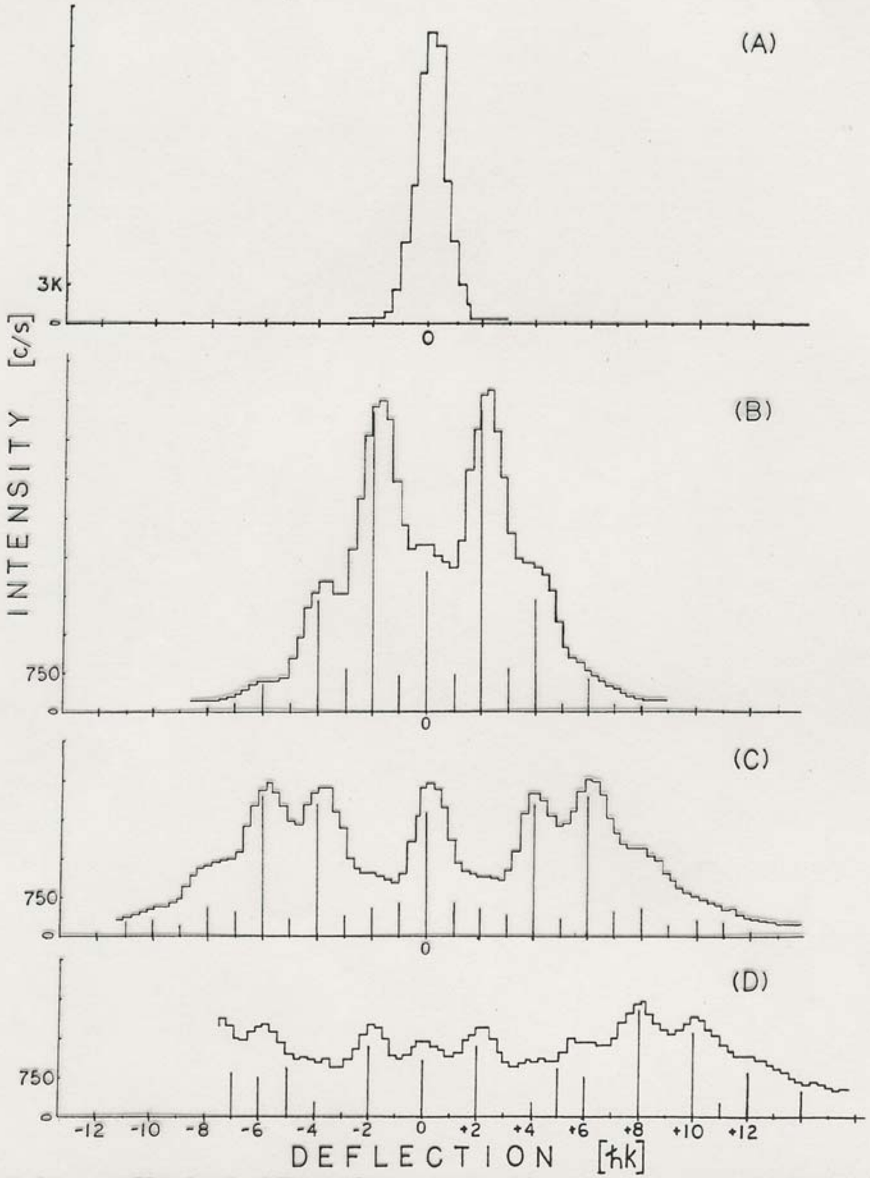
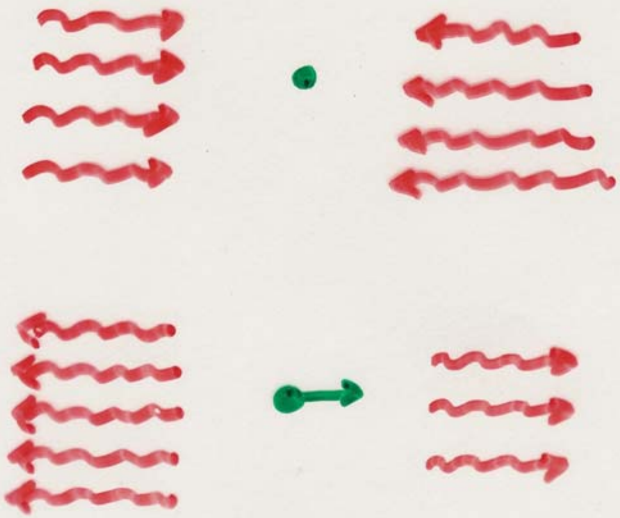


FIG. 1. Atomic beam profiles for the following laser powers: (a) 0, (b) 5, (c) 10, and (d) 20 mW. Vertical bars under data depict momentum transfer imparted by the field, i.e., a computer deconvolution accounting for the atomic beam profile, velocity distribution, and spontaneous emission recoil after the interaction. The height of each bar at position n is proportional to the probability that an atom gains $n\hbar k$ momentum.

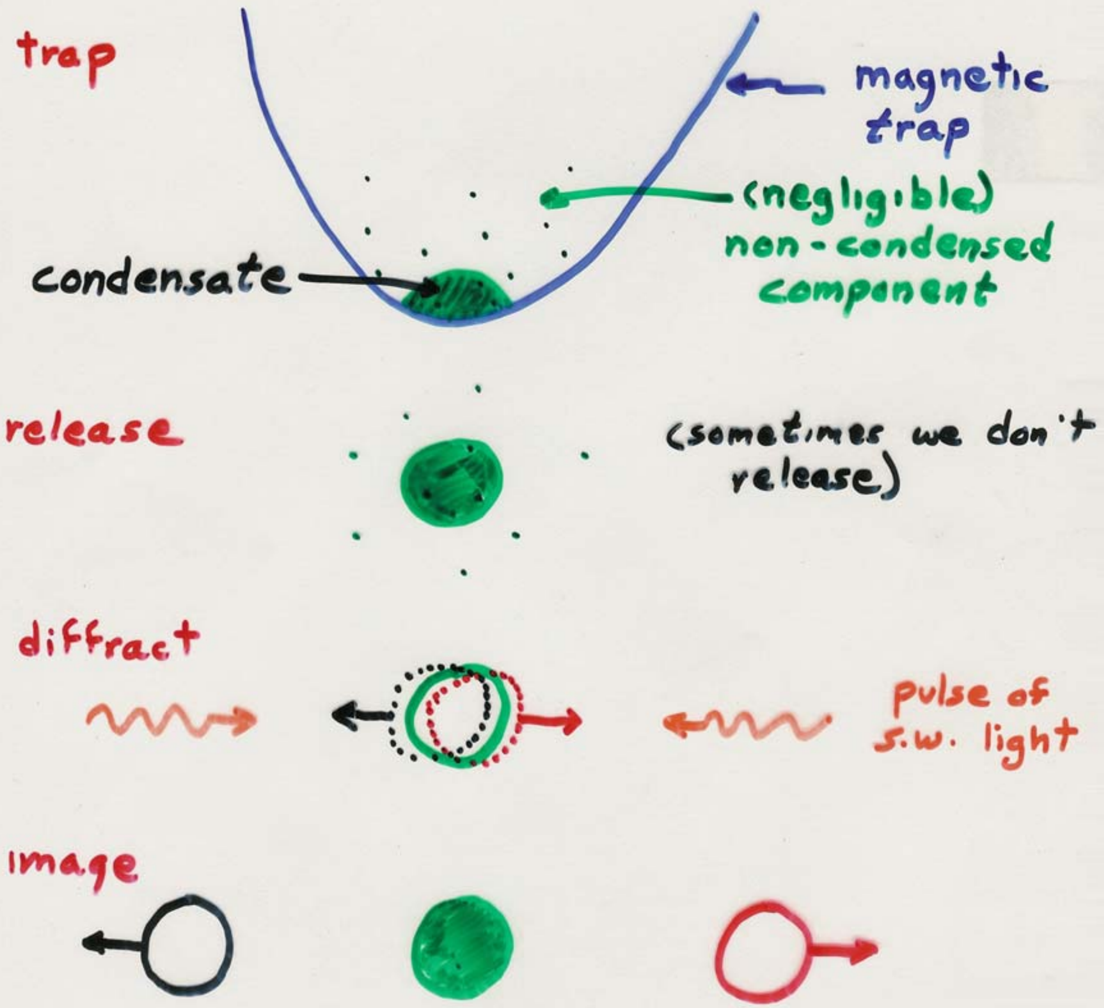
For atoms at rest (BEC)

passage through a *thin* grating is analogous to receiving a *short* standing wave pulse:



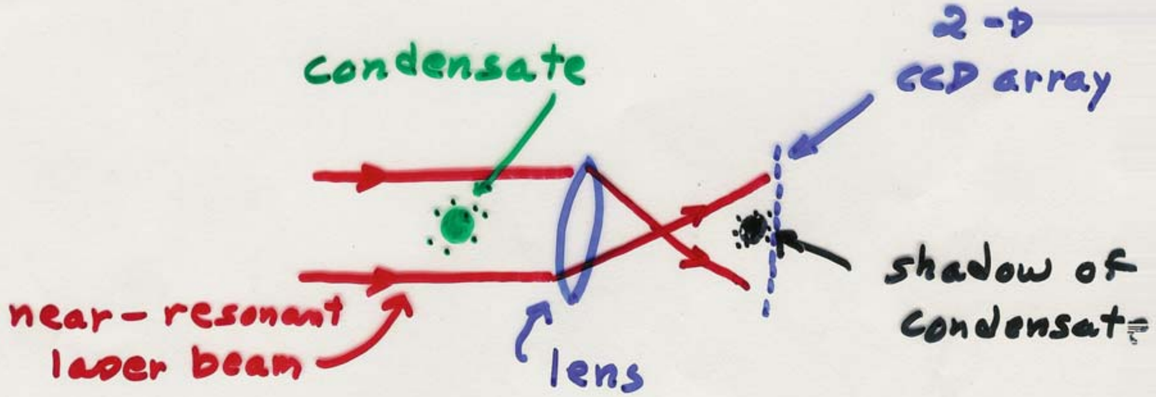
and

General Experimental Procedure



absorption imaging after free-flight. converts momentum distribution into a spatial distribution. Line-of-sight is vertical.

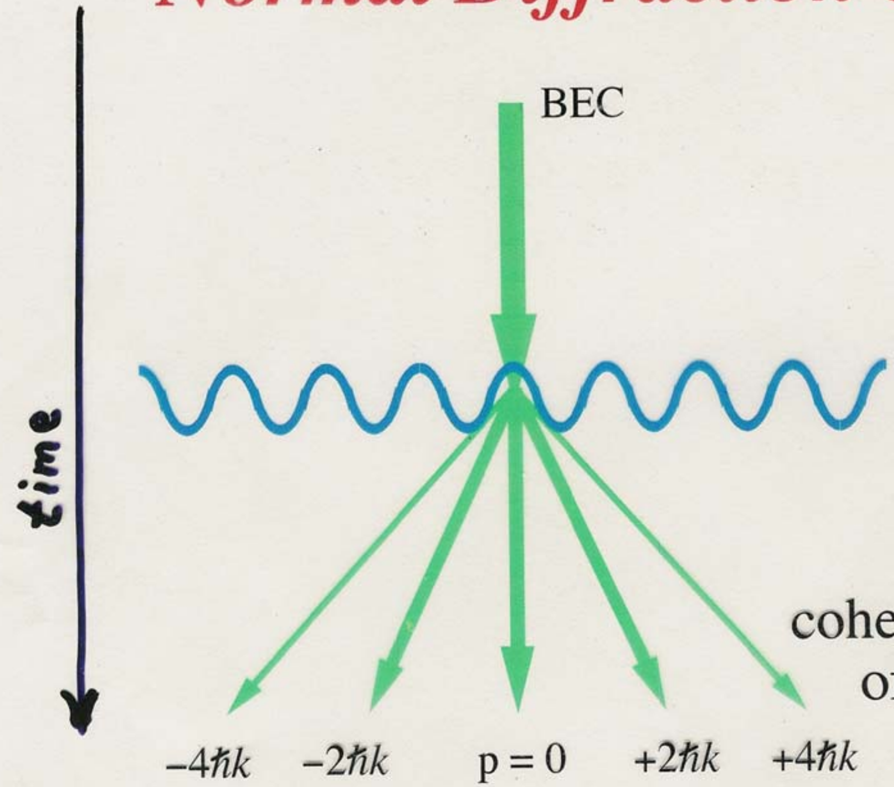
Imaging the condensate



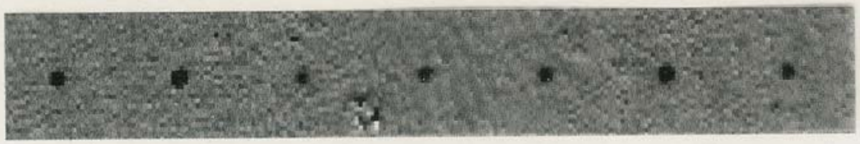
This imaging procedure destroys the BEC.

Other techniques - phase contrast imaging - can preserve most of the condensate (MIT)

Normal Diffraction of BEC

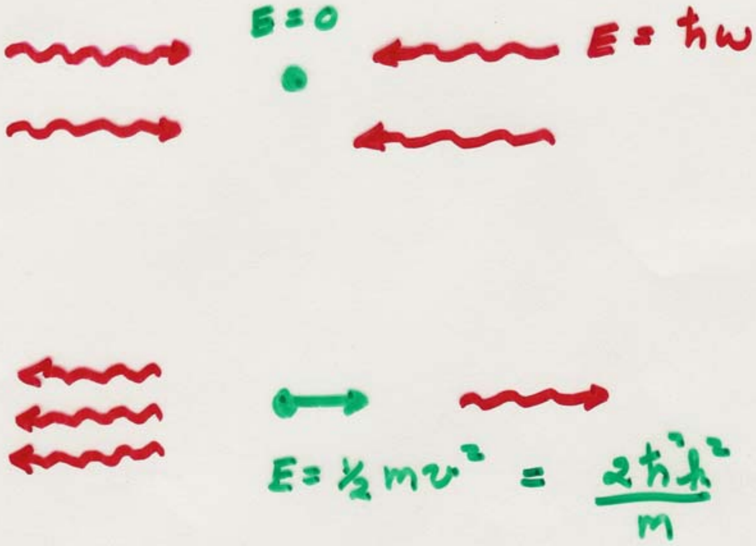


short pulse



long pulse

But – diffraction appears not to conserve energy:



Initial energy = $N\cdot\hbar\omega$

Final energy = $N\cdot\hbar\omega + \frac{2\hbar^2k^2}{m}$

How is this possible?

The light is *pulsed*!

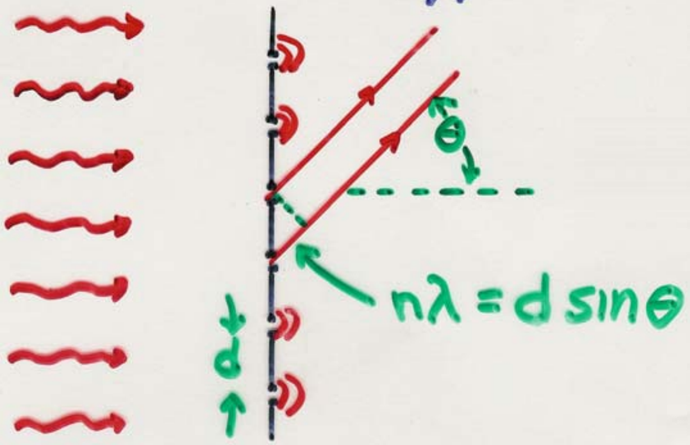
A short pulsed standing wave diffracts atoms because of the Fourier spread of frequencies

Short: $\tau_{\mathbf{pulse}} << \frac{h}{E_{\mathbf{rec}}}$ $\qquad E_{\mathbf{rec}} \equiv \frac{h^2 k^2}{2M}$

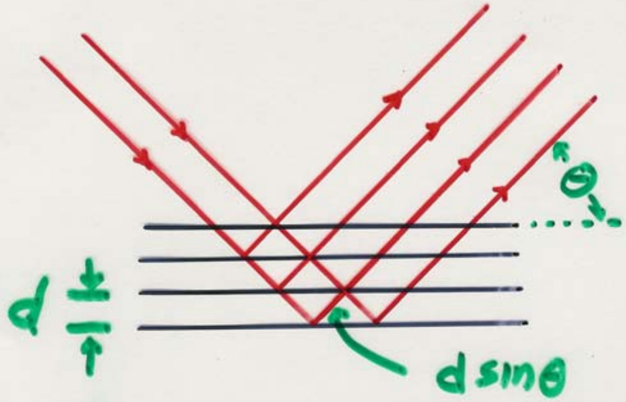
If the pulse is long $\quad (\tau_{\mathbf{pulse}} >> h/E_{\mathbf{rec}})$

there is too little Fourier spread to allow energy conservation, so there is no diffraction – unless the Bragg condition is fulfilled.

"Normal" diffraction vs. Bragg diffraction



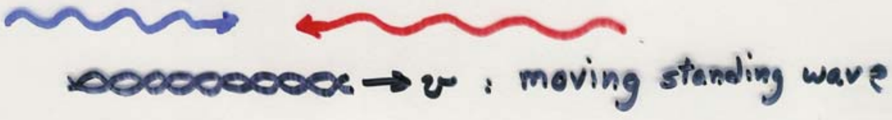
$n\lambda = d \sin\theta$



$d \sin\theta$

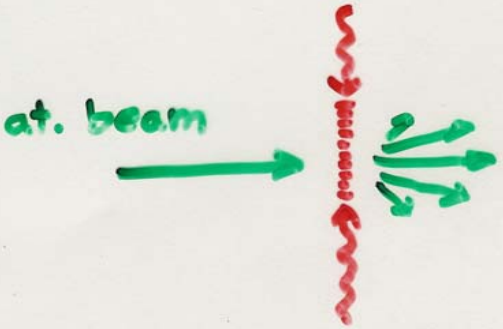
$n\lambda = 2d \sin\theta$

Bragg requires specular reflection, a condition on incident angle.



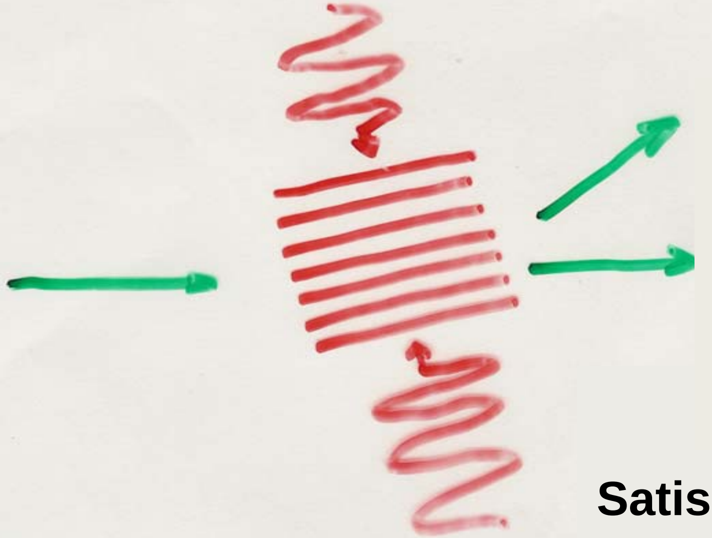
$\rightarrow v$: moving standing wave

Diffraction and Bragg reflection of an atomic beam from a static grating (a standing light wave) - MIT, 1980s



"thin" grating

Diffracts into multiple orders, both directions



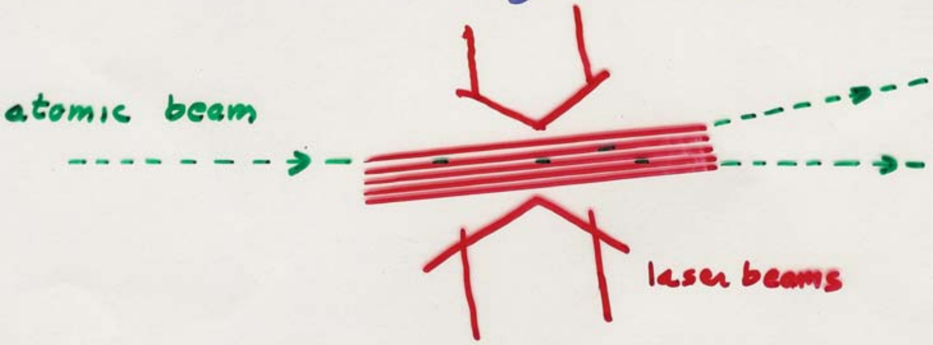
"thick" grating

Specular reflection from planes.

Single order

Satisfies condition on angle of incidence

A standing-wave light field makes a periodic set of planes from which to reflect deBroglie waves

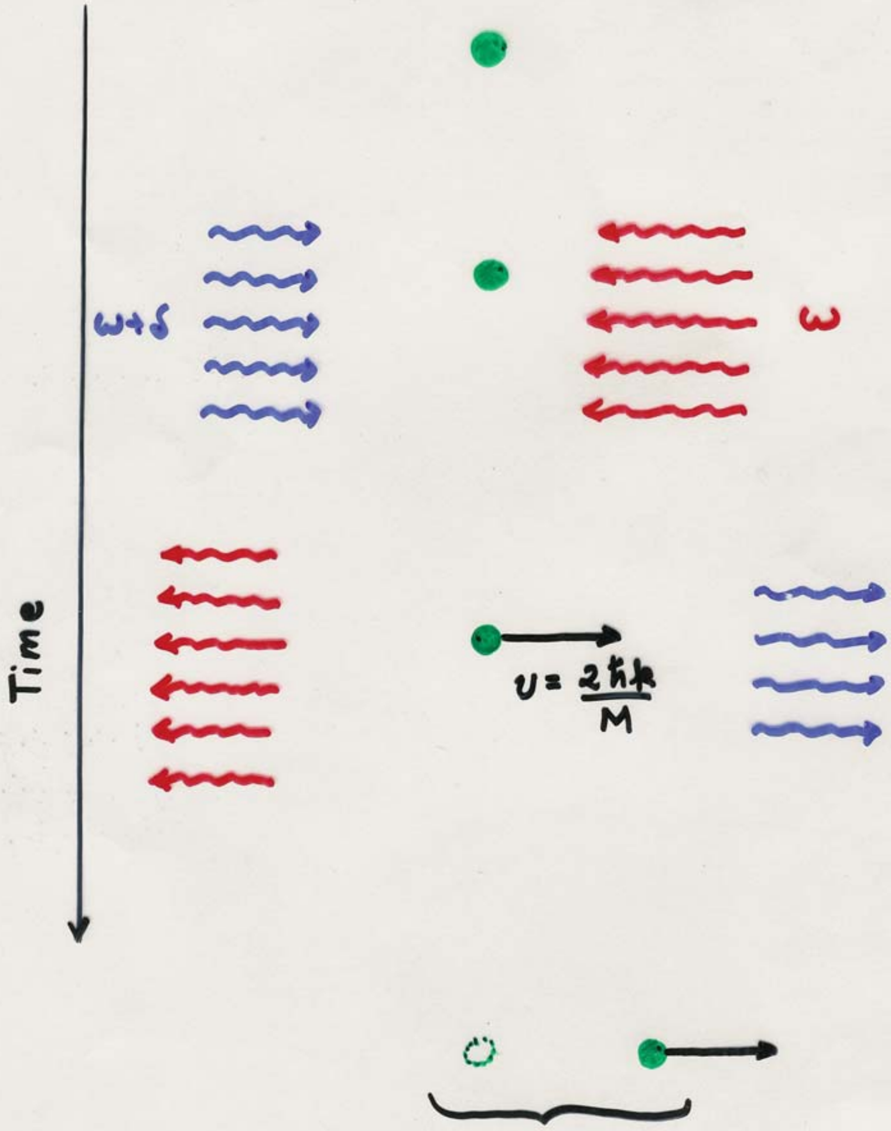


Martin, Oldaker, Miklich + Pritchard (MIT-1988) for incoherent atoms.

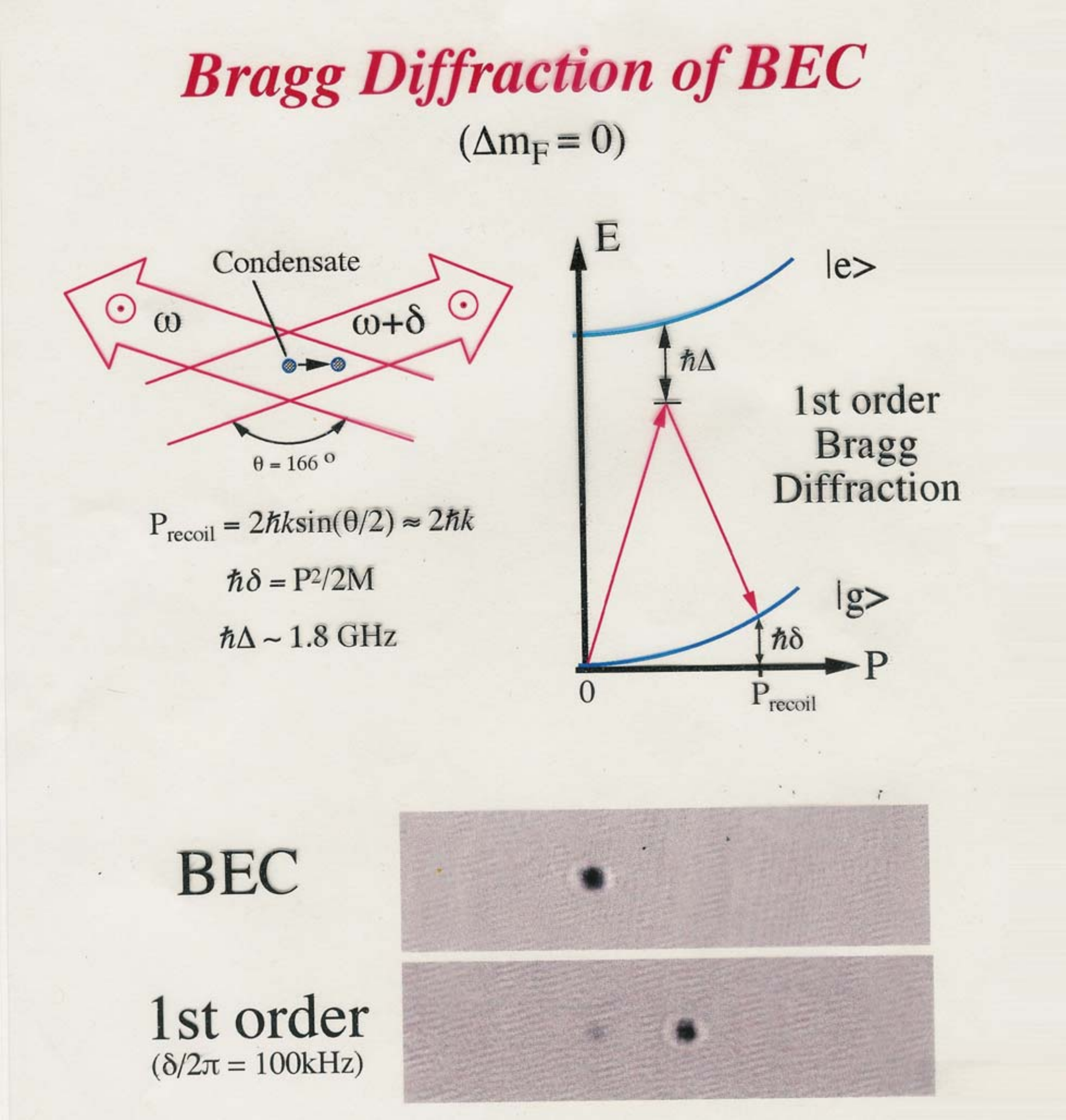
for a BEC at rest:

move a pulsed standing wave past the atoms (Kozuma, Deng, Hagley, Wen, Lutwak, Helmerson, Rolston + Phillips - NIST Gaithersburg 1999)

Bragg "reflection" of a BEC



We make an image of the atoms after they have separated.



Bragg Diffraction of BEC

$(\Delta m_F = 0)$

$P_{recoil} = 2\hbar k \sin(\theta/2) \approx 2\hbar k$

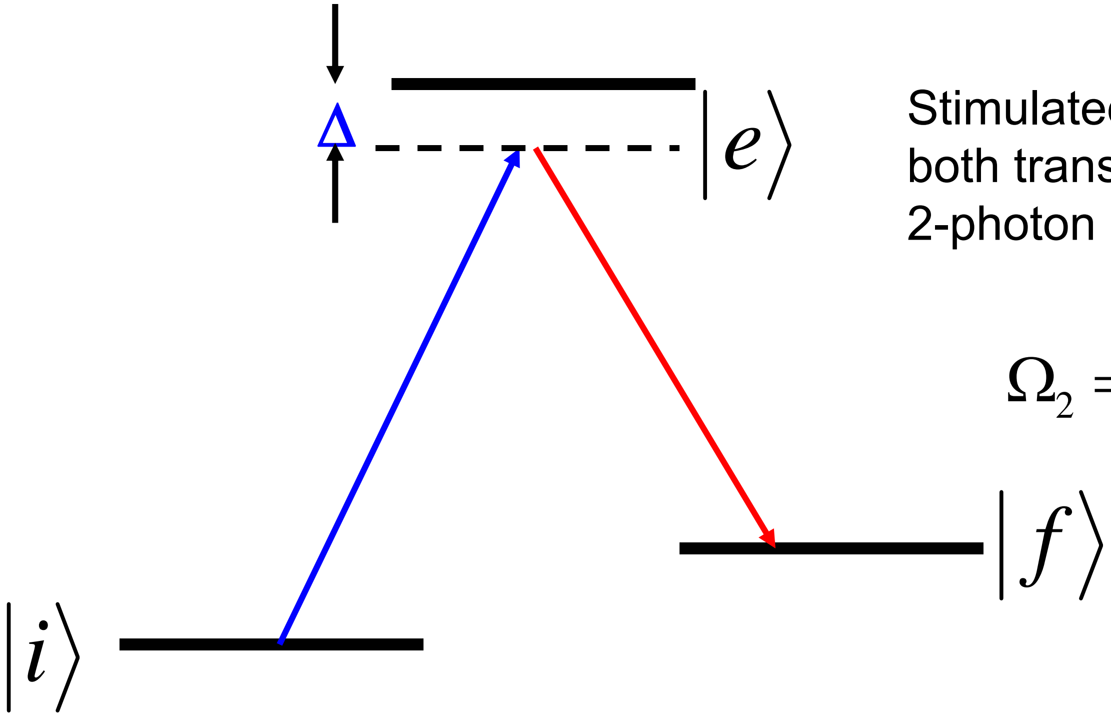
$\hbar\delta = P^2/2M$

$\hbar\Delta \sim 1.8$ GHz

M. Kozuma, L. Deng, E. Hagley, J. Wen, R. Lutwak, K. Helmerson, S. Rolston, WDP (1999)

Raman scattering

Bragg scattering of atoms by light can be thought of as a Raman process, where light scattering leaves an atom in a different state, and the scattered light is a different color. Usually, this involves a change of *internal* state for the atom.

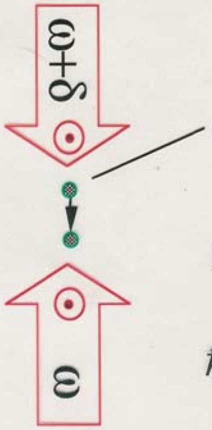


Stimulated Raman involves both transitions being driven. 2-photon Rabi frequency is

$$\Omega_2 = \frac{\Omega_1^2}{2\Delta}$$

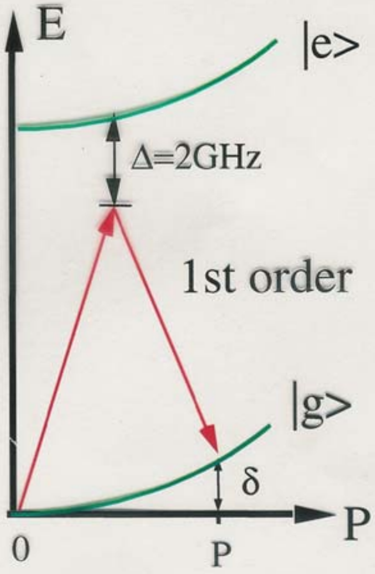
Spontaneous Raman is also possible

Bragg diffraction of a BEC



$P = 2\hbar k$

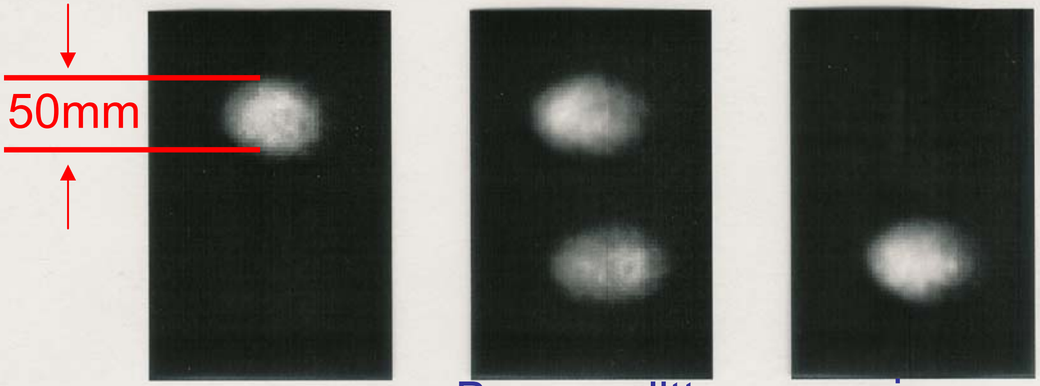
$$\hbar\delta = \frac{P^2}{2M}$$



No pulse

"π/2" pulse

"π" pulse



Beam-splitter

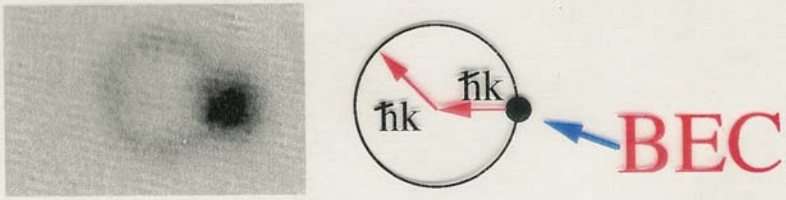
mirror

The Mechanical Effects of Light

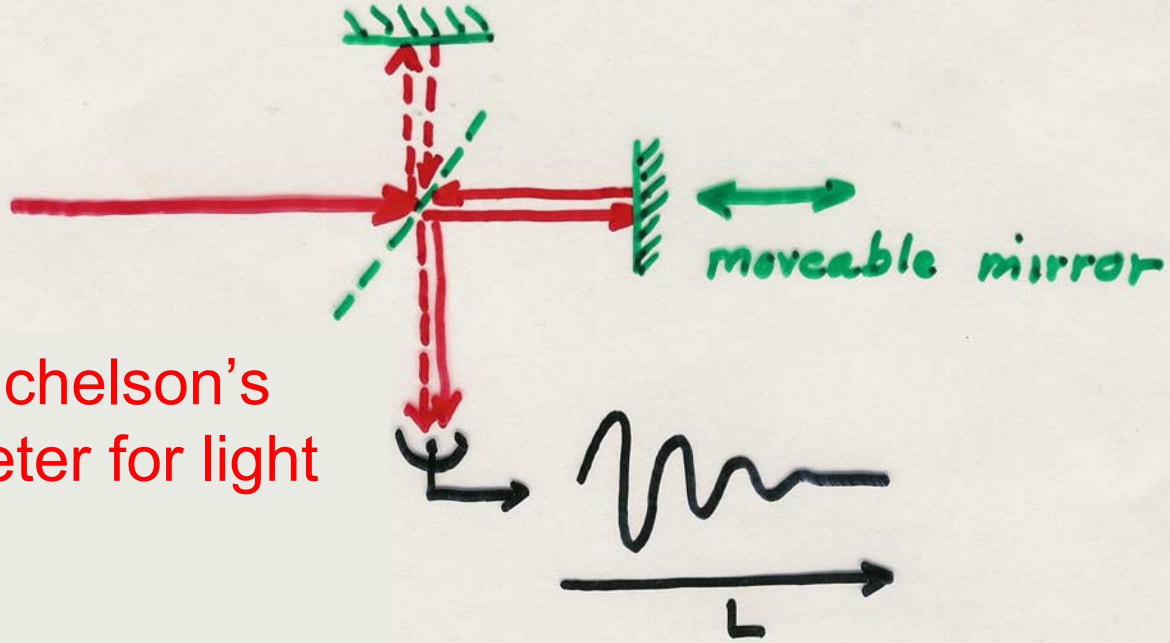
Cold cloud
of atoms

Dipole force
absorption
followed by
stimulated emission

Scattering force
absorption
followed by
spontaneous emission



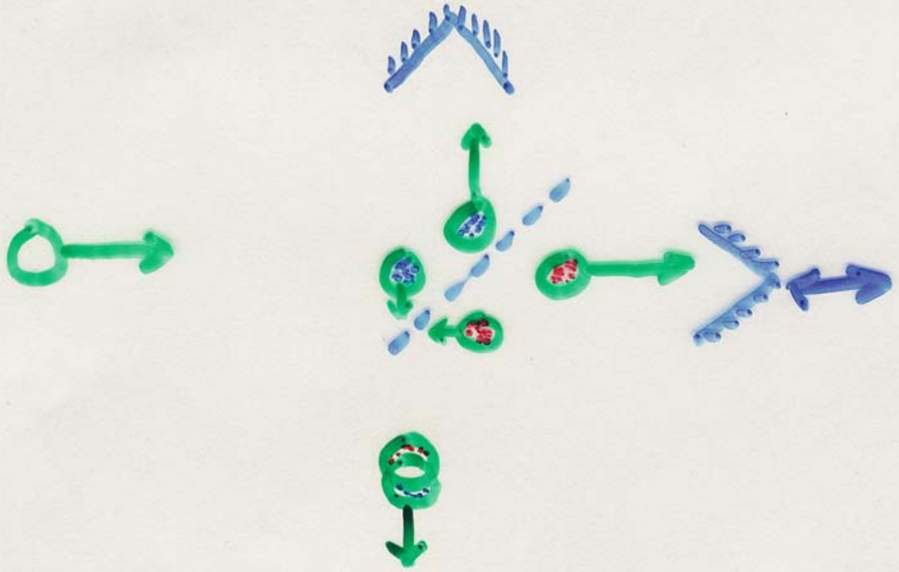
Techniques like Bragg reflection are coherent (if we avoid spontaneous emission) and may be used in studying the intrinsic coherence of a condensate.



Recall Michelson's interferometer for light

Path-length difference, L, over which interference happens gives the Coherence length of the light.

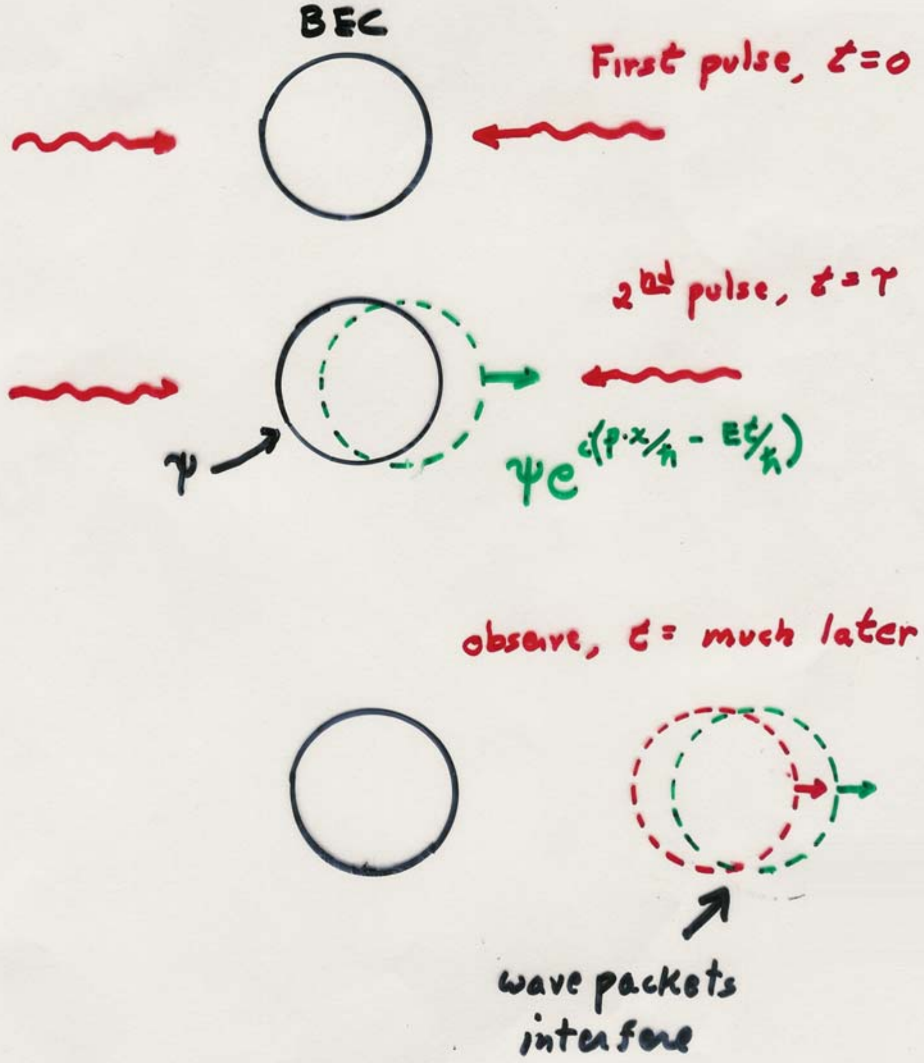
The NIST BEC coherence experiment is equivalent to:

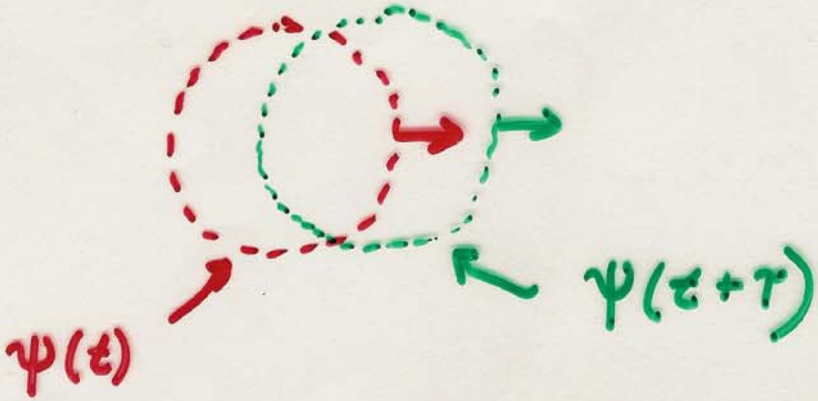


a Michelson interferometer where the condensate is split and recombined after a variable delay. We look at fringes and fringe visibility as the path-length difference changes.

Measuring the coherence of the BEC

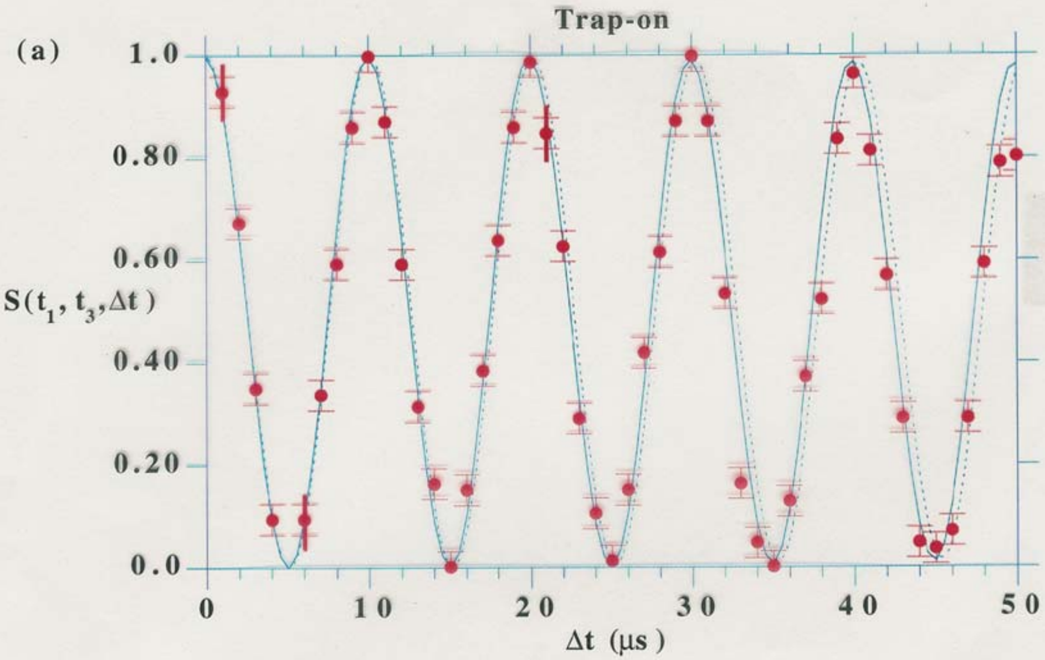
Hagley, Deng, Kozuma, Trippenbach, Band, Edwards, Doery
Julienne, Helmerson, Rolston, Phillips (NIST 1999)

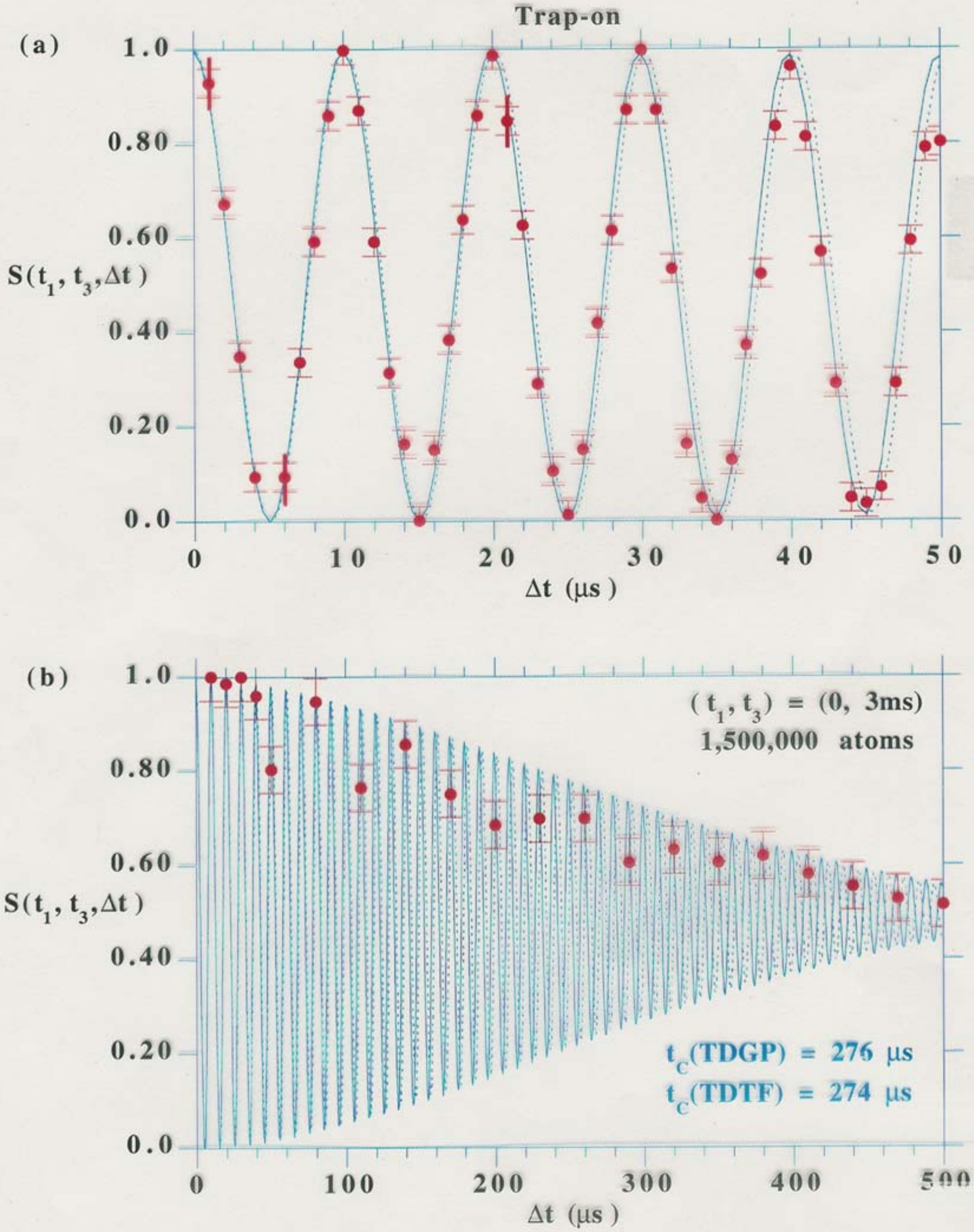




$$\text{Image density} \sim \left|\psi(t) + \psi(t+\tau)\right|^2$$

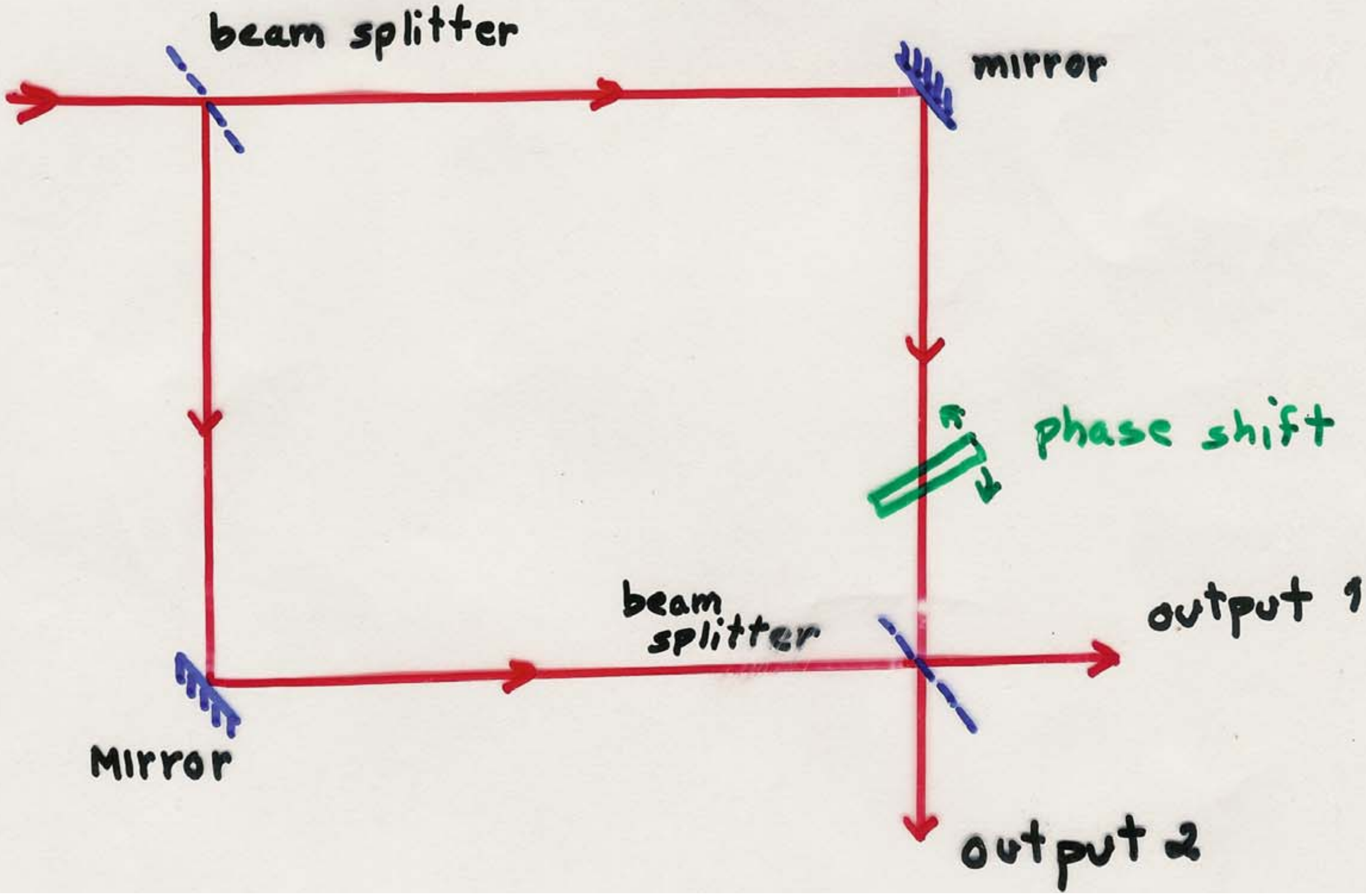
$$\sim |\psi(t)|^2 + |\psi(t+\tau)|^2 + \underbrace{\psi^*(t)\,\psi(t+\tau)}_{\sim\ \text{correlation function}\ g_1(\tau)} + \text{c.c.}$$


Trap-on
(a)
1.0
0.80
0.60
0.40
0.20
0.0
S(t1, t3, Δt)
0
10
20
30
40
50
Δt (μs)

Trap-on
(a)
1.0
0.80
0.60
0.40
0.20
0.0
$S(t_1, t_3, \Delta t)$
0
10
20
30
40
50
Δt (μs)
(b)
1.0
0.80
0.60
0.40
0.20
0.0
$S(t_1, t_3, \Delta t)$
(t_1, t_3) = (0, 3ms)
1,500,000 atoms
t_C(TDGP) = 276 μs
t_C(TDTF) = 274 μs
0
100
200
300
400
500
Δt (μs)

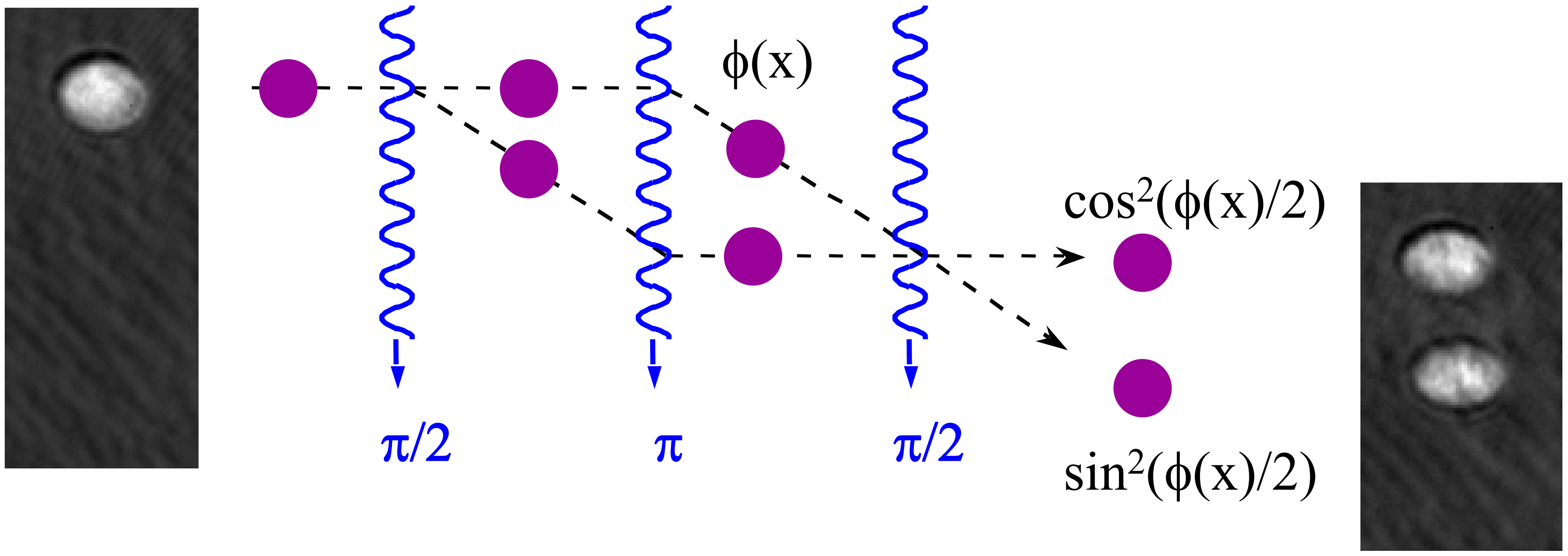
A reminder:

Optical Mach-Zehnder Interferometer



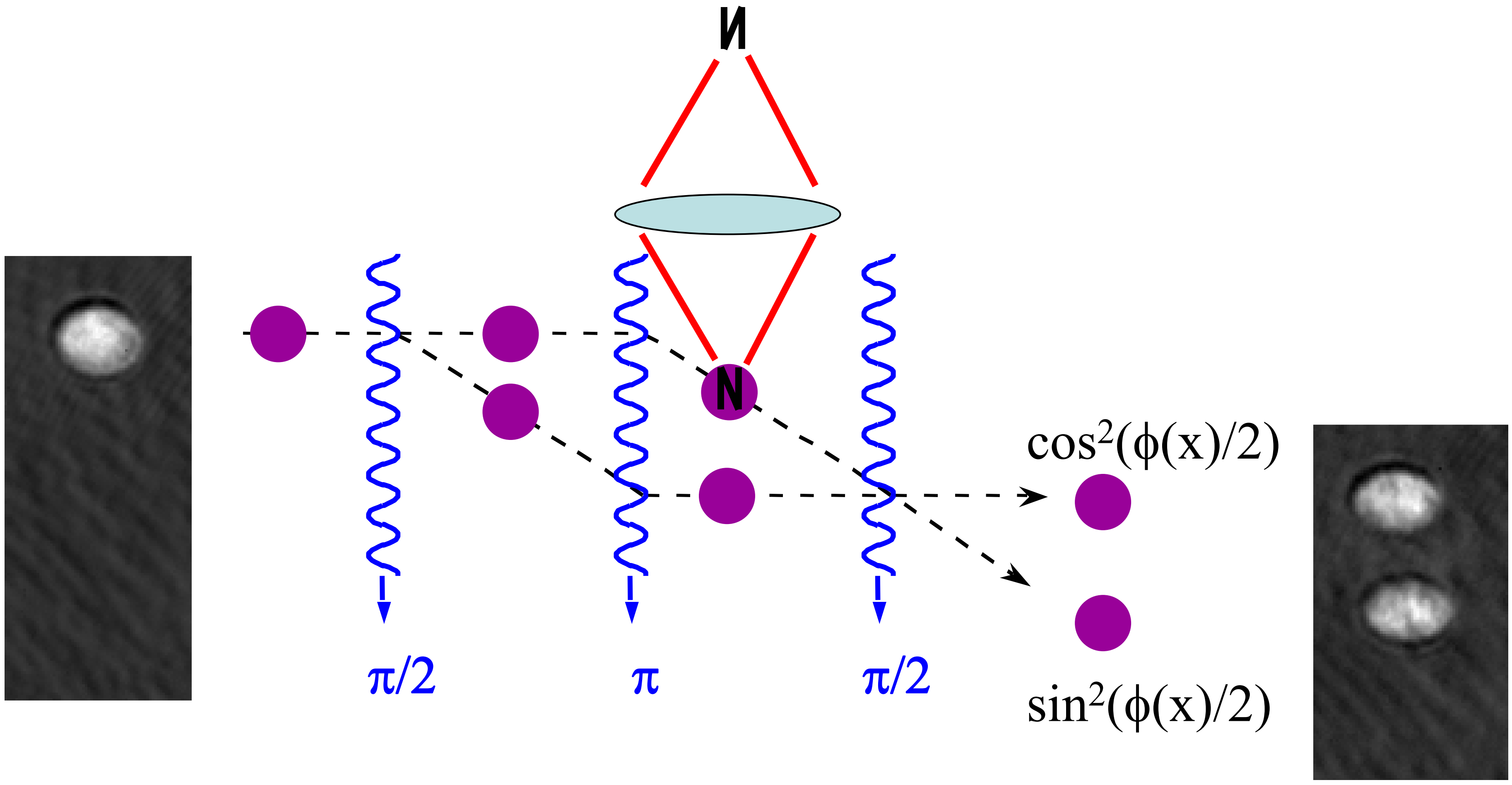
Atom Interferometer

Use Bragg diffraction as 50/50 beamsplitter ($\pi/2$ pulse) and mirror (π pulse) between 0 and 2*hk* momentum states

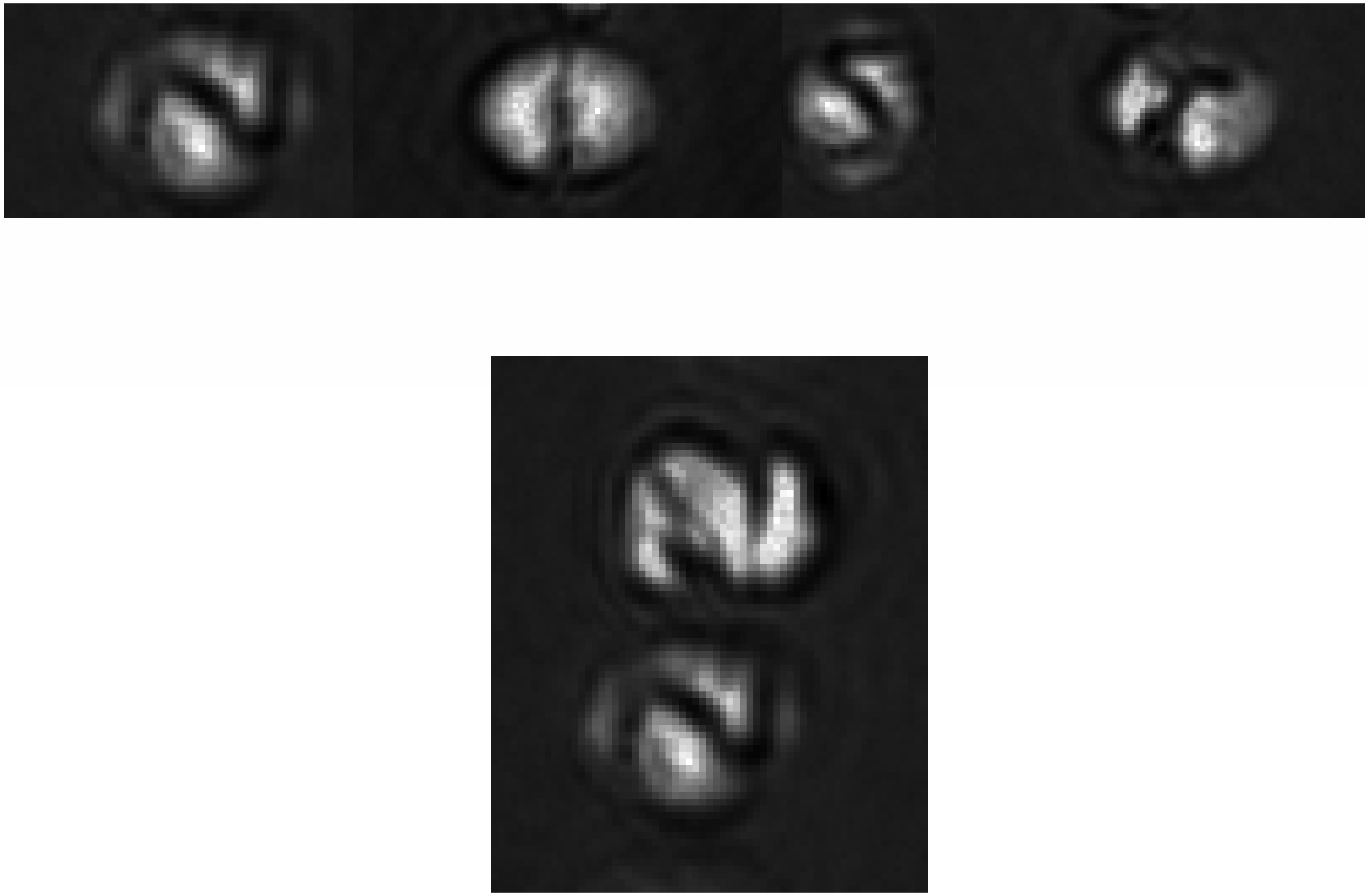


Atom Interferometer

Use Bragg diffraction as 50/50 beamsplitter (π/2 pulse) and mirror (π pulse) between 0 and 2*hk* momentum states

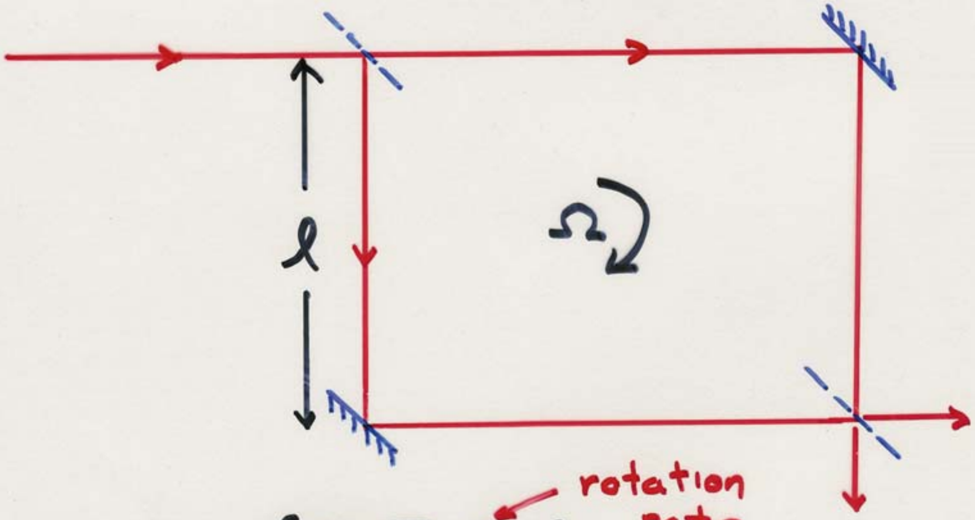


Arbitrary Phase Patterns



An application of atom interference:

Sagnac Interferometer



rotation rate

$$\delta \simeq T \cdot \Omega \, \ell$$

$T \sim \ell / v$

path difference for rotation

time in interferometer

$$\delta \sim \frac{\Omega \ell^2}{v} \sim \frac{\Omega A}{v}$$

$$\text{sensitivity} = \delta/\lambda = \frac{\Omega A}{v\lambda}$$

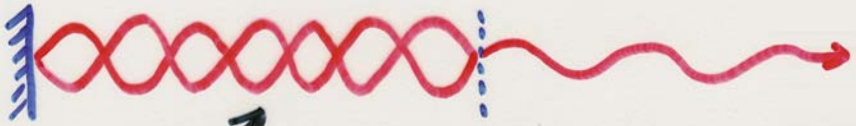
$$(v\lambda)_{part} = \frac{h}{m}$$

$$(v\lambda)_{light} = c^2/\nu$$

$$\frac{(\delta/\lambda)_{part}}{(\delta/\lambda)_{light}} = \frac{mc^2}{h\nu} = \frac{E_{rest}}{E_{photon}} \approx 10^{11}$$

The laser/atom laser analogy

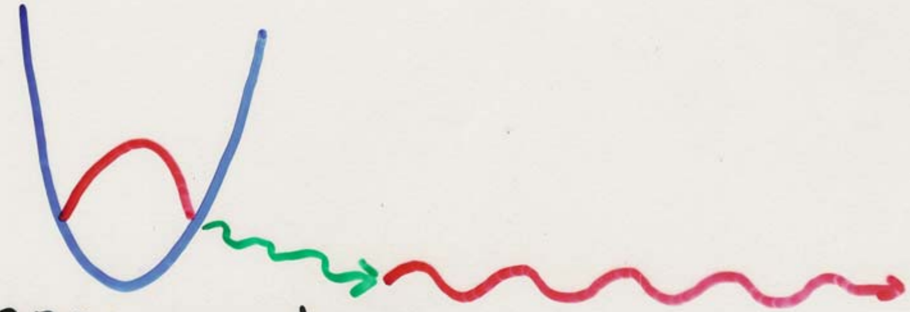
Optical Laser



many photons in a single mode

coupled out as a coherent light beam

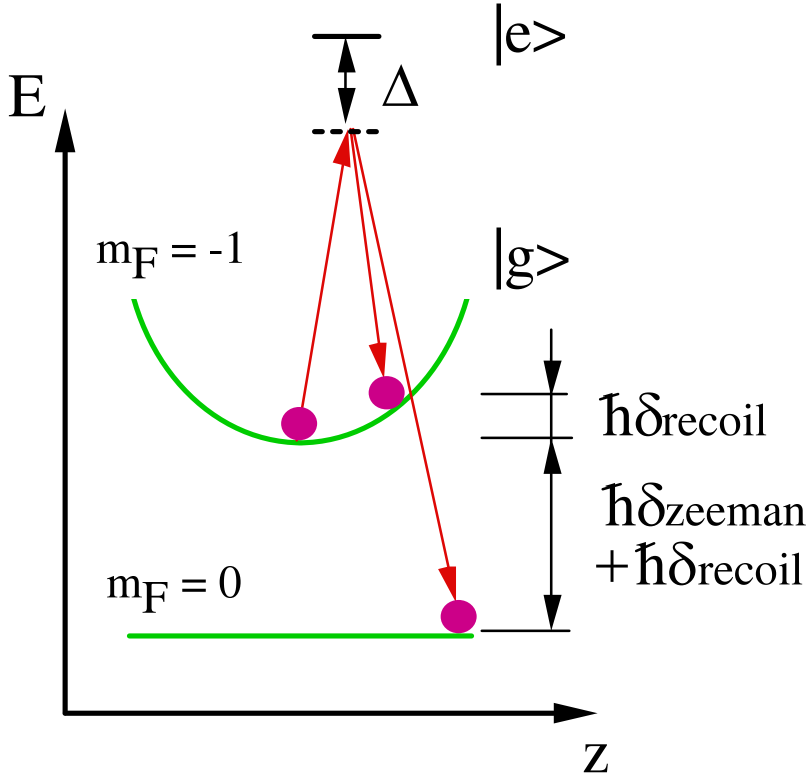
atom laser

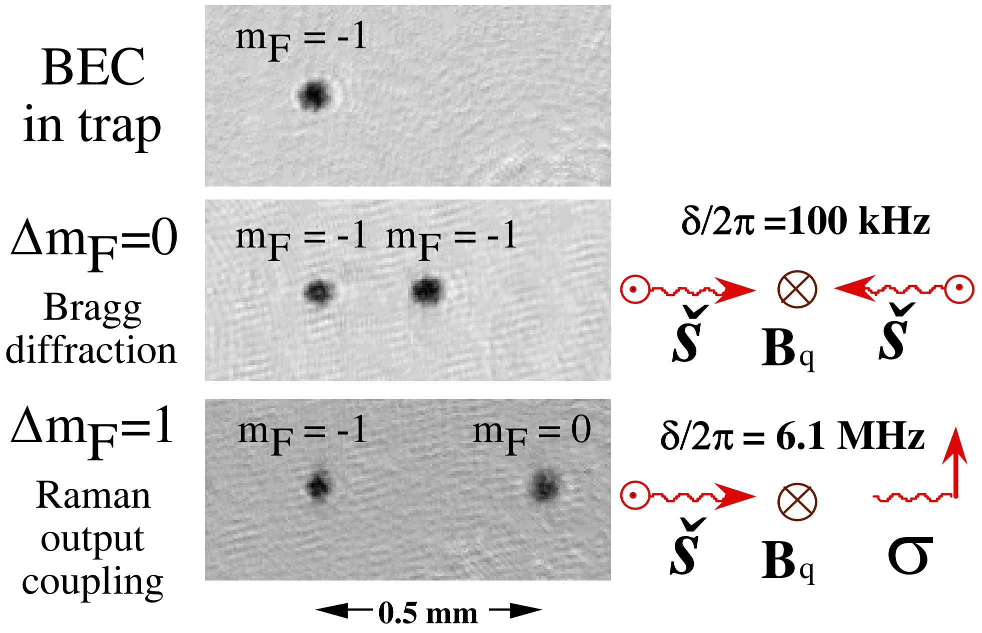


BEC: many atoms in a single trap state

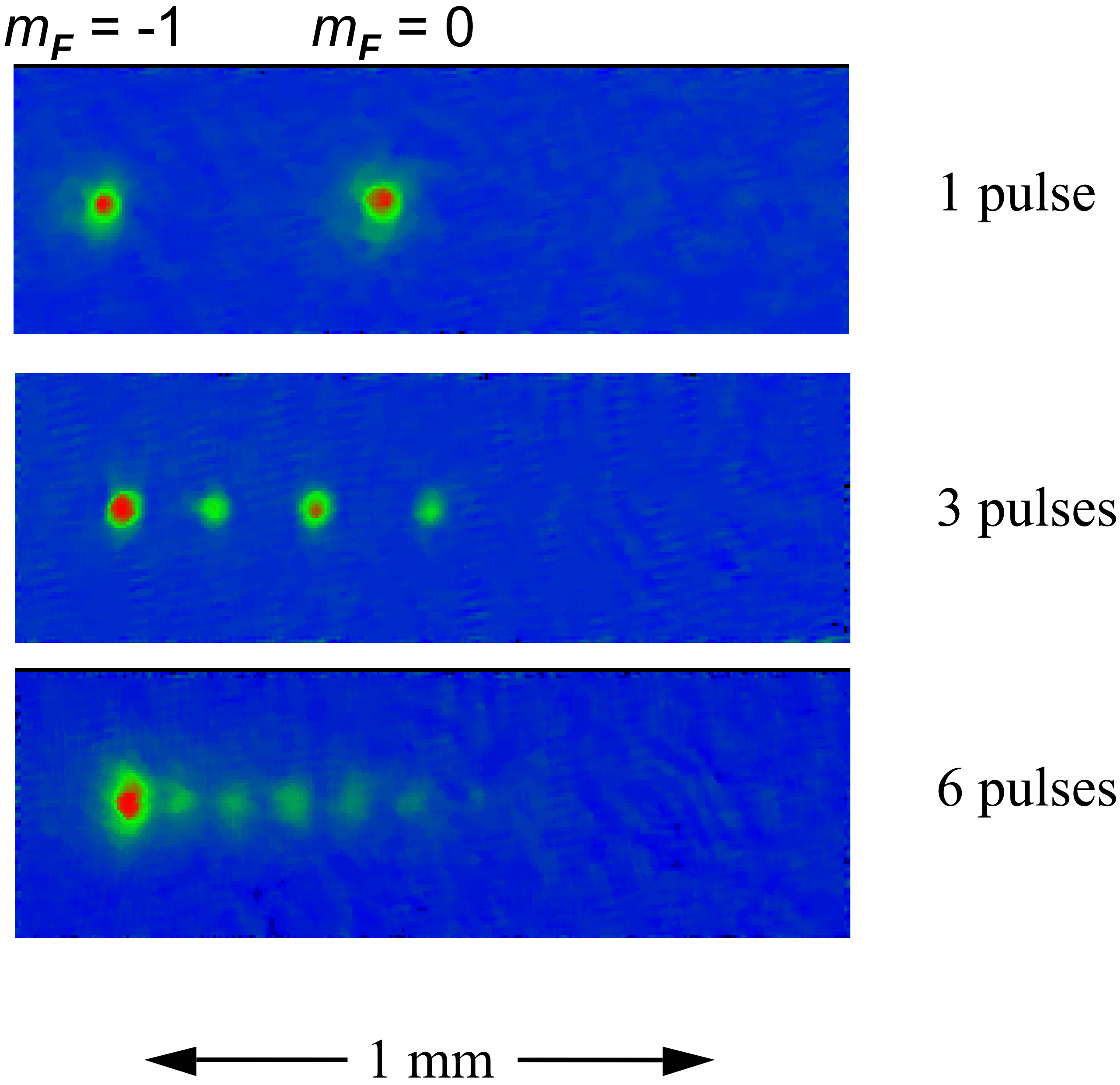
coupled out as a coherent matter-wave beam

Raman Output Coupler



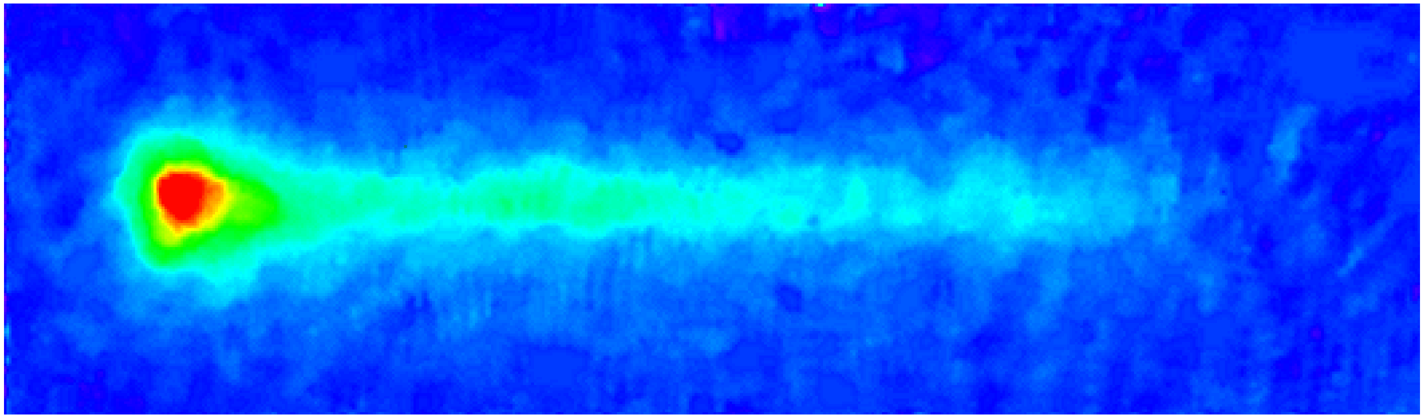


Repeated Raman output coupling



Quasi-Continuous Atom Laser

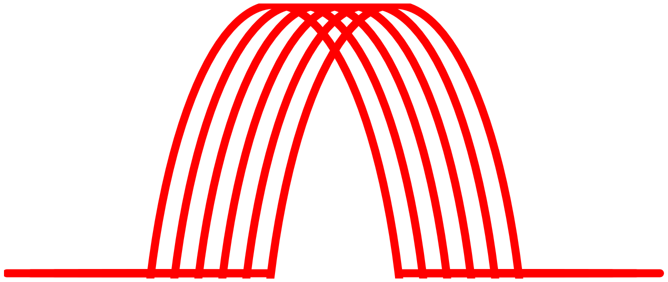
E. Hagley *et al.*, Science 283, 1706 (1999).



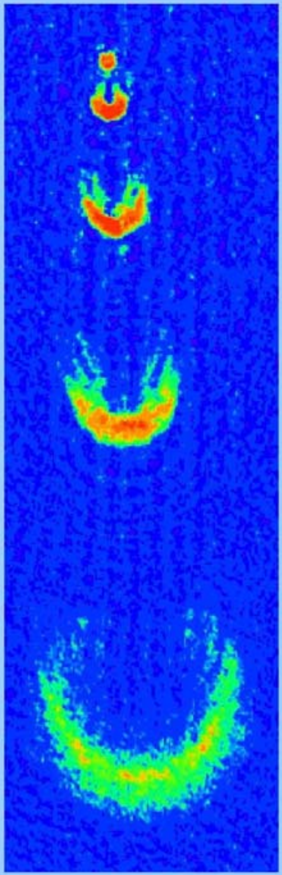
140 pulses (1 μs duration) at 20 kHz (7 ms total)

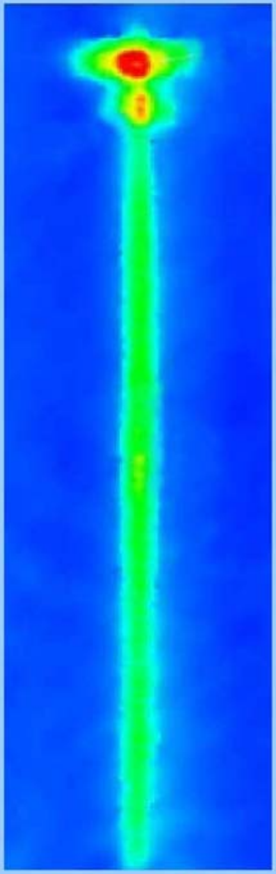
Pulse length ~ 34 μm

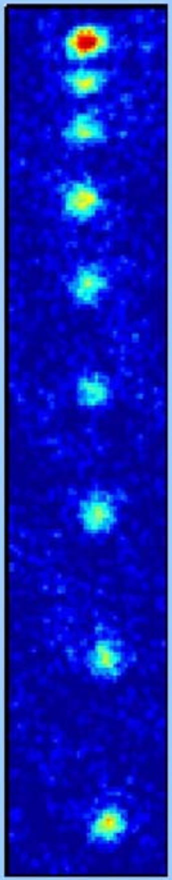
Pulse separation = 2.9 μm

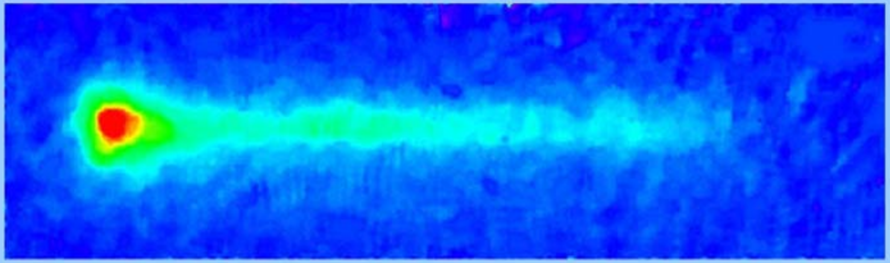


Atom Lasers


MIT

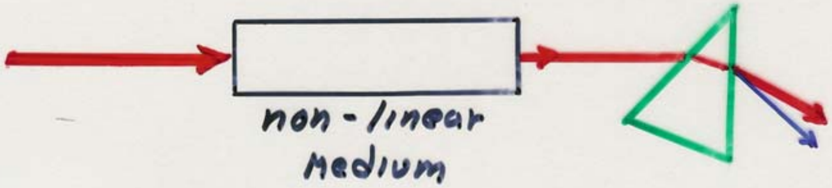

Munich


Yale


NIST-Gaithersburg

One of the first new phenomena to come from the high intensity and high coherence of optical lasers was

non-linear optics

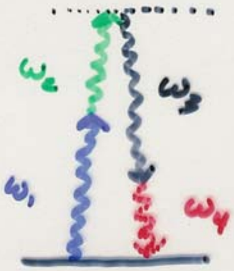


2nd harmonic generation : P. Francken ca. 1961
(This is similar to rectified 60Hz yielding 120 Hz, 180 Hz, etc.)

non-linearity (index depends on intensity or conductance depends on voltage) causes the harmonic generation.

Can we do this with atom waves?

Optical 4WM:



energy conservation:

$$\omega_4 = \omega_1 + \omega_2 - \omega_3$$

momentum conservation:

$$\vec{k}_4 = \vec{k}_1 + \vec{k}_2 - \vec{k}_3$$

(caution: notation for 1, 2, 3, 4 may not be consistent.

The 3 input waves create an oscillating polarization

$$P(x,t) \sim \chi_3 \cdot E_1(x,t)\, E_2(x,t)\, E_3^*(x,t)$$

↑ 3rd order susceptibility

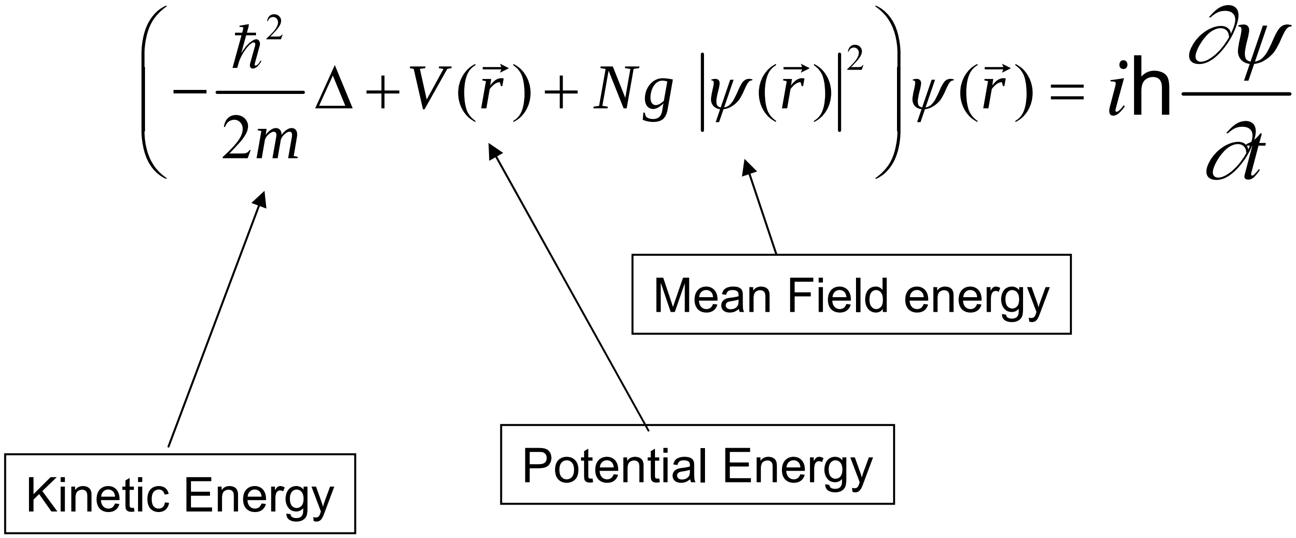
this oscillating polarization radiates the 4th wave.

In the G-P equation there is a term in

$$\frac{\partial}{\partial t}\psi(x,t) \sim g(\psi^*\psi)\,\psi(x,t)$$ which has

the same structure and gives rise to generation of a new momentum component when ψ has the proper 3 components.

Gross-Pitaevski equation (or non-linear Schrödinger's equation) :



$$\left(-\frac{\hbar^2}{2m}\Delta + V(\vec{r}) + Ng\,\left|\psi(\vec{r})\right|^2\right)\psi(\vec{r}) = i\mathrm{h}\frac{\partial\psi}{\partial t}$$

Mean Field energy

Kinetic Energy

Potential Energy

4-Matter-Wave-Mixing at NIST

Theory: **M. Trippenbach, Y. Band, P. Julienne**
Expt.: **L. Deng, E. Hagley, J. Wen, K. Helmerson, S. Rolston, WDP**
(earlier ideas by Meystre)

One way to understand 4WM:

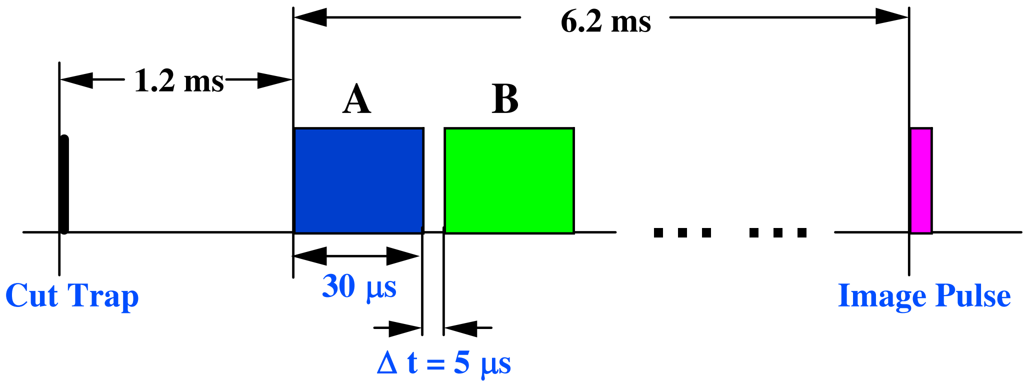
- Two deBroglie waves interfere to create a standing matter wave-equivalent to a refractive-index grating.

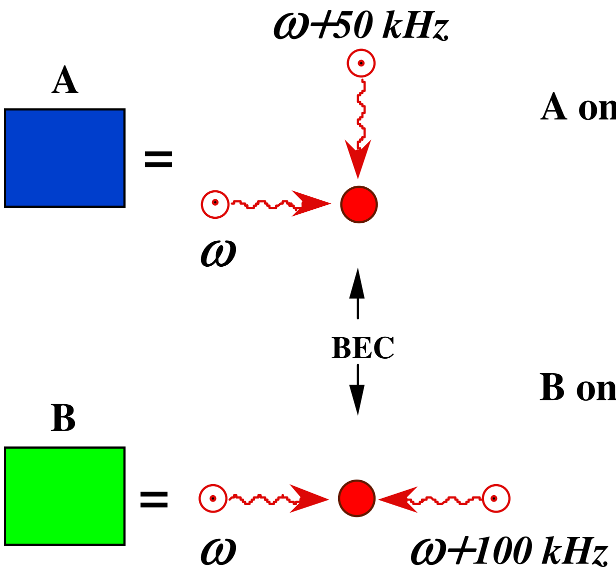
- A 3rd deBroglie wave Bragg-reflects from this grating, and the reflected wave is the

fourth deBroglie wave – a new matter wave arising from the non-linearity (mean-field effect)

phase matching = satisfaction of Bragg condition

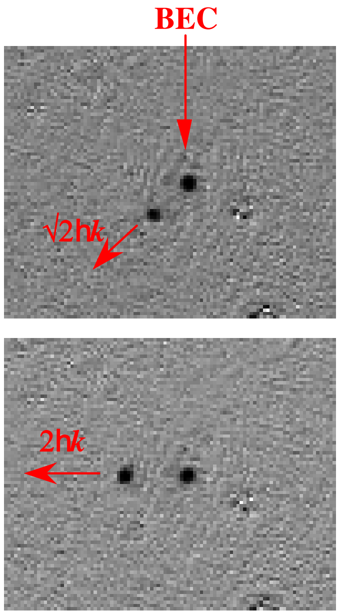
Four Wave Mixing Experiment





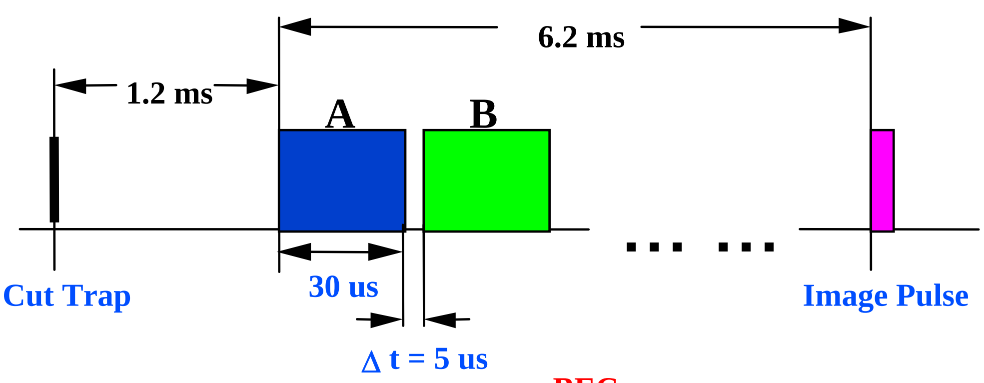
A only -->

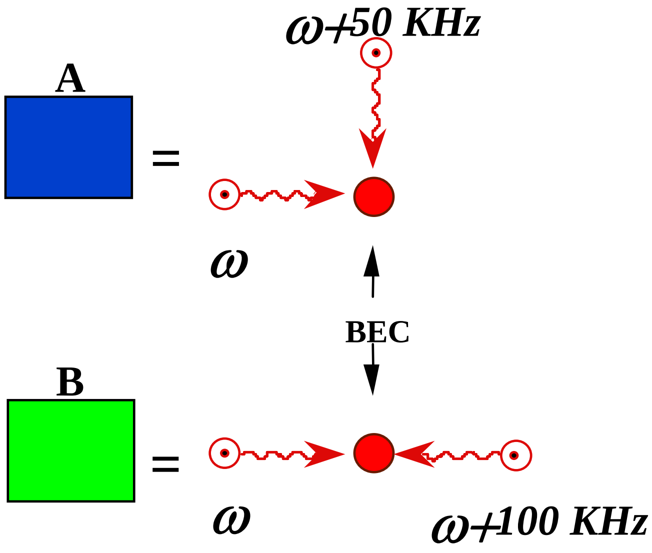
B only -->

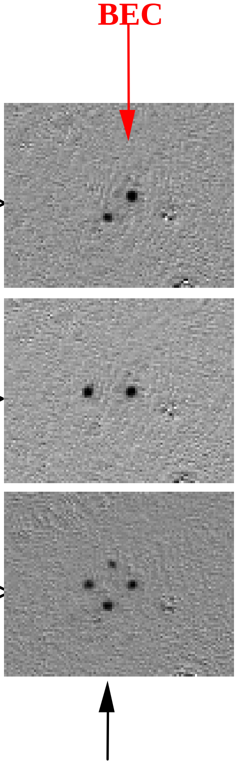


A then B ----> ?

Four Wave Mixing Experiment





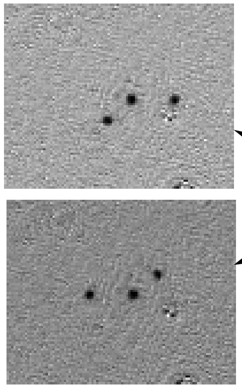


A no B ------>

B no A ------>

A then B ---->

Bragg Condition Satisfied
34% Reflected

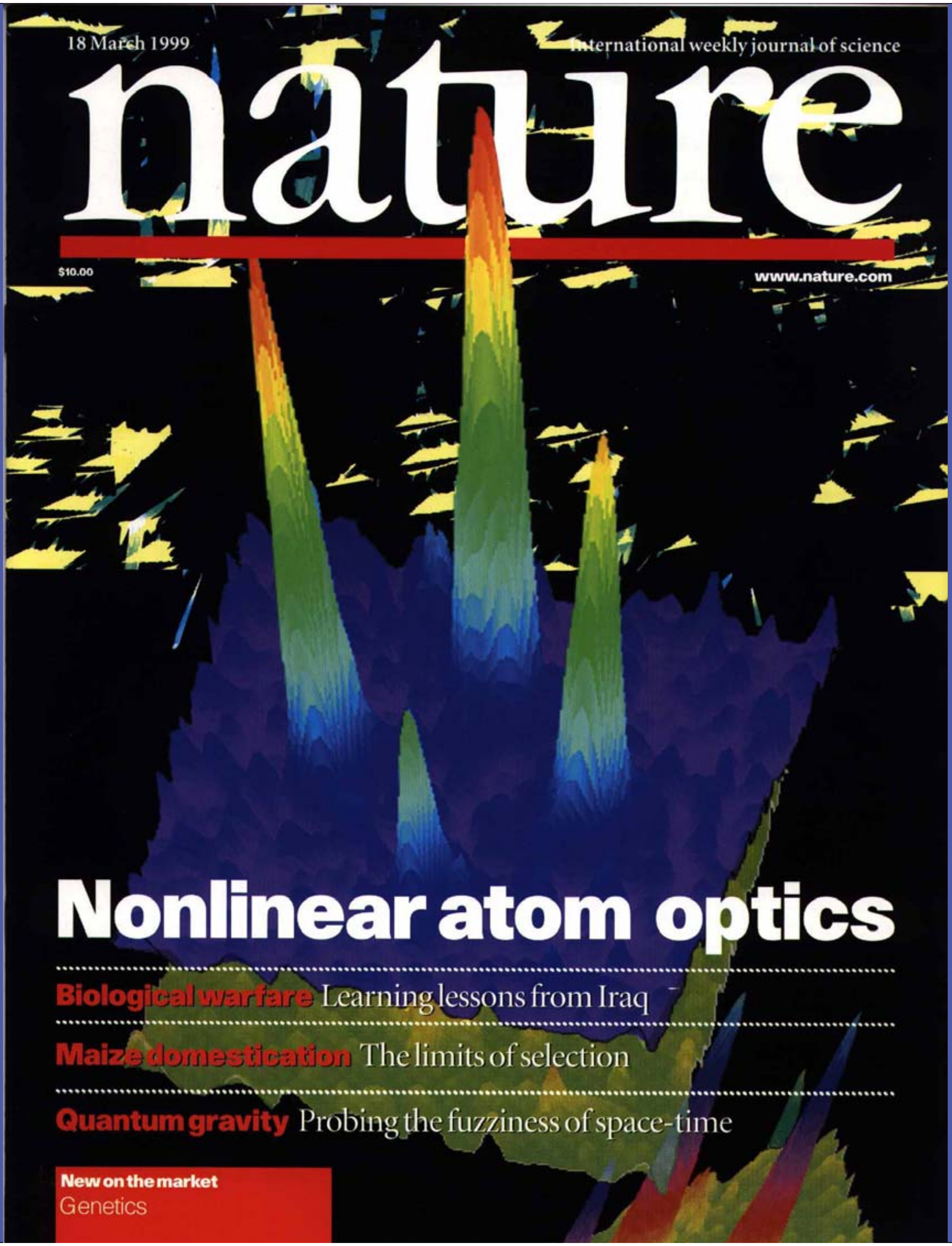


A then B (ω-100 KHz)--->

A(ω-50 KHz) then B ---->

Bragg Condition NOT Satisfied, Nothing Reflected

In all images there were 2 million atoms in original BEC with $_{max}$ = 84 Hz



18 March 1999

International weekly journal of science

nature

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Nonlinear atom optics

Biological warfare Learning lessons from Iraq

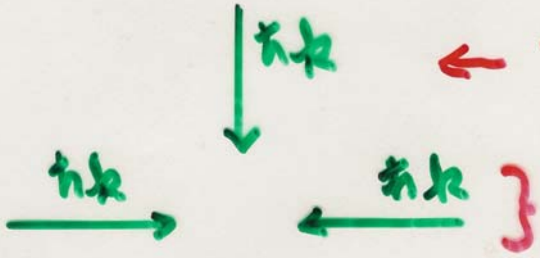
Maize domestication The limits of selection

Quantum gravity Probing the fuzziness of space-time

New on the market
Genetics

4-Wave Mixing

reference frame for degenerate 4WM



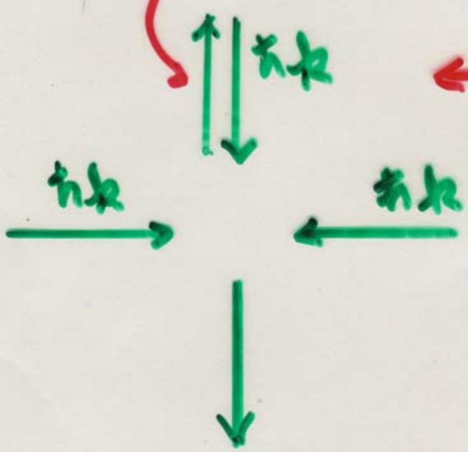
this is amplified

these two form a gain medium

4-Wave Mixing

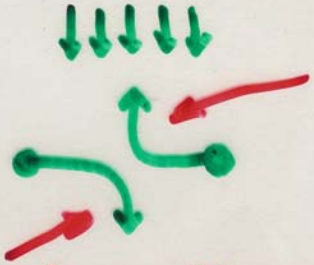
reference frame for degenerate 4WM

new, 4th wave needed to conserve E, $\vec{p}$



this is amplified

these two form a gain medium



Energy, momentum conservation

stimulated scattering in the presence of incident wave

“Quantum” atom optics

The fourth wave and the amplified wave are correlated--for every atom in the fourth wave, there is an amplified atom. This is similar to the quantum correlation between twin photons in parametric downconversion. These atoms are entangled. Can this entanglement be useful? This remains to be seen.

Other Stuff

Atoms in optical lattices
    Analogs to solid state systems

Atoms as qubits for quantum information

Cold, quantum degenerate fermions

Collisions of cold, quantum degenerate atoms

…

THE END