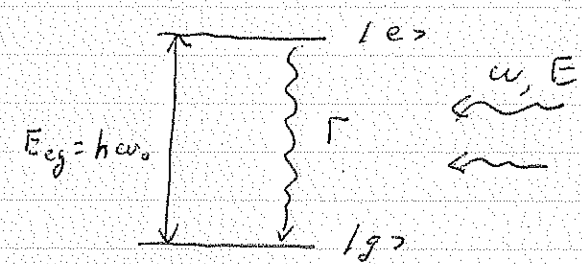
Two level

electric field $E\cos(\omega t - kz)$



$|\psi\rangle = a|g\rangle + b|e\rangle$

$$i\hbar \frac{d|\psi\rangle}{dt} = \mathcal{H}|\psi\rangle$$

$$\Omega = \frac{d_{eg}E}{\hbar}$$

supposed real

$$\mathcal{H} = \mathcal{H}_0 + V_{AL} = \hbar\omega_0|e\rangle\langle e| + \frac{\Omega}{2}\left(e^{-i\omega t}|e\rangle\langle g| + e^{+i\omega t}|g\rangle\langle e|\right)$$

damping $|e\rangle \xrightarrow{\Gamma} |g\rangle$

Absorption, stimulated emission with Probabilities

$$P_{abs} = \frac{2\pi}{\hbar}|\langle e|V_{AL}|g\rangle|^2 \rho(E_e - E_g - \hbar\omega)$$

$$P_{em} = \frac{2\pi}{\hbar}|\langle g|V_{AL}|e\rangle|^2 \rho(E_g - E_e + \hbar\omega)$$

Interference:

analogy with electromagnetism

$\vec{E}, \vec{B}$ measure intensity $I \propto |\vec{E}|^2, |\vec{B}|^2$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

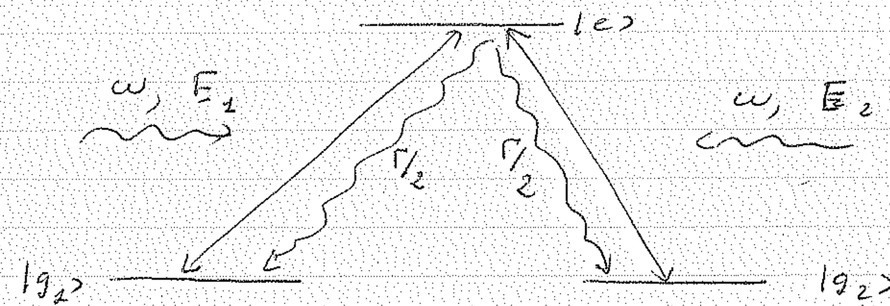
$$\vec{I} = |\vec{E}_1 + \vec{E}_2|^2 = |\vec{E}_1|^2 + |\vec{E}_2| + 2\,\mathrm{Re}(\vec{E}_1 \cdot \vec{E}_2)$$

Fermi golden rule

(2)

Interference in absorption

Three-level system



degenerate in energy

$$E_{eg_1} = E_{eg_2} = \hbar\omega_0$$

$$|\psi\rangle = \boxed{a_1|g_1\rangle + a_2|g_2\rangle} + b|e\rangle$$

$$\mathcal{H} = \mathcal{H}_0 + V_{AL} = \hbar\omega_0|e\rangle\langle e| - \frac{\Omega_1}{2}\left(|e\rangle\langle g_1| e^{-i\omega t} + e^{+i\omega t}|g_1\rangle\langle e|\right)$$
$$- \frac{\Omega_2}{2}\left(e^{-i\omega t}|e\rangle\langle g_2| + e^{+i\omega t}|g_2\rangle\langle e|\right)$$

$$\Omega_1 = \frac{d_{g_1e}E_1}{\hbar} \qquad \Omega_2 = \frac{d_{g_2e}E_2}{\hbar}$$

non-coupled

$$|NC\rangle = \frac{1}{\sqrt{\Omega_1^2+\Omega_2^2}}\left[\Omega_2|g_1\rangle - \Omega_1|g_2\rangle\right] = \frac{1}{\Omega_t}\left[\Omega_2|g_1\rangle - \Omega_1|g_2\rangle\right]$$

$\boxed{\text{dark state}}$

$$\Omega_t = \sqrt{\Omega_1^2+\Omega_2^2}$$

$$P_{abs} \propto |\langle e|V_{AL}|NC\rangle|^2$$

$$\langle e|V_{AL}|NC\rangle = \langle e|V_{AL}|\frac{\Omega_2}{\Omega_t}|g_1\rangle - \langle e|V_{AL}|\frac{\Omega_1}{\Omega_t}|g_2\rangle$$
$$= \frac{\Omega_2}{\Omega_t}\left(-\frac{\Omega_1}{2}\right)e^{-i\omega t} - \frac{\Omega_1}{\Omega_t}\left(-\frac{\Omega_2}{2}\right)e^{-i\omega t} = 0$$

Orthogonal state

$$\overset{\text{coupled}}{|c\rangle} = \frac{1}{\Omega_t}\Big[\Omega_1 |g_1\rangle + \Omega_2 |g_2\rangle\Big]$$

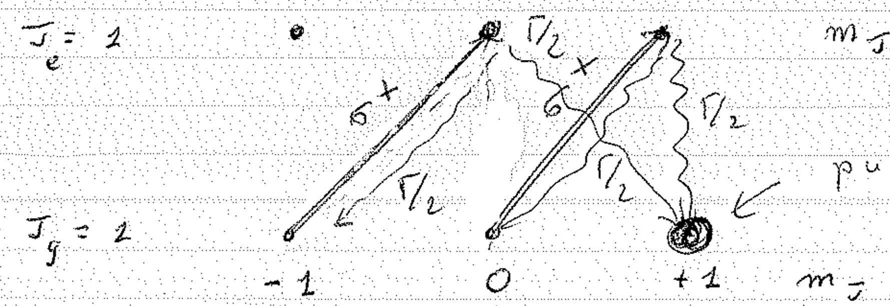
$$\langle NC | c\rangle = 0$$

$$\langle e | V_{AL} | c\rangle = \frac{1}{\Omega_t}\,\Omega_1\left(-\frac{\Omega_1}{2}\right) e^{-i\omega t} + \frac{1}{\Omega_t}\,\Omega_2\left(-\frac{\Omega_2}{2}\right) e^{-i\omega t}$$

$$= -\frac{\Omega_t}{2}\, e^{-i\omega t} \neq 0$$

It is not unique case of dark state

Also through optical pumping

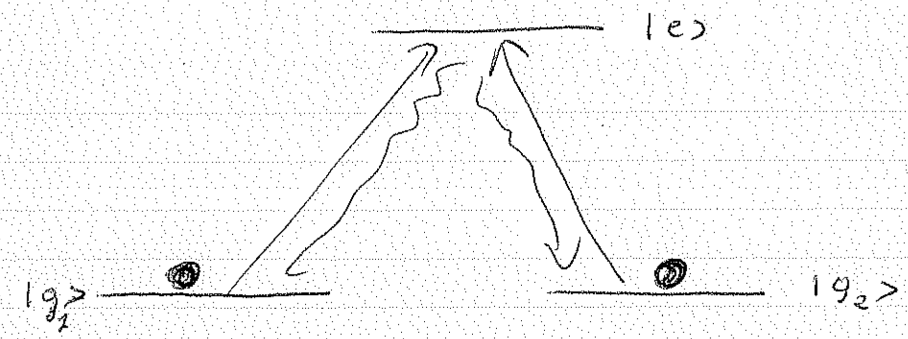


pumping into dark state

Population pumping

No coherence, no coherent evolution between states composing the pumped superposition

Preparation of noncoupled states "coherent population trapping"



$\alpha|NC\rangle + \beta|C\rangle$

$$|g_1\rangle = \alpha|NC\rangle + \beta|C\rangle = \frac{1}{\Omega_t}\left[\Omega_2 |NC\rangle + \Omega_1 |C\rangle\right]$$

$$\alpha = \langle g_1 | NC\rangle = \frac{\Omega_2}{\Omega_t}$$

$$\beta = \langle g_1 | C\rangle = \frac{\Omega_1}{\Omega_t}$$

$$|g_2\rangle = \alpha|NC\rangle + \beta|C\rangle = \frac{1}{\Omega_t}\left[-\Omega_1 |NC\rangle + \Omega_2 |C\rangle\right]$$

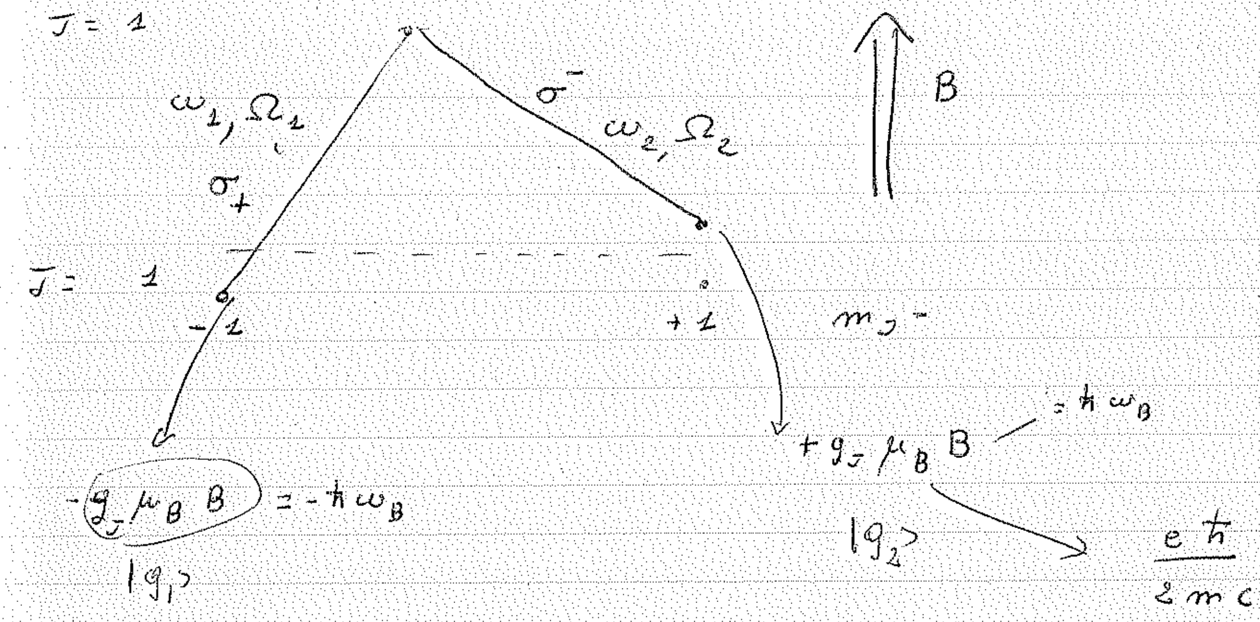
$$\alpha = \langle g_2 | NC\rangle = -\frac{\Omega_1}{\Omega_t}$$

$$\beta = \langle g_2 | NC\rangle = \frac{\Omega_2}{\Omega_t}$$

Absorption probability $|\beta|^2 = \frac{\Omega_2^2}{\Omega_t^2}$ $\quad \frac{N}{2}\frac{\Omega_1^2}{\Omega_t^2}$

Non absorption probability $|\alpha|^2 = \frac{\Omega_2^2}{\Omega_t^2}$ $\quad \frac{N}{2}\frac{\Omega_2^2}{\Omega_t^2}$

Filtering process



$$V_{AL} = -\frac{\Omega_1}{2}\left[|e\rangle\langle g_1| e^{-i\omega_1 t} + |g_1\rangle\langle e| e^{+i\omega_1 t}\right]$$
$$-\frac{\Omega_2}{2}\left[|e\rangle\langle g_2| e^{-i\omega_2 t} + |g_2\rangle\langle e| e^{+i\omega_2 t}\right]$$

$$\langle e|V_{AL}|NC\rangle = \frac{\Omega_2}{\Omega_t} e^{+i\omega_0 t}\langle e|e\rangle\langle g_1|g_1\rangle e^{-i\omega_1 t} e^{+i\frac{g_J \mu_B B}{\hbar} t}\left(-\frac{\Omega_1}{2}\right)$$

$$-\frac{\Omega_1}{\Omega} e^{i\omega_0 t}\langle e|e\rangle\langle g_2|g_2\rangle e^{-i\omega_2 t} e^{-i\omega_B t}\left(-\frac{\Omega_2}{2}\right) =$$

$$= -\frac{\Omega_1\Omega_2}{2\Omega_t}\left(e^{i\boxed{(\omega_0+\omega_B-\omega_1)}t} - e^{i\boxed{(\omega_0-\omega_B-\omega_2)}t}\right)$$

$$|e(t)\rangle = e^{-i\omega_0 t}|e\rangle$$
$$|g_1(t)\rangle = e^{+i\frac{g_J \mu_B B}{\hbar}t}|g_1\rangle$$
$$|g_2(t)\rangle = e^{-i\frac{g_J \mu_B B}{\hbar}t}|g_2\rangle$$

$$= 0$$

resonance term

$$|NC(t)\rangle = \frac{1}{\Omega}\left[\Omega_2 |g_1\rangle e^{+i\omega_B t} - \Omega_1 |g_2\rangle e^{-i\omega_B t}\right]$$

$$\sigma = |NC(t)\rangle\langle NC(t)| \cong \begin{pmatrix} \sigma_{11} & \sigma_{12} & - \\ \sigma_{21} & \sigma_{22} & - \\ - & - & \sigma_{ee} \end{pmatrix}$$

$$\sigma_{12} = \langle g_1 | NC(t)\rangle\langle NC(t) | g_2\rangle =$$

$$= -\frac{\Omega_1 \Omega_2}{\Omega^2_t} e^{-i2\omega_B t}$$

coherence

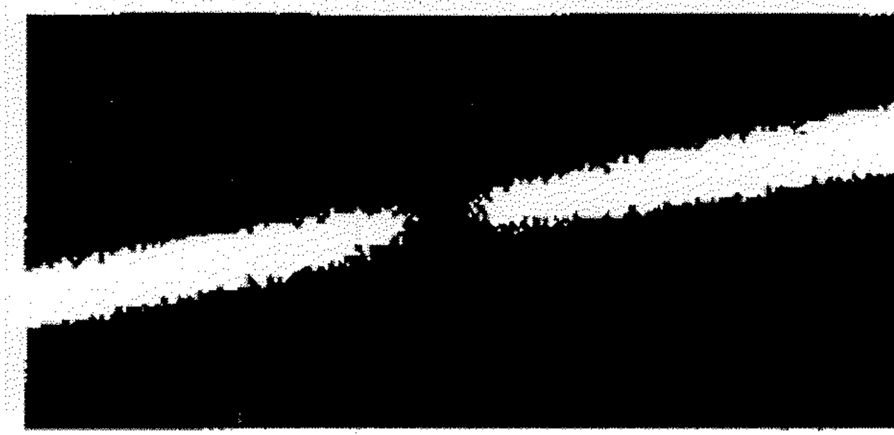
small decoherence

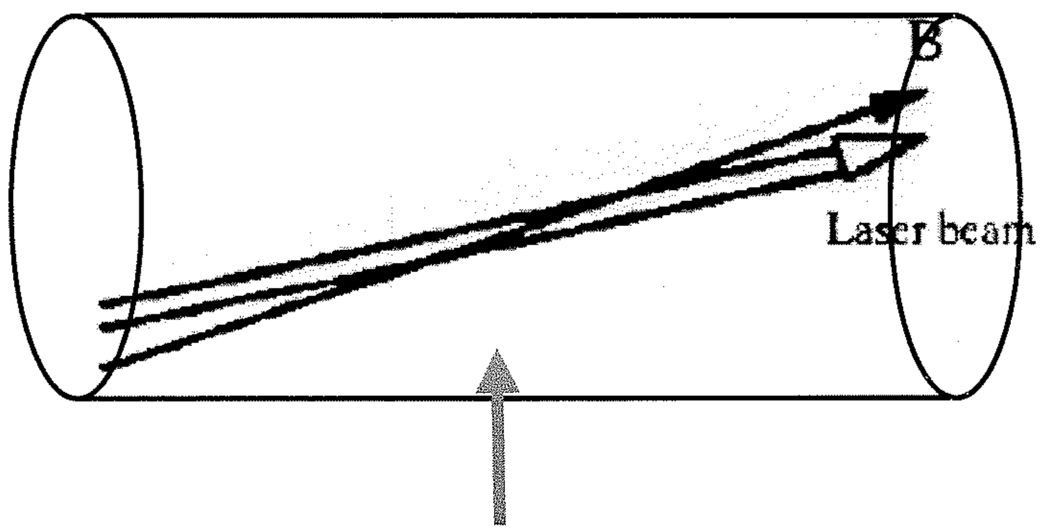
very long lifetime

$\boxed{\text{No cell}}$

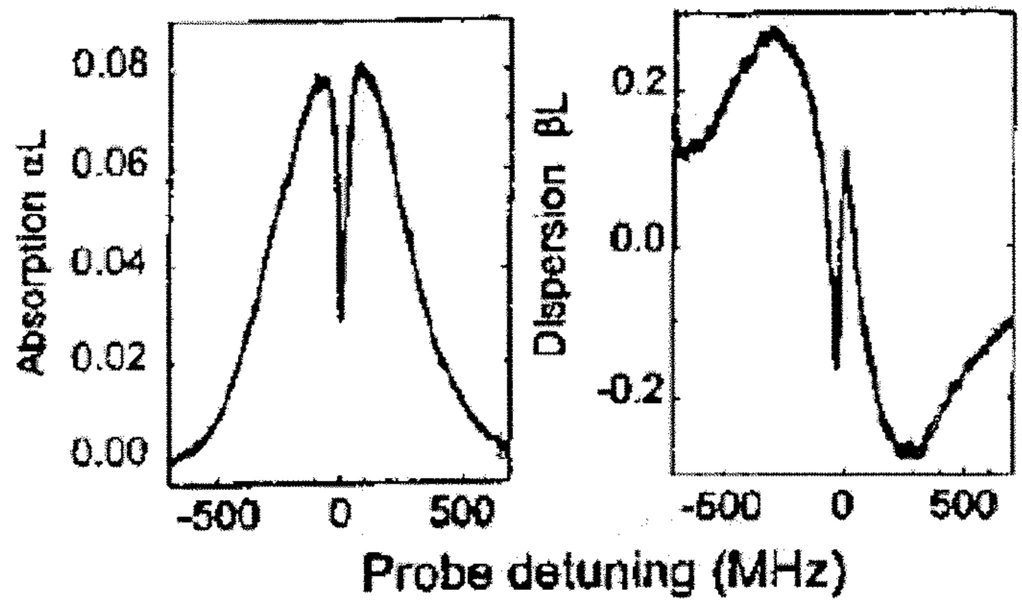
Absorption
followed by
spontaneous
emission

Dark line fluorescence from Na cell

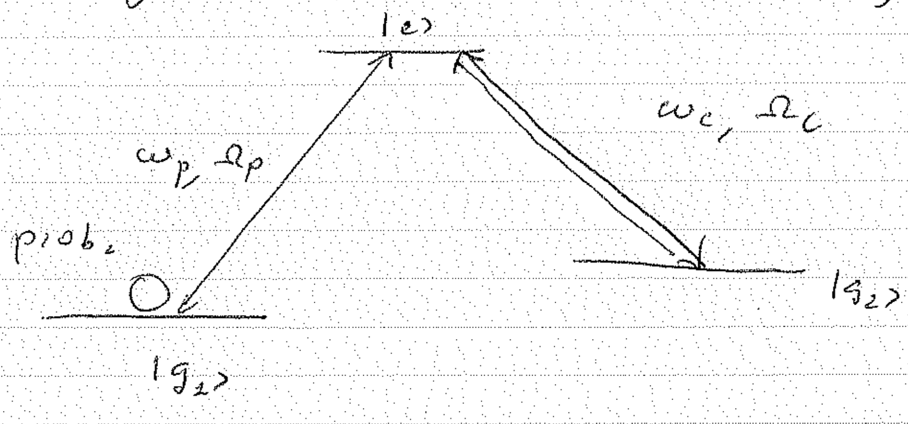




B_{res} where
ω_2-ω_1= ground state splitting


Absorption αL
0.08
0.06
0.04
0.02
0.00
-500
0
500
Dispersion βL
0.2
0.0
-0.2
-500
0
500
Probe detuning (MHz)

Electromagnetic induced transparency



$\Omega_c \gg \Omega_p$

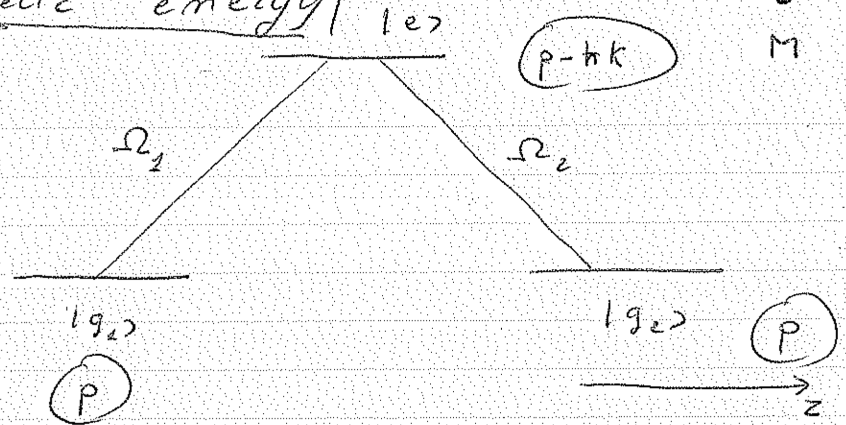
$$|NC\rangle = \frac{1}{\Omega_t}\left[\Omega_c |g_1\rangle - \Omega_p |g_2\rangle\right]$$

$\Omega_p \to 0$, $\Omega_p \ll \Omega_c$ $\quad |NC\rangle \to |g_1\rangle$

"adiabatic following"

Refractive index modified too

$\boxed{\text{Kinetic energy}}$



$\overset{\vec{v}}{\bullet}$ M $\quad \overset{E_1}{\longleftarrow} \quad \omega_1 - kv$

$\overset{E_2}{\longleftarrow} \quad \omega_2 - kv$

Photon transport momentum $\hbar\vec{k}$

$$\mathcal{H} = \mathcal{H}_0 + V_{AL} = \frac{\hat{p}^2}{2M} + \underbrace{\hbar\omega_0 |e\rangle\langle e| + V_{AL}}_{\text{internal degree of freedom}}$$

$\uparrow$ external degrees of freedom

$$|NC(t)\rangle = \frac{1}{\Omega_t}\left[\Omega_2\, e^{-i\frac{p^2}{2M}\frac{t}{\hbar}}|g_1\rangle + \Omega_1\, e^{-i\frac{p^2}{2M}\frac{t}{\hbar}}|g_2\rangle\right]$$

$$|e(t)\rangle = e^{-i\omega_0 t}\, e^{-i\frac{(p-\hbar k)^2}{2M}t}|e\rangle$$

$$\langle e(t)|V_{AL}|NC(t)\rangle =$$

$$= -\frac{\Omega_1}{2} e^{+i\omega_0 t}\, e^{i\frac{(p-\hbar k)^2}{2M}\frac{t}{\hbar}} \frac{\Omega_2}{\Omega_t}\langle e|e\rangle\langle g_1|g_1\rangle e^{-i\omega t} e^{-i\frac{p^2}{2M}\frac{t}{\hbar}}$$

$$+ \frac{\Omega_2}{2} e^{-i\omega t}\, e^{+i\omega_0 t}\, e^{+i\frac{(p-\hbar k)^2}{2M}\frac{t}{\hbar}}\langle e|e\rangle\langle g_2|g_2\rangle \frac{\Omega_1}{\Omega_t} e^{-i\frac{p^2}{2M}\frac{t}{\hbar}}$$

$$= \left[-\frac{\Omega_1\Omega_2}{2\Omega_t} + \frac{\Omega_2\Omega_1}{2\Omega_t}\right] e^{i\left(\hbar\omega_0 - \hbar\omega + \frac{(p-\hbar k)^2}{2M} - \frac{p^2}{2M}\right)\frac{t}{\hbar}}$$

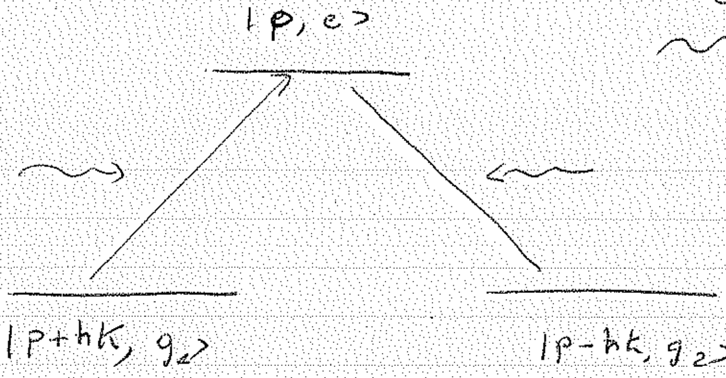
$$\omega_0 - \omega + \cancel{\frac{p^2}{2M}} + \frac{\hbar^2 k^2}{2M} - \frac{pk}{M} - \cancel{\frac{p^2}{2M}} = 0$$

$$\omega = \omega_0 + \frac{\hbar^2 k^2}{2M} - vk$$

$vk \leftarrow$ Doppler shift

$\frac{\hbar^2 k^2}{2M} \leftarrow$ recoil shift

J. Hall



$$|NC(t)\rangle = \frac{1}{\Omega_t}\left[\Omega_2\, e^{-i\frac{(p+\hbar k)^2}{2M}\frac{t}{\hbar}}|g_1\rangle - \Omega_1\, e^{-i\frac{(p-\hbar k)^2}{2M}\frac{t}{\hbar}}|g_2\rangle\right]$$

$$= \frac{1}{\Omega_t}\, e^{-i\frac{p^2}{2M}\frac{t}{\hbar}}\, e^{-\frac{\hbar k^2}{2M}t}\left[e^{-i\frac{pk}{M}t}\,\Omega_2|g_1\rangle - \Omega_1\, e^{+i\frac{pk}{M}t}|g_2\rangle\right]$$

Frequency level $\omega = \omega_0 + \frac{\hbar^2 k^2}{2M}$

Perfect dark state only at $p = 0$

$$|NC(t)\rangle = e^{-\frac{\hbar k^2}{2M}t}\left[\frac{\Omega_2}{\Omega_t}|g_1\rangle - \frac{\Omega_1}{\Omega_t}|g_2\rangle\right]$$

velocity selective coherent population trapping