1 Dec. 2005

Some leftover points from the exam.

problem #4: dipole trap

$$I(r,z) = I(0,z)\, e^{-2r^2/w^2(z)}$$

$$w(z) = w_0 \left[1 + (z/z_0)^2\right]^{1/2}$$

$$z_0 = \pi w_0^2 / \lambda$$

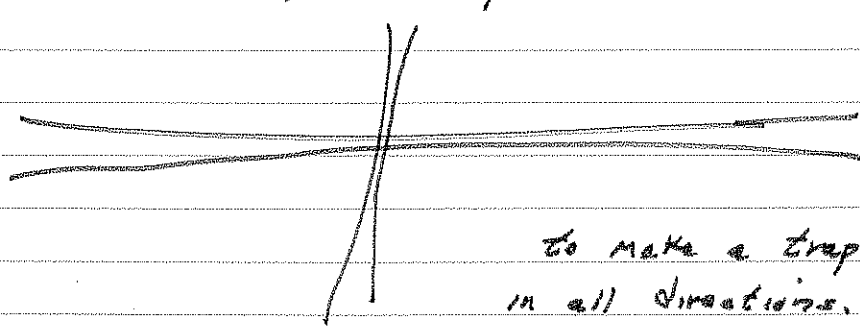
z_0 is the Rayleigh length note that it grows like w_0^2, so unless the focus is <u>very</u> tight, z_0 will be very long.

only when $w_0 \approx \lambda$ is $z_0 \approx w_0$ — this is near the diffraction limit.

for most, modest, waists w_0, z_0 is long, $\gg w_0$ so

$\omega_z \ll \omega_r$ (longitudinal trapping weak c.f. radial trapping, as we saw in 4d)

practically, to compensate for this, we often cross the dipole traps



to make a trap that is tight in all directions.

Another common trick is to use different frequencies or polarizations to avoid standing wave interference in the trap.

Is the scattering force important?
(you were told to ignore it in calculating the trapping)

compare trapping force

$F = -\nabla U$ to scattering force $F = \frac{\hbar k \Gamma}{2} \frac{I/I_0}{1 + I/I_0 + (2\delta/\Gamma)^2}$

for $(2\delta/\Gamma)^2 \gg I/I_0$ we find (do this yourself)

$$\frac{\hbar\delta}{2} \frac{2\, z/z_0^2}{1+(z/z_0)^2} \frac{I(0)/I_0}{1+(2\delta/\Gamma)^2} = \frac{\hbar k \Gamma}{2} \frac{I(0)/I_0}{1+(2\delta/\Gamma)^2}$$

$$z \cong \pi^2 \left(\frac{\Gamma}{\delta}\right) \frac{w_0^2}{\lambda^2} \cdot z_0 \qquad \text{assuming } z \ll z_0$$

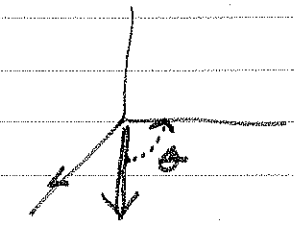
for the given parameters $z = 0.2\, z_0$

so, in this case the scattering force is <u>not</u> very important.

and (part 4d) increased detuning makes it less important

for sufficient detuning we have a Far Off Resonance Trap (FORT) and can ignore both scattering force and heating.

Problem 5



rotating frame

$$\theta = \int_0^T \Omega(t)\,dt = \pi/2 \quad \text{for}$$

a "$\pi/2$ pulse" θ is called the pulse area.

after the pulse the atom evolves freely ($\Omega = 0$)
(free induction decay)

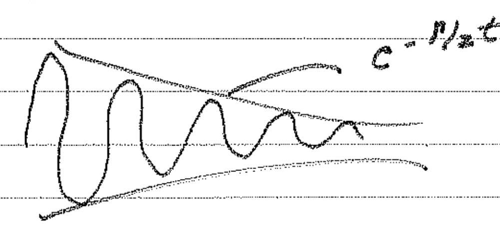
$\dot{u} = \delta v - \Gamma/2\, u$
$\dot{v} = -\delta u - \Gamma/2\, v$
$\dot{w} = -\Gamma/2 - \Gamma w \Rightarrow$ ~~[illegible]~~ $\dot{w} = -\Gamma(w + \frac{1}{2})$

but $\delta = 0$ (if it were not, the Bloch vector would rotate)

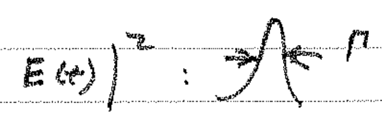
and $u = 0$ after the pulse

in the lab frame, the Bloch vector is rotating at ω_0
and decaying as $\Gamma/2$ (half the population decay rate)

the field goes like the Bloch vector - the dipole moment



power spectrum is

$|F.T.\ E(t)|^2$: 

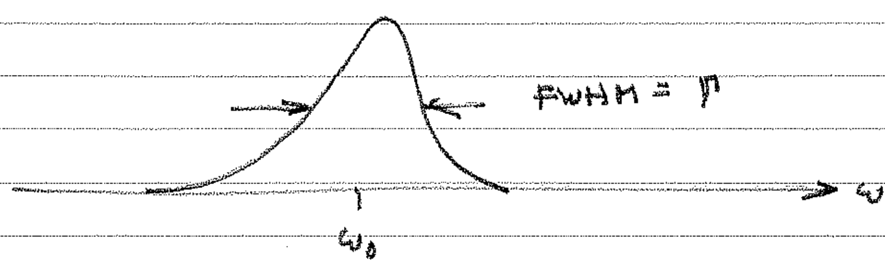
quick demonstration

$$E(t) \sim e^{-i\omega_0 t} \, e^{-\Gamma/2 \, t}$$

$$FT(E(t)) \quad \int e^{i\omega t} E(t)\, dt = \int dt \, e^{i(\omega-\omega_0)t} \, e^{-\Gamma/2 \, t}$$

$$\cong \frac{1}{i(\omega-\omega_0) - \Gamma/2}$$

$$P(\omega) \sim \left| \frac{1}{i(\omega-\omega_0)-\Gamma/2} \right|^2 \sim \frac{1}{(\omega-\omega_0)^2+(\Gamma/2)^2} \sim \frac{1}{1+(2\delta/\Gamma)^2}$$

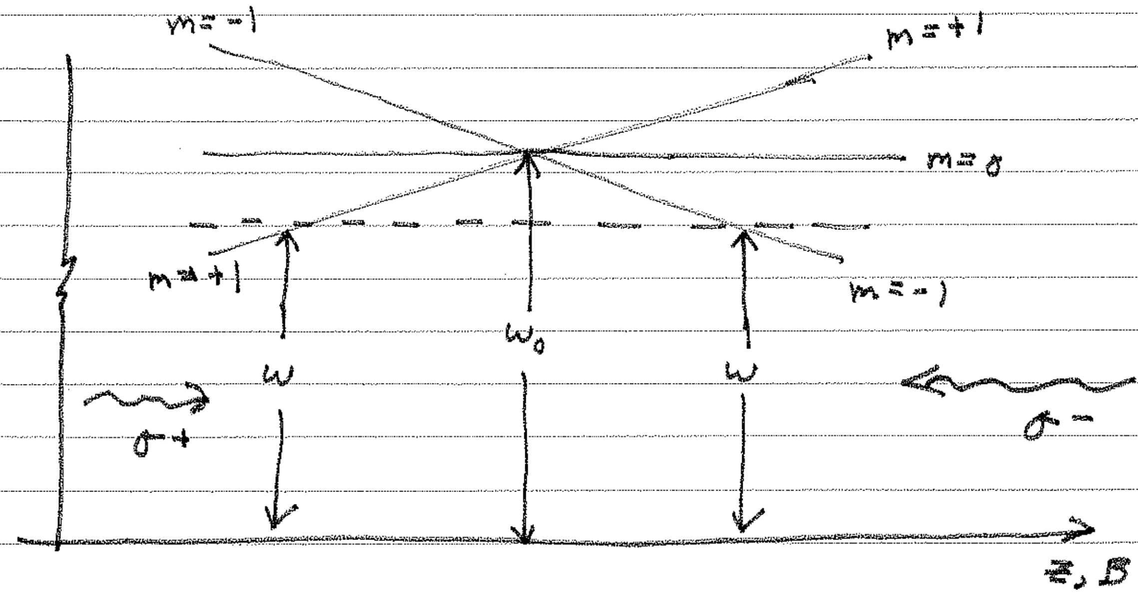


The emission spectrum after a pulse (any pulse) is the same as the low-intensity absorption spectrum (i.e., w/o power broadening)

Problem 6 Radiation-Pressure force trap

(also known as magneto-optical trap (MOT))
– the "workhorse" of laser cooling.

the principle of the trap (6c)



radiation pressure forces the atom back to center compare radiation pressure to magnetic force:

$$F_{r.p} = \frac{4\hbar k \left(2\delta/\Gamma\right) I/I_0 \, \beta z}{1 + I/I_0 + 4\delta^2/\Gamma^2}$$

take $I/I_0 = 1$, $\delta/\Gamma = 1$

$$F_{rp} = \frac{4\hbar k \; \beta z}{4} = \hbar k \beta z$$

magnetic force: $\frac{dE}{dz} = \frac{d}{dz}\left(\hbar \beta z\right) = \hbar \beta$

set $F_{rp} = F_{Zeeman}$: $\hbar k \beta z = \hbar \beta$

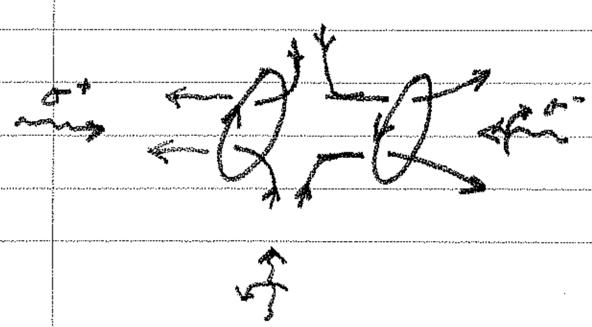
$kz = 1$

$z = \lambda\!\!\!^{-}$

for $z > \lambda\!\!\!^{-}$ $F_{rp} > F_{Zeeman}$ (even if atom were always in the excited state)

Note also that the magnetic force is anti-trapping for the trapping choice of polarization.

The MOT works great in 3D with the same choice of polarization — helicity of light is opposite field pointing toward the source of light



How deep is the MOT?

roughly: $F = \hbar k \beta z$

for a constant ~~[struck]~~ spring constant over 5 mm

$U = \frac{1}{2} \hbar k \beta z^2$

for $\beta = 2\pi \cdot 14$ MHz/cm

$$\frac{U}{k_B} = \frac{\frac{1}{2} \cdot 6\times10^{-34}\,\mathrm{J\cdot s} \cdot \frac{2\pi}{6\times10^{-7}\,\mathrm{m}} \cdot 14\times10^{6}\,\mathrm{s^{-1}}/10^{-2}\,\mathrm{m} \cdot 25\times10^{-6}\,\mathrm{m}^2}{1.38\times10^{-23}\,\mathrm{J/K}}$$

$$= 133\,\frac{10^{-34}}{10^{-32}} = 1.3\,\mathrm{K}$$

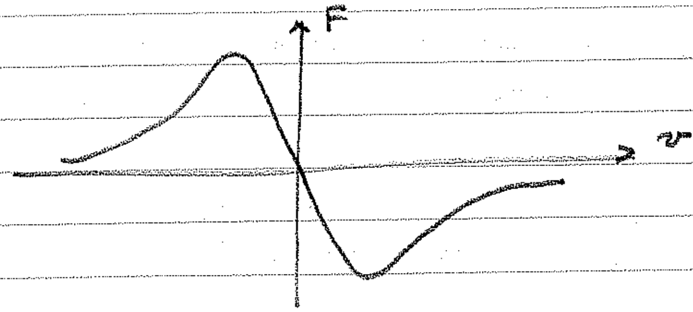
this is for $I = I_0 = 6\,\mathrm{mW/cm^2}$ for Na.

compare to 1 W focused to 10 μm for ~0.5 K in the dipole trap.

this is why the MOT is so great – low power, large depth, big volume.

Now let us return to laser cooling as in optical molasses, or cooling and trapping as in a MOT.

These processes work very well to take reasonably slow atoms and to bring them rapidly to very low velocities.



But — how do atoms get cold enough to feel the force of molasses?

for optimum detuning $\delta = \Gamma/2$, the force is strong only for $|kv| \lesssim \Gamma$

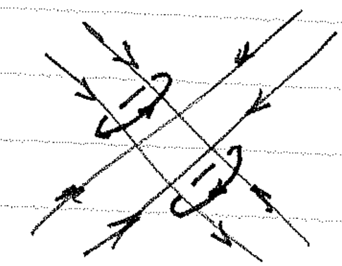
for Na $\Gamma/k = 6\ m/s$ but v_{rms} at "typical" temperatures is $500\ m/s$

so, few atoms have "coolable" velocities

There are several approaches that are used in practice

- deceleration of a beam: chirp or Zeeman
- collection from vapor

collection from vapor.



magneto-optical trap

atoms with $v \lesssim v_{capture}$ are slowed + trapped in the MOT - they collect there.

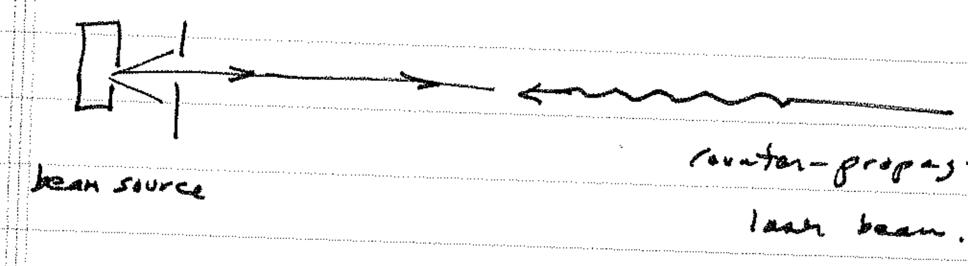
speed distribution $\sim v^2 e^{-v^2}$ integrates to available atoms $\sim v^3_{capture}$, so a larger volume, giving larger capture velocity increases the number of atoms rapidly.

balance of rate of capture $\sim$ density of vapor and loss due to background $\sim$ density of vapor.

so # of atoms is roughly independent of vapor density, but speed of filling goes up.

sometimes, the pressure is reduced after filling.

beam deceleration



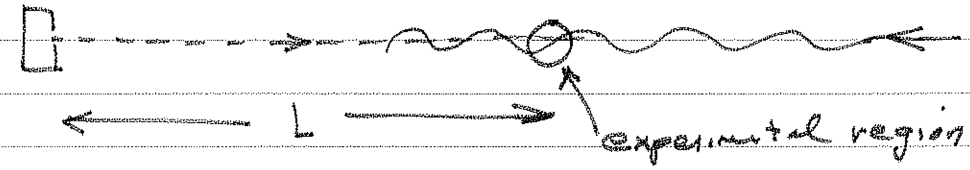
wide range of velocity $F(\bar{v})dv \sim v^3 e^{-v^2}$
for flux

width ~ ave velocity. — small fraction is resonant, small deceleration possible before going out of resonance.

2 solutions : • chirp frequency of laser up as velocity decreases

• change magnetic field along length of beam to compensate changing Doppler shift.

chirp cooling :



start with laser frequency at $\omega = \omega_0 - kv_0$

where v_0 is the "starting" velocity

change frequency linearly in time to achieve constant rate of deceleration.

$$\omega(t) = \omega_0 - kv_0 + \dot{\omega}\cdot t$$

$$\dot{\omega} = k\dot{v}$$

$$\dot{v} = \frac{F}{m} = \frac{\hbar k}{m}\frac{\Gamma}{2}\frac{I/I_s}{1+I/I_s+\left(2\delta'/\Gamma\right)^2}$$

where δ' = the effective detuning for a given velocity at a given time t

there will be a significant force only for velocities in the vicinity of

$$v(t) = v_0 - \frac{\dot{\omega}}{k}\cdot t$$

but $\dot{\omega}$ must satisfy some condition $\dot{v}_{max} = \frac{\hbar k}{m}\frac{\Gamma}{2}$

for Na, e.g.

$$\dot{v}_{max} = \frac{\hbar k}{m}\frac{\Gamma}{2} = v_{rec}\cdot\frac{\Gamma}{2}$$

$v_{rec} \approx 3\ cm/s \qquad \Gamma/2 = 1/32\ ns$

$$\dot{v}_{max} = \frac{3\times10^{-2}\ m/s}{3.2\times10^{-8}\ s} \approx 10^{6}\ m/s^2$$

If $v_0 = 10^{3}\ m/s$ (a hot Na oven)

the stopping time is $v_0/a_{max} = 10^{-3}\ s$

stopping distance, $S = v_0^2/2a = 0.5\ m$

But you cannot decelerate at a_{max} – it takes $I/I_s \to \infty$ and it is unstable:

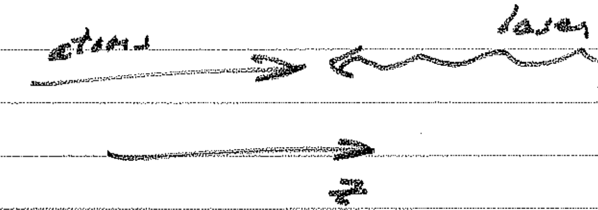
If, during some time interval, an atoms absorbs less than the ave. no. of photons, it will get out of resonance and absorb even fewer photons.

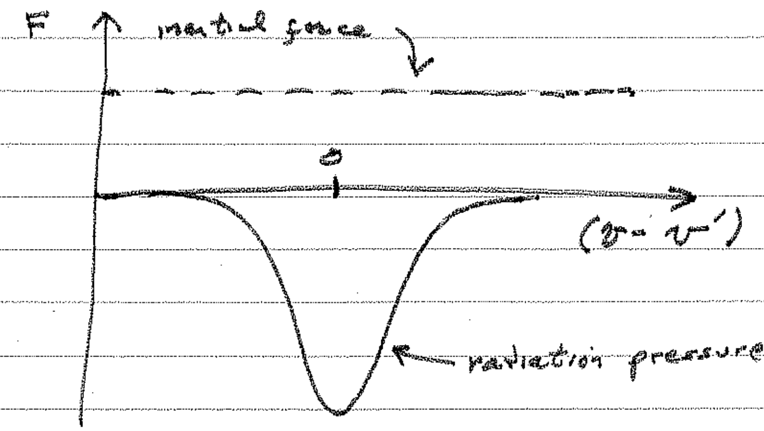
solution: set $\dot{v}$ to less than the maximum allowed for the intensity used.

then, the average velocity will self-set to the one that detunes it enough to absorb photons at the rate needed to accelerate. If an atom absorbs a few more than the ave. no. in some interval, it will go further from resonance and self-correct in the next interval. If it absorbs fewer than ave, it will be closer to resonance and self correct by absorbing more in the next interval.

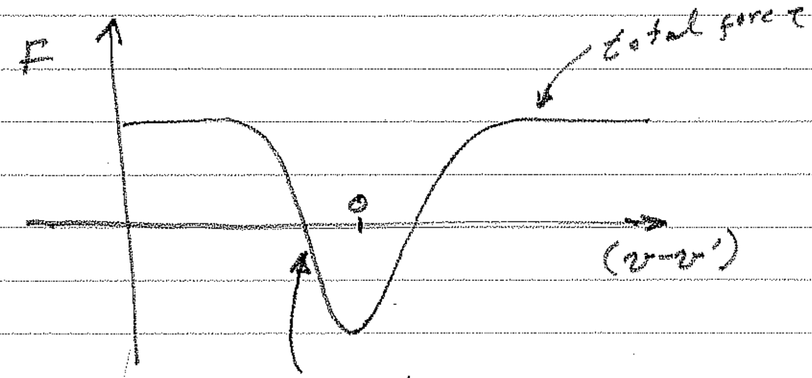
The atoms do not just decelerate, they <u>cool</u> (velocity distribution narrows)

Go to a reference frame decelerating at $\dot{\omega}/k$. In this frame, there is a radiation pressure force and an inertial force

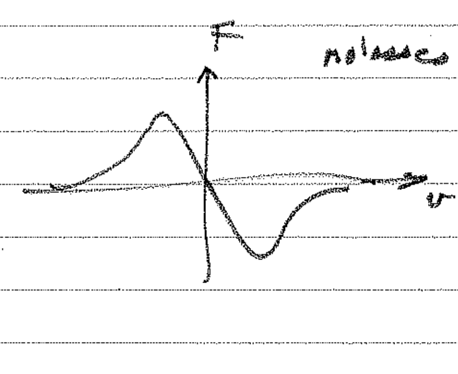




v' is the instantaneous velocity of the frame in which the Doppler-shifted laser frequency is $\omega' = \omega_0$



atoms will be damped to here, just like in molasses



the atoms slowed by this "chirp cooling" can be easily captured in molasses or a MOT.

If the length L between the source and the experiment is just long enough that

$L = S = v_0^2/2a$, then atoms with v_0 at the source will be brought to rest at the expt. after $\tau = v_0/a$

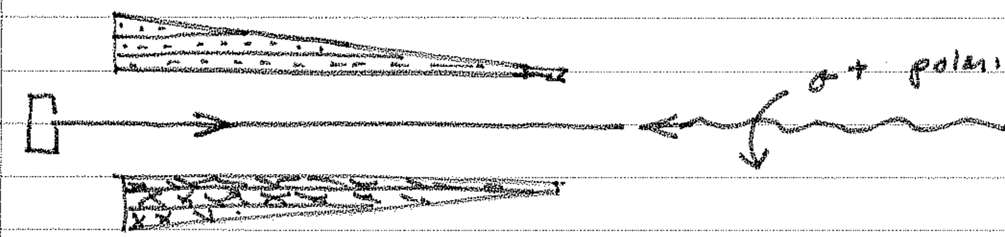
slower atoms, closer to the expt. will be stopped at earlier times at the expt.

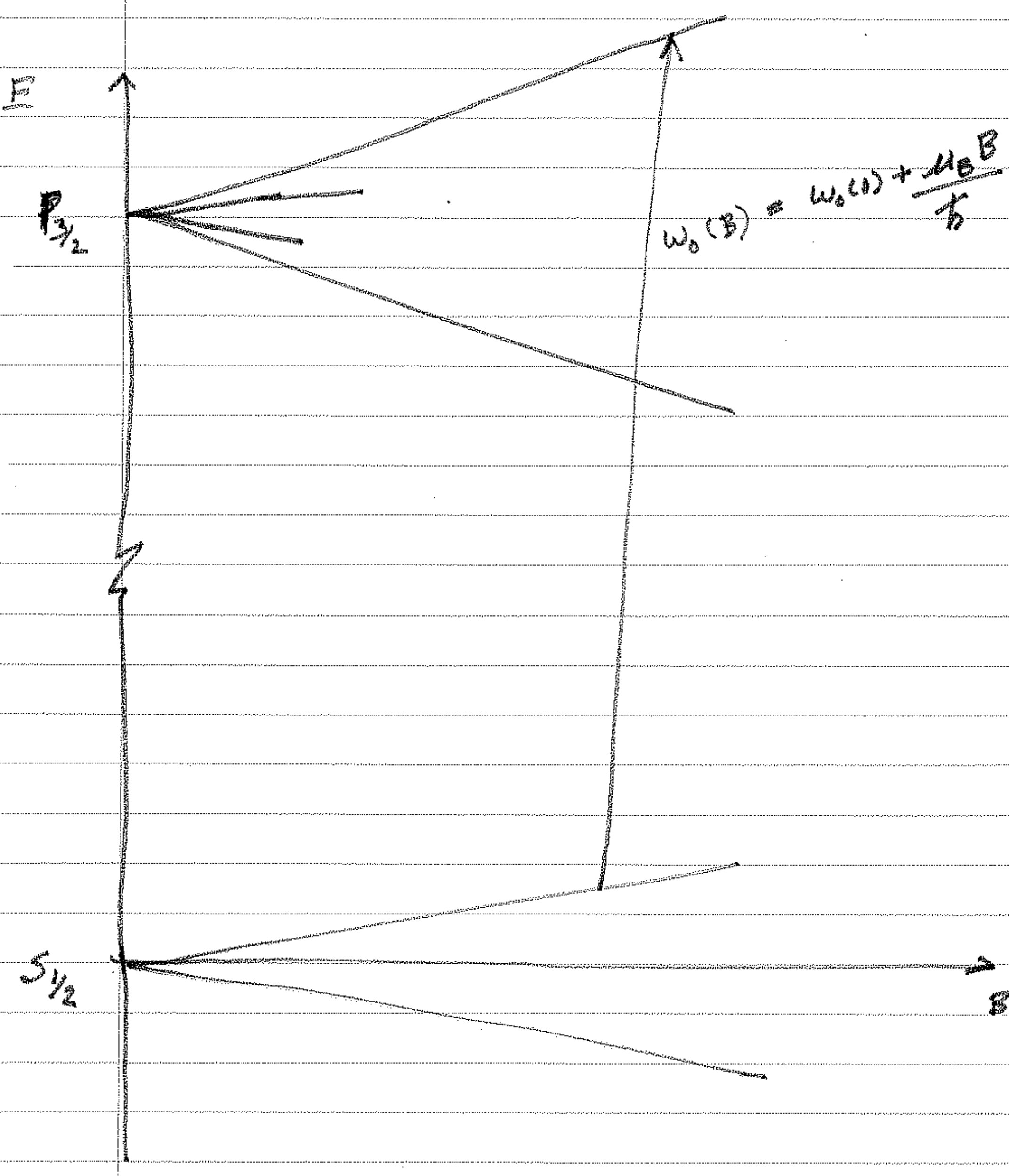
If $L > S$, then atoms over a distance between the source and expt. are brought to rest at a distribution of distances from the source.

If the chirp stops short of ω_0, the atoms will be at some $v > 0$ and will drift into the MOT or molasses. Then the chirp can start over, so the process produces pulses of slow atoms.

Continuous beam deceleration: the Zeeman cooler

compensate the changing Doppler shift, as atoms decelerate, with a spatially varying magnetic field.

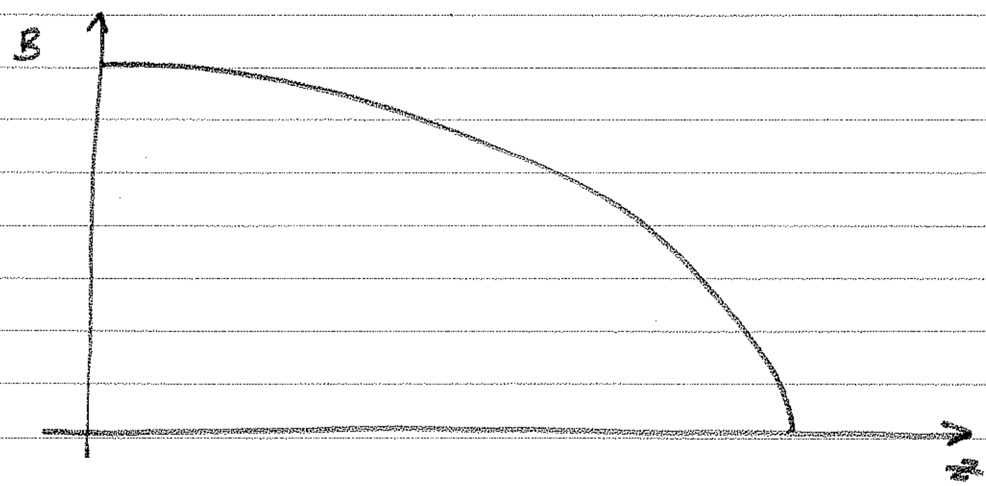

σ+ polarized


E
$P_{3/2}$
$\omega_0(B) = \omega_0(0) + \frac{\mu_B B}{\hbar}$
$S_{1/2}$
B

To make the Zeeman shift cancel Doppler shift:

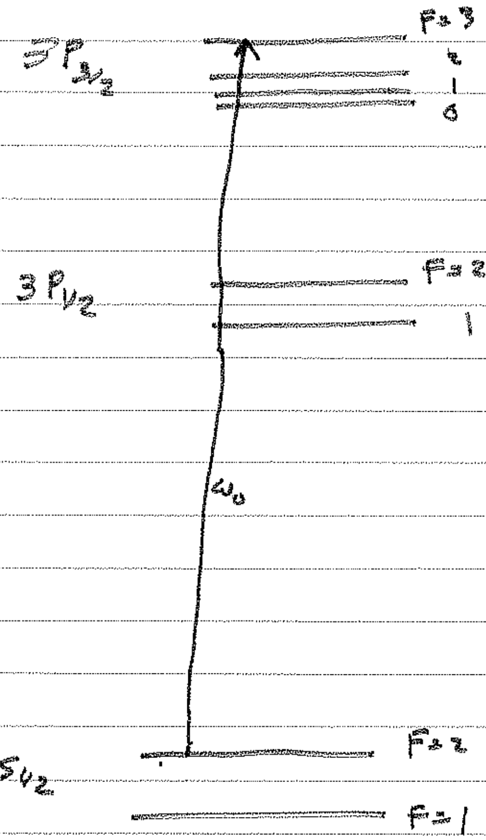
$$k v(z) = \frac{\mu_B B(z)}{\hbar} \qquad v(z) = \sqrt{v_0^2 - 2az}$$

$$B(z) = \frac{\hbar k}{\mu_B}\left(v_0^2 - 2az\right)^{1/2}$$



as before a should not be as big as a_{max}.

A practical problem with cooling Na and other alkali atoms.



$3S_{1/2}$ F=2 → $3P_{3/2}$ F′=3 is "closed."

but off-resonant excitation to F′=2 allows decay to F=1 – optical pumping – almost all population goes to F=1 and cooling stops.

solution: for Zeeman cooling σ^+ polarization helps prevent off-resonant excitation.

in general: another laser at $3S_{1/2}$ F=1 → $3P_{3/2}$ F′=2 the "repumper"

optical pumping is a common phenomenon in laser excitation.