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Some leftover points from the exam.

Problem #4: Dipole Trap

\[ I(r, \theta) = I(0, \theta) e^{-\frac{r^2}{W(z)}}, \]

\[ W(z) = W_0 \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right]^{1/2} \]

\[ z_0 = \frac{\pi W_0^2}{\lambda} \]

\( z_0 \) is the Rayleigh length. Note that it grows like \( W_0^2 \), so unless the focus is very tight, \( z_0 \) will be very long.

Only when \( W \ll \lambda \) is \( z_0 \approx W_0 \) - this is near the diffraction limit.

For most modest beams \( W \), \( z_0 \) is long, \( z_0 \ll W_0 \).

\( W_0 < \lambda \) (longitudinal trapping weak, e.g., radial trapping, as we saw in 4c)

Practically, to compensate for this, we often cross the dipole traps to make a trap that is tight in all directions.
Another common trick is to use different frequencies or polarizations to avoid standing wave interference in the trap.

Is the scattering force important?
(you were told to ignore it in calculating the trapping)

compare trapping force

\[ F = -\nabla U \quad \text{to scattering force} \quad F = \frac{\hbar k^2}{2} \frac{1}{1 + \left( \frac{2\kappa}{\pi} \right)^2} \]

for \( \frac{2\kappa}{\pi} \ll 1 \), we find \( F \approx 0 \) (do this yourself)

\[ \frac{x_0^2}{1 + \left( \frac{2\kappa}{\pi} \right)^2} \frac{\pi}{\kappa} = \frac{\hbar k^2}{2} \frac{1}{1 + \left( \frac{2\kappa}{\pi} \right)^2} \]

\[ x_0 = \frac{\pi}{\kappa} \frac{\hbar k^2}{2} \cdot x_0 \quad \text{assuming} \quad x_0 \ll x_0 \]

for the given parameters \( x_0 = 0.2 x_0 \)

so, in this case the scattering force is not very important.

and (part 9d) increased detuning makes it less important

for sufficient detuning we have a Far Off Resonance Trap (FORT) and can ignore both scattering force and heating.
Problem 5

Rotating frame

\[ \theta = \int_0^T \gamma(t) \, dt = \frac{\pi}{2} \text{ for} \]

a "\( \pi/2 \) pulse" \( \theta \) is called the pulse area.

After the pulse the atom evolves freely \((\dot{\theta} = 0)\)
(same induction decay)

\[
\begin{align*}
\dot{\gamma} &= \gamma - \frac{\pi}{2} \gamma \\
\dot{\omega} &= -\gamma \omega - \frac{\pi}{2} \omega \\
\omega &= -\frac{\pi}{2} - \gamma \omega & \Rightarrow \omega &= -\gamma (\omega + \frac{\pi}{2})
\end{align*}
\]

but \( \delta > 0 \) (if it were not, the Bloch vector would rotate)

and \( \omega = 0 \) after the pulse.

In the lab frame, the Bloch vector is rotating at \( \omega \)
and decaying as \( \frac{\gamma}{2} \) (half the population decay rate)

The yield goes like the Bloch vector - the dipole moment.

\[ c \propto \frac{1}{\lambda^2} \]

Power spectrum is

\[ |\text{F.T.} \, E(t)|^2 \propto \lambda^{-2} \]
Quick demonstration:

\[ E(t) = e^{-i\omega_0 t - \frac{t^2}{2}} \]

\[ \text{FT}(E(t)) \int e^{i\omega t} E(t) dt = \int e^{-i\omega_0 t - \frac{t^2}{2}} \]

\[ \frac{1}{\sqrt{\pi}} \frac{1}{(\omega - \omega_0)^2 + (\Gamma/2)^2} \sim \frac{1}{1 + \left(\frac{2\Delta \nu}{\Gamma}\right)^2} \]

The emission spectrum after a pulse (any pulse) is the same as the low-intensity absorption spectrum (i.e., w/o power broadening).
Problem 6: Radiation Pressure force trap

(also known as magneto-optical trap (MOT))
- The "workhorse" of laser cooling.

The principle of the trap ($E$-c)

[Diagram of radiation pressure forces an atom back to center]
compare radiative forces to magnetic force:

\[ F_{\text{rad}} = \frac{4 \pi \hbar (2\beta)^3}{3 \hbar \omega + 9 \beta^2} \]

\[ F_{\text{mag}} = \frac{4 \pi \hbar \beta^2}{4} = \hbar \beta \]

magnetic force:
\[ \frac{\Delta E}{\Delta \gamma} = \frac{1}{\gamma^3} (\hbar \beta^2) \cdot \hbar \beta \]

set \[ F_{\text{rad}} = F_{\text{mag}} : \quad \hbar \beta \gamma^2 = \hbar \beta \]
\[ \gamma = 1 \]
\[ \omega = \omega_0 \]

for \( \omega > \omega_0 \), \( F_{\text{rad}} > F_{\text{mag}} \) (even if atom were always in the excited state)

Note also that the magnetic force is anti-trapping for the trapping choice of polarization.

The MOT works great in 3D with the same choice of polarization: helicity of light is opposite field pointing toward the source of light.

\[ \theta \leftarrow \frac{\lambda}{4} \leftrightarrow \frac{\lambda}{2} \rightarrow \]
How deep is the MOT?

roughly: \[ F = \frac{1}{2} k \beta z \]

spring constant

for a constant \( k \) over \( 5 \text{ mm} \)

\[ U = \frac{1}{2} k \beta \beta z^2 \]

for \( \beta = 2 \pi \cdot 14 \text{ MHz/cm} \)

\[ U = \frac{1}{2} \cdot 2 \pi \cdot 14 \times 10^6 \frac{4}{3} \times 5 \times 10^{-10} \text{ N} \cdot 25 \times 10^{-6} \text{ m}^2 \]

\[ = \frac{81.6 \times 10^{-26}}{6} \times 10^{-10} \text{ m} \]

\[ < 133 \frac{10^{-39}}{10^{-32}} = 1.3 \text{ K} \]

this is for \( J = 3F = 6 \text{ mK} \) for Na.

compare to 1W focussed to 10 \( \mu \text{m} \) for \( 0.5 \text{ K} \)

in the dipole trap.

this is why the MOT is so great - low power,

large depth, big volume.
Now let us return to laser cooling as in optical molasses, or cooling and trapping as in a MOT.

These processes work very well to take reasonably slow atoms and to bring them rapidly to very low velocities.

![Graph showing a force vs. velocity relationship.]

But — how do atoms get cold enough to feel the force of molasses?

For optimum detuning, \( \delta = \pi/2 \), the force is strong only for \( |\Delta \omega| \leq \Gamma \).

For Na, \( \Gamma/\Delta \omega = 6 \text{ m/s} \) but turns at "typical" temperatures at 500 m/s.

So, cold atoms have "coolable" velocities.

There are several approaches that are used in practice:

- Deceleration of a beam: chirp or Zeeman
- Collection from vapor
collection from vapor.

atoms with $v \leq v_{\text{max}}$ are slowed & trapped in MOT, not - they collect there.

speed distribution $\sim v^2 e^{-v^2}$ integrates to available atoms $\sim v_{\text{max}}^3$, so a larger volume, giving larger capture velocity increases the number of atoms rapidly.

balance of rate of capture vs density of vapor and loss due to background density of vapor.

so # of atoms is roughly independent of vapor density, but speed of filling goes up.

Sometimes, the pressure is reduced after filling.
beam deceleration

... beam source

... beam beam.

wide range of velocity $F(v) dv = v^3 e^{-v^2}$

for $v \gg \lambda$

width $\nu$ ave velocity. - small frequency is resonant, small deceleration possible before going out of resonance.

2 solutions: 1 chirp frequency $\nu$ brom up as velocity decreases

- change magnetic field along length of beam to compensate changing Doppler shift.
chirp cooling:

$\gamma = \frac{h^2}{2m^2} \left(1 + \frac{\Delta}{\Delta_0}\right)$

where $\Delta$ is the "starting" velocity

change frequency linearly in time to achieve constant rate of deceleration,

$\dot{\omega}(t) = \omega_0 - \frac{\Delta}{t}$

$\omega = \frac{h^2}{2m^2} \left(1 + \frac{\Delta}{\Delta_0}\right)$

$\Delta = \frac{F}{m} = \frac{h^2 k^2}{m^2} \frac{I(t)}{1 + I_0 \left(\frac{\Delta}{\Delta_0}\right)}$

where $\Delta$ is the effective detuning for a given velocity at a given time $t$.

there will be a significant force only for velocities in the vicinity of

$\omega(t) = \omega_0 - \frac{\Delta}{t}$

but it must satisfy some condition $\omega_{\text{max}} = \frac{h^2 k^2}{m^2} \frac{I(t)}{1 + I_0 \left(\frac{\Delta}{\Delta_0}\right)}$
for Na, e.g.

\[ v_{\text{max}} = \frac{\hbar}{m_e} \frac{\pi}{2} = v_{\text{esc}} \frac{\pi}{2} \]

\[ v_{\text{esc}} = 3 \text{ cm/s} \quad \frac{1}{\text{a}} = \frac{\sqrt{2}}{32 \text{ ns}} \]

\[ \frac{3 \times 10^{-2} \text{ m/s}}{3.2 \times 10^{-8} \text{ s}} \approx 10^6 \text{ m/s}^2 \]

If \( v_a = 10^3 \text{ m/s} \) (a hot Na atom)

the stopping time is \( v_{\text{av}} = \frac{v_0}{v_{\text{av}}} = 10^{-3} \text{ s} \)

stopping distance, \( S = \frac{v_0^2}{2a} \approx 0.5 \text{ m} \)

But you cannot accelerate at \( v_{\text{max}} \) - it takes \( \frac{v_0}{v_{\text{av}}} \) and it is unstable:

If, during some time interval, an atom absorbs less than the ave. no. of photons, it will get out of resonance and absorb even fewer photons.

**Solution**: set \( N \) to less than the maximum allowed for the intensity used.

Then, the average velocity will self-set to the one that defines it enough to absorb photons at the rate needed to accelerate. If an atom absorbs a few more than the ave. no. in some interval, it will go farther from resonance and self-correct in the next interval. If it absorbs fewer than ave, it will be closer to resonance and self correct by absorbing more in the next interval.
The atoms do not just decelerate, they cool (velocity distribution narrows.

In the reference frame accelerating at $\alpha t^2$. In this frame, there is a radiation pressure force and an inertial force.

\[ F = \text{inertial force} \]

\[ \gamma' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]

\[ \gamma' \] is the instantaneous velocity of the frame in which the Doppler-shifted laser frequency is $\omega'$.

\[ \omega' = \omega_0 \]

\[ F = \text{radiation pressure} \]

\[ F = \text{total force} \]

Atoms will be clamped to here, just like in molasses.
The atoms slowed by this "chirp cooling" can be easily captured in MOT or a MOT.

If the length $L$ between the source and the experiment is just long enough that

$$L = 5 = \frac{2a}{\gamma a},$$

then atoms with $N_0$ at the source will be brought to rest at the end after $T = 2a/\gamma$.

Slower atoms, closer to the epicenter, will be stopped at earlier times at the epicenter.

If $L > 5$, then atoms over a distance between the source and epicenter are brought to rest at a distribution of distances from the source.

If the chirp stops short of $L_0$, the atoms will be at some $v > 0$ and will drift into the MOT or elsewhere. Then the chirp can start over, so the process produces pulses of slow atoms.

Continuous beam deceleration: the Zeeman cooler:

compensate the changing Doppler shift, so atoms decelerate with a spatially varying magnetic field.
\[ w_0(B) = w_0(0) + \frac{u_B B}{\hbar} \]
To make the Zeeman shift cancel Puppe's shift:

\[ kT(z) = \frac{\mu_B B(z)}{\hslash} \quad \nu(z) = \sqrt{\nu_0^2 - 2aE} \]

\[ B(z) = \frac{\hbar k}{\mu_B} \left( \nu_0^2 - 2aE \right)^{\frac{1}{2}} \]

As before, \( a \) should not be as big as \( a_{\text{max}} \).
A practical problem with cooling Na and other alkali atoms.

\[ \begin{align*}
3P_{3/2} & \quad F = 2 \\
5S_{1/2} & \quad F' = 3
\end{align*} \]

\[
\text{Closed.} \quad \text{but off-resonant excitation to F'=2 allows decay to F=1.}
\]

Optical pumping — almost all population goes to F=1 and cooling steps.

Solution: for Zeeman cooling or polarization helps prevent off-resonant excitation.

In general: another laser at 5S_{1/2} F=1 \rightarrow 3P_{3/2} F'=2 is the "repumper."

Optical pumping is a common phenomenon in laser excitation.