Define a second moment to be a correlation of intensities, or a four-field correlation

$$g^{(2)}(\tau) = \frac{\langle \bar{I}(t)\bar{I}(t+\tau)\rangle}{\langle \bar{I}\rangle^2} = \frac{\langle E^*(t)E^*(t+\tau)E(t+\tau)E(t)\rangle}{\langle E^*(t)E(t)\rangle^2}$$

If stationary, $g^{(2)}(-\tau) = g^{(2)}(\tau)^*$

Cauchy-Schwartz inequality imply

$$2\bar{I}(t_1)\bar{I}(t_2) \leq \bar{I}(t_1)^2 + \bar{I}(t_2)^2$$

$$\langle (I(t_1)^* - I(t_2))^2\rangle \geq 0$$

$$\langle I(t_1)^2\rangle + \langle I(t_2)^2\rangle - 2\langle I(t_1)I(t_2)\rangle \geq 0$$

for stationary light,

$$2\langle I(t)^2\rangle - 2\langle I(t)I(t+\tau)\rangle \geq 0$$

$$\langle I(t)^2\rangle \geq \langle I(t)I(t+\tau)\rangle$$

$$g^{(2)}(0) \geq g^{(2)}(\tau)$$

Similar arguments can show $g^{(2)}(0) \geq 1$

$$g^{(2)}(\tau) \geq 0, \quad \tau \neq 0$$

applies for any light beam for which there is a time or ensemble average.

Classical statistical behavior. compare to EPR, local-realism

Examples: Doppler or collision broadened light. (anything that is a sum over independent radiators.

$$\langle E^*(t)\,E^*(t+\tau)\,E(t+\tau)\,E(t)\rangle = \cancel{\sum_{i=1}^{\text{atoms}} \langle E_i^*(t)\,E_i^*(t+\tau)\,E}$$

$$= \sum_{i}^{\text{atoms}} \langle I_i(t)\, I_i(t+\tau)\rangle$$

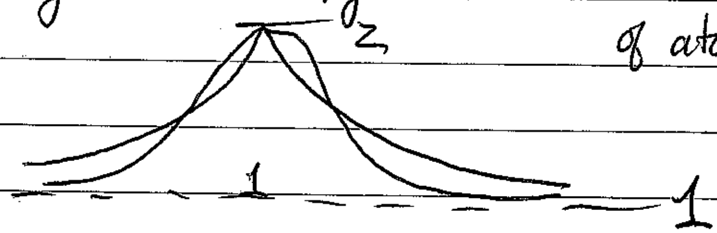
$$\underbrace{+ \sum_{i\neq j}^{\text{atoms}} \langle I_i(t)\, I_j(t+\tau)\rangle + \sum_{i\neq j} \langle E_i^*(t)\,E_j^*(t+\tau)\,E_i(t+\tau)\,E_j(t)\rangle}$$

dominated by these large pair-wise sums.

assume stationary,

$$\langle E^* = N_{\text{atoms}}^2 \left(\langle I_i(t)\rangle^2 + |\langle E_i(t)\,E_i(t+\tau)\rangle|^2 \right.$$

$$g^{(2)}(\tau) = 1 + |g^{(1)}(\tau)|^2$$ (assumes very large number of atoms contributing



Sine wave:

$$g^{(2)}(\tau) = 1$$ → intensity intensity correlations

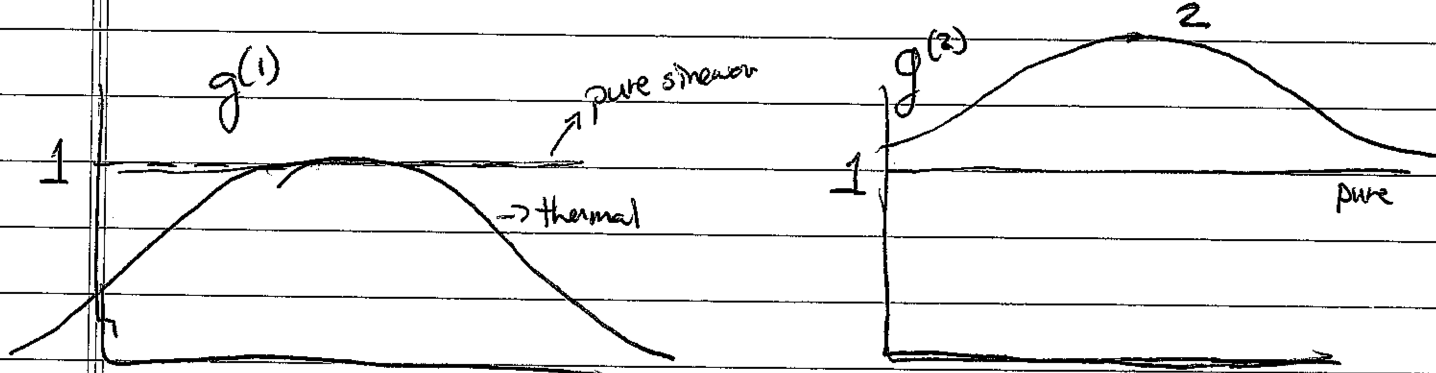
Note: even for filtered thermal or collision broadened light, measured on time scales $t \ll \tau_c$, there is a difference from deterministic sine-wave light.

$g^{(1)}(\tau)$ $\tau \ll \tau_c$ $= \underline{1}$ thermal

$g^{(1)}(\tau)$ $= 1$ → sine wave.

$g^{(2)}(\tau)$ $\tau \ll \tau_c$ $= 2$ thermal

$g^{(2)}(\tau)$ $= 1$ → sine wave



excess fluctuations always present for incoherent summed light!

Example astronomical applications of $g^{(1)}$ + $g^{(2)}$

→ starlight transverse coherence:

~~take R ≈ ... to be the size of the Earth (By double-slit interferometer)~~

star diameter $d_{star} \simeq$ $\qquad$ 1 lt. year $\simeq 10^{16}$ m

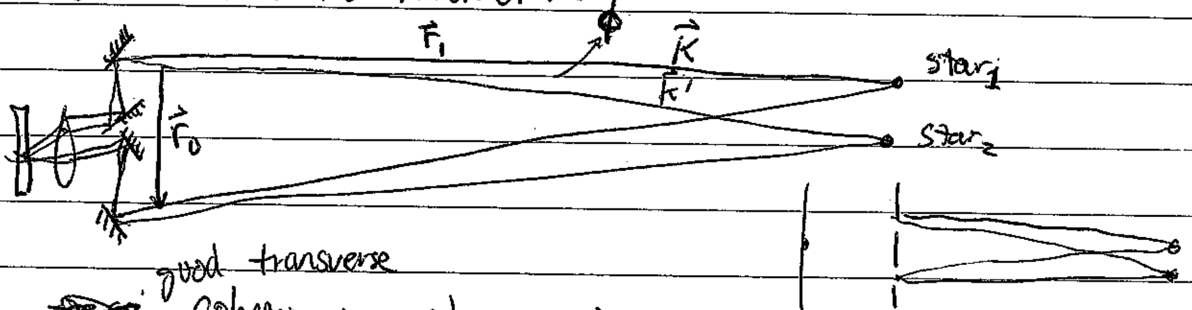
binary star spacing $\Delta d_{star} \simeq$ $\qquad$ $d_{star} \simeq 10^{9}$ m

$\lambda \simeq 10^{-7}$ m

transverse coherence length $\simeq R\left(\frac{\lambda}{a}\right) \simeq$ 1m

---

Michelson stellar interferometer.



good transverse coherence: plane waves.

$$I \propto \langle E^* E \rangle = \langle | E_k (e^{ik\cdot r_1} + e^{ik\cdot r_2}) + E_{k'} (e^{ik'\cdot r_1} + e^{ik'\cdot r_2}) \rangle$$

$$\propto \langle 2|E_k|^2 + |E_{k'}|^2 + |E_k|^2 \left(e^{ik\cdot(\vec{r}_1-\vec{r}_2)} + e^{-ik(\vec{r}_1-\vec{r}_2)} \right)$$

$$+ |E_{k'}|^2 \left(e^{ik'(\vec{r}_1-\vec{r}_2)} + e^{-ik'\cdot(\vec{r}_1-\vec{r}_2)} \right) \rangle$$

(assumes light is filtered so $\nu_k = \nu_k'$, so no relative time dependence.)

Assuming thermal light. $\langle E_k \rangle = 0$

$\langle E_k^* E_{k'} \rangle = 0$ (uncorrelated sources)

$$I \propto 2I_0 \left(2 + \cos(k\cdot(r_1 - r_2)) + \cos(k' \cdot (r_1' - r_2')) \right)$$

$$= 4I_0 \left(1 + \cos(k+k')\cdot\left(\frac{\vec{r}_1 - \vec{r}_2}{2}\right) \times \underbrace{\cos(\vec{k} - k')}_{} \cdot \frac{(r_1 - r_2)}{2} \right)$$

difference in k-values, ie angle

$\vec{r}_1' - \vec{r}_2' = \vec{r}_1 - \vec{r}_2 = \vec{r}_0$

$$= 4I_0 \left(1 + \cos\left((\vec{k} + \vec{k}') \cdot \frac{\vec{r}_0}{2}\right) \cos\left((\vec{k} - \vec{k}') \cdot \frac{\vec{r}_0}{2}\right) \right)$$

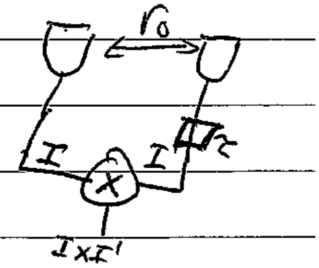
$\Delta k \simeq \phi k \qquad I = 4I_0 \left(1 + \cos\left((\vec{k} + \vec{k}') \cdot \frac{\vec{r}_0}{2}\right) \cos\left(\frac{\pi r_0 \phi}{\lambda}\right) \right) \qquad \Delta r_0 \gg \lambda$ → sensitive.

→ can be used to measure the beating of the two interferences against each other → get the angle

There is a drawback: $\vec{k} + \vec{k}'$ is a high fringe term, very sensitive to atmospheric turbulence.

Hanbury Brown - Twiss.

merely place two photodetectors separated by r_0



$$\cancel{I_i = |E_k|^2 + |E_{k'}|^2 + E_k E_{k'}^* e^{i(k-k')r}}$$

$$I_i = \left| E_k \left(e^{ik\cdot\vec{r}_1} \right) + E_{k'} e^{i\vec{k}'\vec{r}_1'} \right|^2 = |E_k|^2 + |E_{k'}|^2 + E_k E_{k'}^* e^{i(k\cdot r_1 + k' r_1')}$$

$+ c.c.$

$$\langle I(r_1,t)\,I(r_2,t)\rangle = \langle\Big(|E_k|^2 + |E_{k'}|^2 + E_k E_{k'}^* e^{i(k\cdot r_1 - k'\cdot r_1')} + E_k^* E_{k'} e^{-i(k\cdot r_1 - k' r_1')}\Big)$$

$$\times\ \Big(|E_k|^2 + |E_{k'}|^2 + E_k E_{k'}^* e^{i(k\cdot r_2 - k' r_2')} + E_k^* E_{k'} e^{-i(k\cdot r_2 - k' r_2')}\Big)\rangle$$

$$= \langle\left(|E_k|^2 + |E_{k'}|^2\right)^2 + |E_k|^2|E_{k'}|^2\left(e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2) - \vec{k}'(\vec{r}_1{}' - \vec{r}_2{}')} + \text{c.c.}\right)\rangle$$

terms that average to zero have been dropped.

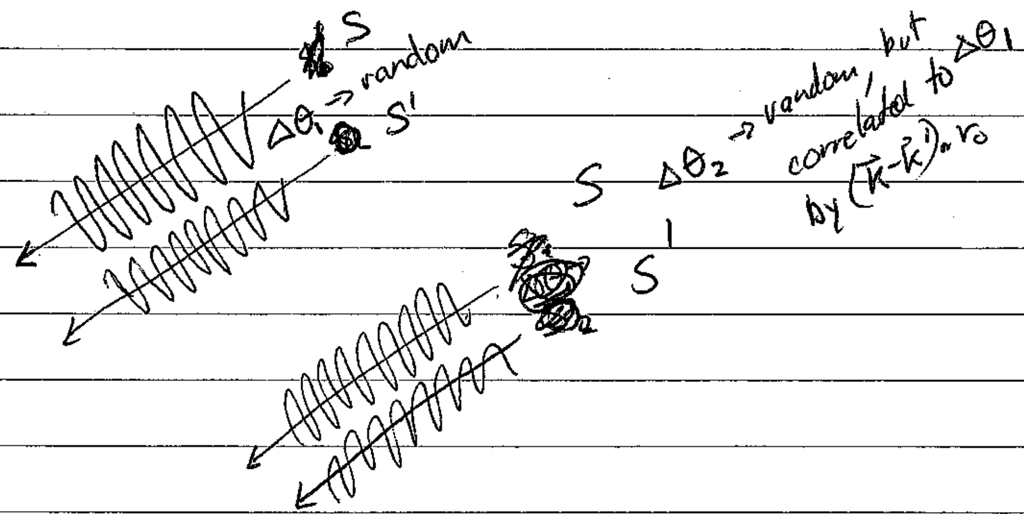
$$= I_1 I_2\, g^{(2)}(\vec{r}_1, \vec{r}_2, \cancel{0})$$

$$\langle I(r_1,t)\,I(r_2,t)\rangle = \overbrace{\langle\left(I_1 + I_2{}'\right)^2\rangle + I_1 I_2{}' \cos\left((\vec{k}-\vec{k}')\vec{r}_0\right)}$$

interference! and no $\vec{k}+\vec{k}'$ term!

→ this caused a stir: multiplication is in the photo current electronics. How can they "interfere"?

Noise-correlation interferences.



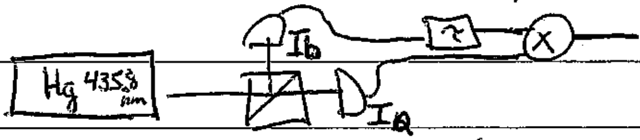
When I_1 is large, I_2 is large & vice versa for $(\vec{k}-\vec{k}')\cdot r_0 = 2\pi$

Will be true for any amount of $g^{(2)}$

~~How do we deal with quantum light?~~

~~replace $E \longrightarrow \hat{E} = \sum_k \mathcal{E}_k \hat{e}_k (a_k$~~

effects of detector bandwidth: the original H-B-T experiment used light from mercury lamp at 435 nm and used a 50-50 beam splitter.



For ideal detectors (infinite bandwidth) this will be

$$\langle \bar{I}_a(t) \bar{I}_b(t+\tau) \rangle = \frac{1}{4} \langle \bar{I}_1(t) \bar{I}_2(t+\tau) \rangle$$

$$= \frac{1}{4} \langle \bar{I}_1 \rangle^2 \, g^{(2)}(\tau)$$

(actually the experiment is setup to ~~extract~~ subtract the average)

$$\propto g^{(2)}(\tau) - 1$$

however if the detectors have finite bandwidth (as the first ones surely did for H-BT) must average.

e.g. Collisionally broadened light

$$g^{(2)}(\tau) - 1 = |g^{(1)}(\tau)|^2 = e^{-2|\tau|/\tau_c}$$

$$\left\langle g^{(2)}(0) - 1 \right\rangle_T^{BW} = \frac{1}{T^2} \int_0^T dt_a \int_0^T dt_b \, e^{-2|t_a - t_b|/\tau_c}$$

$T \ll \tau_c \rightarrow 0 \; \langle g^{(2)}(0) \rangle^{BW} = 1$

$$= \frac{\tau_c^2}{2T^2} \left(e^{-\frac{2T}{\tau_c}} - 1 + \frac{2T}{\tau_c} \right)$$

for HBT $\tau_c / T \simeq 10^{-5}$

$T \gg \tau_c \rightarrow \frac{\tau_c}{T}$ down by the ratio.

~~Before looking at time correlation of intensities (instead of fields)~~ Look at time-averaged moments of the intensity.

$\langle I(t) \rangle \rightarrow$ average intensity, $\sum_{atoms} I_i$ intensity from each independent source.

$\langle I(t)^2 \rangle_{\Delta T=0} = 2\langle I \rangle^2 \Rightarrow$ true for all random sources of light "chaotic", consequence of assuming collections of atoms radiate random rel. phases.

full fluctuation

$\Delta I^2 = \langle I(t)^2 \rangle - \langle I(t) \rangle^2 = \langle I \rangle^2 \rightarrow$ fluctuations equal the average value.

(remember the thermal distribution of photons?

$\rightarrow \overline{n^2} - \bar{n}^2 = \bar{n}^2$

most likely value is 0, somewhat counter intuitive.)

~~Defg~~ note this is not a measure of $g^{(1)}(\tau)$, this is an equal time measure of I

Intensity + Field fluctuations of chaotic light → probability distributions.

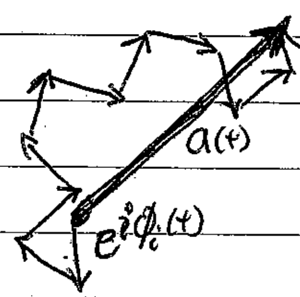
$$\frac{\langle (\bar{I}(t) - \langle \bar{I}(t) \rangle)^2 \rangle}{\langle \bar{I} \rangle^2} = \frac{\langle \bar{I}^2(t) \rangle - \langle \bar{I} \rangle^2}{\langle \bar{I} \rangle^2}$$

$$= g^{(2)}(0) - 1 = 1 \text{ for chaotic light}$$

(large number of random sources)

$= 0$ for pure sinewave.

What is the probability distribution of fields for ~~[illegible]~~ chaotic light?



$E_0 a(t)$ is total field of N radiating sources with random phases.

For N large, this is a 2D random walk problem, with distribution

$$\underbrace{p[a(t)] = \frac{1}{\pi N} e^{-(a^2(t)/N)}}_{\text{Gaussian distribution of Field amplitudes.}}$$

(not to be confused with the Gaussian or Lorentzian time ~~dependence~~ correlation dependence for different kinds of chaotic light) this is the probability distribution averaged over all time of E values.

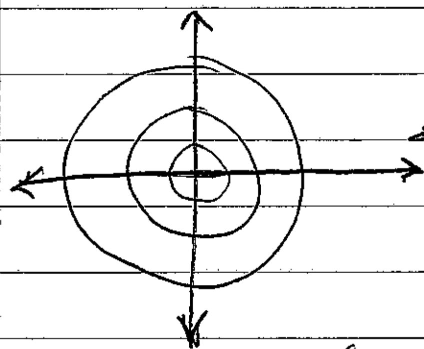
$E_0(t) = E_0\, \hat{a}(t)$ is the amplitude (not $e^{i\omega t}$ time dependence.)

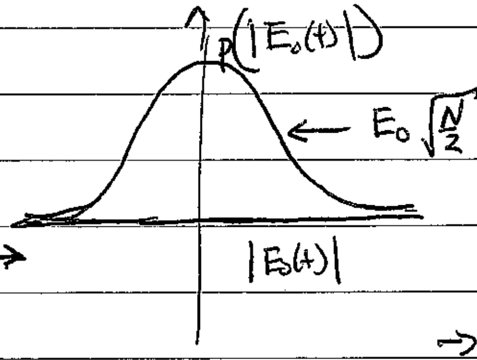
a is complex. $\quad Re[E_0(t)]$ = cos quadrature

$Im[E_0(t)]$ = sin quadrature

$|E_0(t)|$ is the ~~size of~~ mag. of field

$p[\,|E_0(t)|\,] = \rightarrow$ gaussian with most probable value 0!





$\rightarrow$ N randomly phased radiators give an average field of $\sqrt{\frac{N}{2}}\, E_0$

$$\left(\Delta |E_0(t)|^2\right) = \tfrac{1}{2} E_0^2 N$$

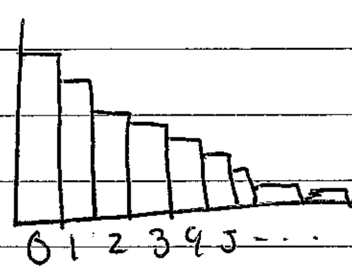
N in-phase radiators give $N E_0$

$$I = \tfrac{1}{2}\epsilon_0 c\,|E(t)|^2, \quad p[I] \text{ is } \frac{1}{\bar{I}}\, e^{-\frac{I(t)}{\bar{I}}}$$

exponential.

again, the most probable value is $I(t) = 0$!

Recall that a thermal distribution of photons looks similar



Or