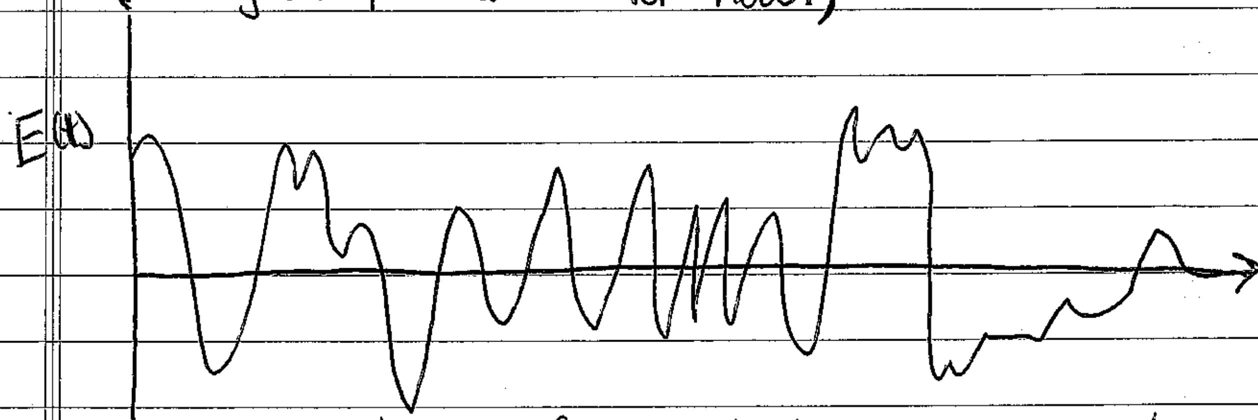
Classical Theory of Coherence:

First: "There is no general agreement on the precise meaning of the term coherence."

Usually "coherence" is used to describe light that shows some kind of stable interference in an experimental set up.

We will try to describe "coherence" properties in terms of correlation functions.

Given a point in space where a light field is passing, how can we characterize this field? (for now, look only at the time dependence of the field at a single point, so-called longitudinal coherence, ignore transverse directions) (Also ignore polarization for now.)



All light comes from material sources: convenient to consider sources when describing idealized sources of light

How can we characterize this EM field?

→ It will be convenient to use statistical argument.

- We could specify $E(t)$ in its entirety.
  - but there aren't any convenient $E(t)$ measurement devices in optical regime.
  - requires lots of detailed information that may not be physically relevant
  - ignores the fact that light from real sources may have statistical properties that simplify discussion.

- We could specify the Fourier transform in its entirety. → $\tilde{E}(\omega)$
  → similar measurement difficulties, need phase information

Detectors measure intensity: $I \propto |\vec{E}|^2$, usually averaged over some bandwidth, $I \propto \langle |\vec{E}|^2 \rangle_\tau$
↳ necessarily throws away some information.

- Specify $I(t)$ → detector bandwidth not large enough to include optical frequencies. → but can be done for lots of sorces of noise
→ provides noise information

Specify $\tilde{I}(\omega)$ this can be done to some degree, eg using optical spectrum analyzers, or FFTing $I(t)$.

→ Note: any time dependence other than sine or cosine implies spectral width to $\tilde{E}(\omega)$, $\tilde{I}(\omega)$.

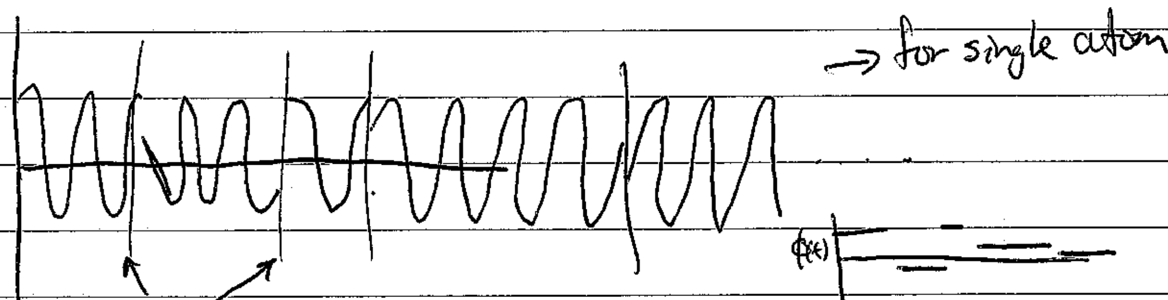
Look at ~~these~~ different types of light.

~~naturaly averaged: $\langle I(t) \rangle_{\Delta T}$ over some time scale (detector bandwidth), always much slower than $\omega_0 = c/\lambda$~~

~~example of $I(t)$ for different sources:~~

Chaotic light - light from a collection of independent sources.

example collisional broadening: elastic collisions (light scattered from atoms that collide.) are very brief, discontinuously adjust the phase.



discontinuous phase jumps occur with probability $p(t) = \frac{1}{\tau_{col}} e^{-(t/\tau_{col})}$, $\tau_0 = 4\sigma_{col}\, n\, V_{th}$

σ_{col} is collisional cross section

typically $\omega_0 \tau_0 \simeq 10^5$ $\qquad$ n is density $V_{th} = \sqrt{\frac{\pi k_B T}{m}}$

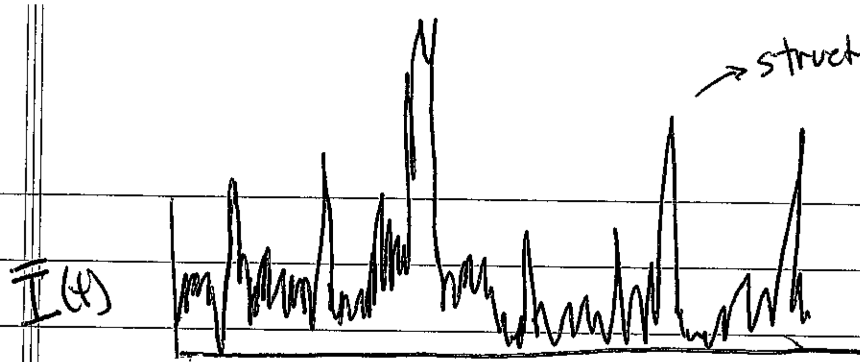
$$E(t) = \sum_{atoms} E_i(t)$$

$$= E_0 e^{-i\omega_0 t} \sum_i e^{i\phi_i(t)} = E_0 e^{-i\omega_0 t} \underbrace{a(t) e^{i\phi(t)}}$$

random walk in phase space.

I is cycle averaged intensity

→ structure depends on averaging time



→ large intensity fluctuations compared to $\langle I \rangle_\infty$

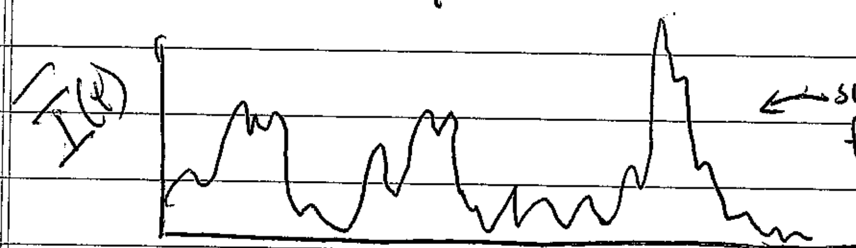
many frequency components.

the Frequency dependence of this light intensity is Lorentzian; wide wings → lots of freq. components.

$$\bar{I}(\omega) \propto \frac{\Gamma + \Gamma_{col}}{\delta^2 + \frac{\Gamma+\Gamma_{col}}{4}\left(1 + \frac{\Gamma_{col}}{\Gamma}\frac{I}{I_s}\right)} \qquad \Gamma_{col} = \frac{1}{\tau_{col}}$$

→ Doppler broadening.

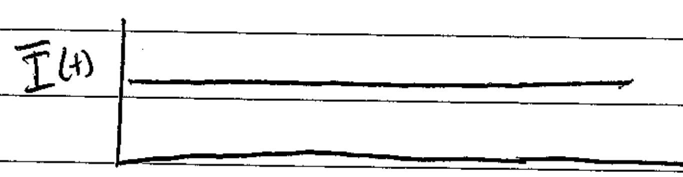
$$E(t) = \sum_i E_0 e^{i\omega_i t} \quad \rightarrow \omega_i \rightarrow \text{thermal distribution}$$



← sum over thermally distributed frequencies

as we know from homework,
→ gives rise to gaussian frequency → smaller wings - less freq. components → smoother

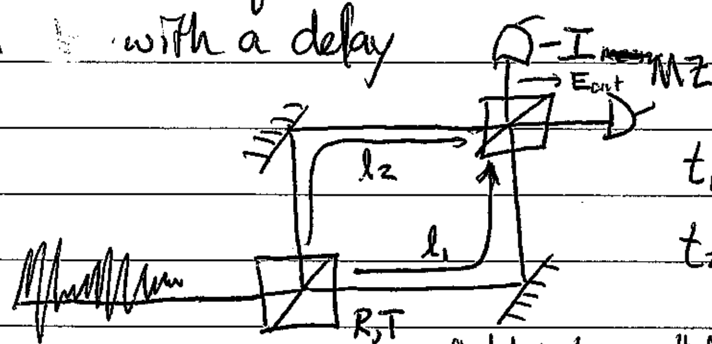
Pure sine wave (classically possible, n.b <u>not</u> possible QM.)



~~FDD~~ FT → δ-function

In practice, $\bar{I}(t)$ fluctuates more rapidly than can easily be detected. plus, we want to know about coherence + interference.

Use an interferometer to self-reference the beam with a delay



$t_1 = t - (l_1/c)$

$t_2 = t - (l_2/c)$

→ reflected + transmitted fraction

$$E_{out} = RT\, E(t_1) + TR\, E(t_2)$$ → path length delay gives access to very short time-scales!

$$\bar{I}_{out_{MZ}} = \tfrac{1}{2}\epsilon_0 c R^2 T^2 \left(|E(t_1)|^2 + |E(t_2)|^2 + 2\mathrm{Re}\left(E^*(t_1) E(t_2)\right) \right)$$

→ average over bandwidth of the detector: assume $R^2 T^2 = (\frac{1}{2})(\frac{1}{2})$

$$\langle \bar{I}_{out_{MZ}} \rangle = \frac{\langle I(t_1)\rangle + \langle I(t_2)\rangle}{4} + \frac{\epsilon_0 c}{4} \mathrm{Re}\langle E^*(t_1) E(t_2)\rangle$$

$\langle I(t)\rangle$ is just the intensity averaged if there were no interferometer. in the limit of infinite averaging time → I_0

$$\langle E^*(t_1) E(t_2)\rangle = \frac{1}{T}\int_T dt_1\, E^*(t_1) E(t_2)$$ for stationary sources

$$\langle E^*(t) E(t+\tau)\rangle = \frac{1}{T}\int_T dt\, E^*(t) E(t+\tau)$$

if T >> correlation times T << drift times → good measure nice measurement, because even in the limit of infinitely slow detectors, we get a correlation measurement out: statistical average over light properties that differ in time by $t = \frac{1}{c}(l_1 - l_2)$

→ define first order coherence:

$$g^{(1)}(\tau) = \frac{\langle E^*(t) E(t+\tau) \rangle}{\langle I \rangle} \qquad 0 \le g^{(1)}(\tau) \le 1$$

For stationary light (independent of t)

$$g^{(1)}(-\tau) = \frac{\langle E^*(t) E(t-\tau) \rangle}{\langle I \rangle} = \frac{\langle E^*(t+\tau) E(t) \rangle}{\langle I \rangle} = g^{(1)}(\tau)^*$$

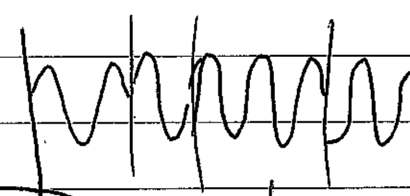
real part of $g^{(1)}$ is the same at $+\tau$ & $-\tau$

$$\tau = \frac{1}{c}(l_1 - l_2)$$

$$\langle I_{\substack{out \\ MZ}} \rangle = \frac{1}{2} \langle I(t) \rangle \left(1 + Re[g^{(1)}(\tau)]\right)$$

examples:

collision broadened:



$$\langle E_i^*(t) E_i(t+\tau) \rangle = E_0^2 \underbrace{e^{-i\omega_0 \tau}}_{\text{coherent part}} \langle e^{i(\phi_i(t+\tau) - \phi_i(t))} \rangle$$

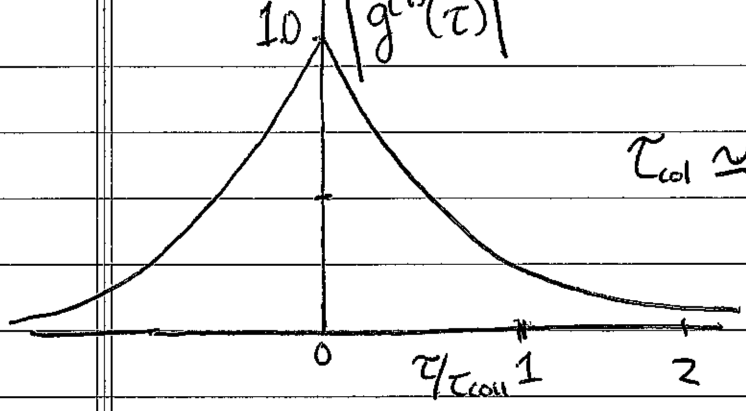
completely random phase after each collision

$$= E_0^2 e^{-(i\omega_0 \tau + \tau/\tau_{coll})}$$

probability that $\phi(t+\tau)$ & $\phi(t)$ are uncorrelated is $1 - \int_0^\tau (p(t))dt$

summing over atoms & divide

$$g^{(1)}(\tau) = e^{-i\omega_0 \tau - |\tau|/\tau_0}$$



$\tau_{col} \simeq$

Tells over what range interference fringes can be seen.

Example:
For Doppler broadened sources:

$$\langle E^*(t)\, E(t+\tau)\rangle = \sum_{atoms} E_0\, e^{-i\omega_i \tau}$$

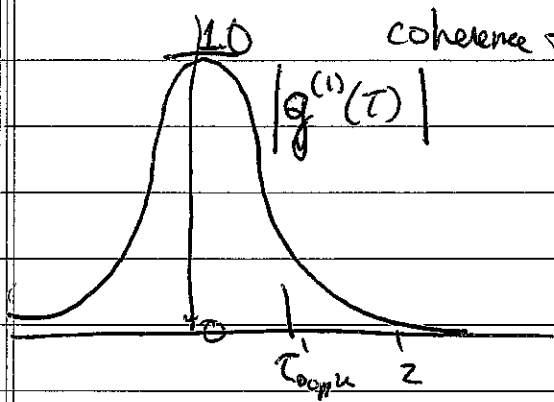
, averaged all other cross terms to zero.

ω_i is averaged over a Doppler spectrum

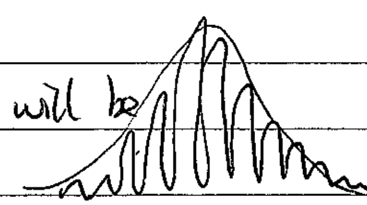
$$g^{(1)}(\tau) = e^{-i\omega_0\tau - \frac{1}{2}\Delta^2\tau^2}$$

$\Delta \to$ width of Doppler spectrum.

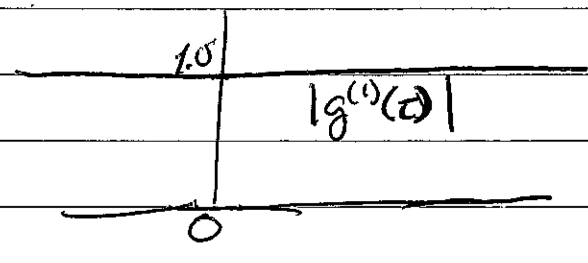
coherence time can be define as $\tau_{Doppler} = \frac{\sqrt{\pi}}{\Delta}$



$\to$ actual fringes will be



Example:
Sine wave $\quad g^{(1)}(\tau) = e^{-i\omega_0\tau}$



There is a connection between 1^{st} order correlation function and the frequency spectrum:
Power

$$\tilde{E}_{\Delta T}(\omega) = \int_{\Delta T} dt \; E(t)\sin(\omega t)$$, assume $\Delta T \gg \tau_{coll}$ or $\tau_{doppler}$

Power spectrum

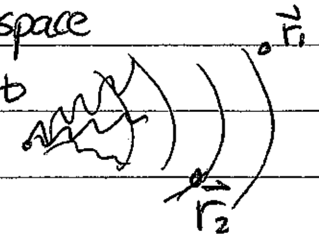
$$f(\omega) = \frac{|E_T(\omega)|^2}{T} = \frac{1}{2\pi T}\int_{\Delta T} dt \int_{\Delta T} dt' \; E^*(t)\, E(t')\, e^{-i\omega(t-t')}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} d\tau \langle E^*(t)\, E(t+\tau)\rangle e^{i\omega\tau}$$

$$F(\omega) = \frac{f(\omega)}{\int_{-\infty}^{\infty} f(\omega)d\omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\tau \; g^{(1)}(\tau)\, e^{i\omega\tau}$$

$\rightarrow$ Wiener-Kintchine theorem.

This can be generalized to different points in space for non-plane-wave EM fields (diffraction to propagation included.



$$g^{(1)}(\vec{r}_1, \vec{r}_2, \tau) = \frac{\langle E^*(\vec{r}_1, t)\, E(\vec{r}_2, t+\tau)\rangle}{\sqrt{\langle |E(\vec{r}_1, t)|^2\rangle \langle |E(\vec{r}_2, t+\tau)|^2\rangle}}$$

$\rightarrow$ independent of t

$\hookrightarrow$ double-slit diffraction and Michelson interferometer are the same.

$\rightarrow$ Transverse coherence.

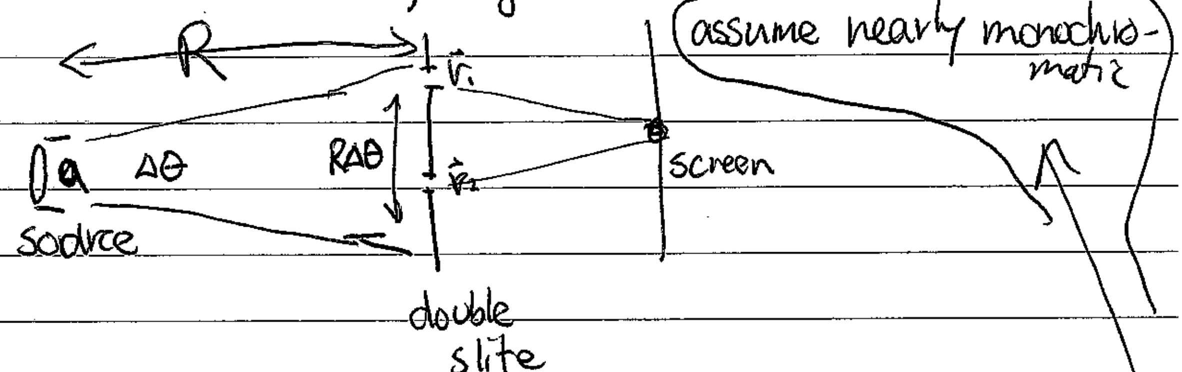
→ we can define a coherence length ~~or~~ ~~coherence volume~~

τ_{coll}, $\tau_{doppler}$ - coherence time

$l = c\,\tau_{coll}$ = coherence length

→ ~~with full definition of $g^{(1)}(\vec{r}_1, \vec{r}_2, \tau)$ we~~

Transverse coherence, e.g. from a star



when will you get interference? ~~when~~ assume equal time measurement to see purely geometric averaging effects $g^{(1)}(\vec{r}_1, \vec{r}_2, \tau)$ $\tau = \frac{|\vec{r}_1 - \vec{r}_2|}{c}$ equivalent to

when

$a\Delta\theta \lesssim \lambda$. Choose $\Delta\theta$ exactly at coherence width

So the area of coherence is $(R\Delta\theta)^2 \sim R^2\left(\frac{\lambda}{a}\right)^2$

Coherence volume $\Delta V = A\,l_{c}$

$$= \left(\frac{c^2}{\nu^2}\right)\frac{R^2}{A^2}$$ → Area of object emitting light.

coherence area scales like R^2 → farther away, better interference.

→ one can approximate deterministic light with a laser.
one can also try to filter thermal light to get coherence → hard to do and you lose power.

examples:

best thermal light	$\Delta\nu \sim 10^8$ Hz
laser light.	$\Delta\nu \sim 0.1$ Hz

Coherence times	thermal	10^{-8} sec
	laser	10 sec
Coherence lengths		3m
		3×10^9 m

First order correlation gives alot of information about the coherence properties of a source, but does not specify everything!
Statistical information

What about correlations of intensities? (like shown in figures of $I(t)$)

→ consider chaotic light.