Lecture #21 10 Nov. 2005

so $k_B T = \frac{D_p}{\alpha}$

(This is from Einstein's 1905 theory of Brownian motion.

Now we will calculate D_p and obtain the temperature.

The fluctuations have two sources: the randomness of spontaneous emission and randomness of absorption.

spontaneous emission: each emission is in a random direction, so the atomic momentum does a random walk.

after N emissions

$$\langle p^2 \rangle = N (\hbar^2 k^2)$$

↑ square of single photon impulse

$$\langle \dot{p}^2 \rangle = R (\hbar k)^2$$

↑ scattering rate

so $D_{p,\,spont} = \frac{1}{2} \langle \dot{p}^2 \rangle = \frac{1}{2} R (\hbar k)^2$

Absorption: fluctuation in the direction from which the photons are absorbed causes a random walk

$$\langle p^2 \rangle = \mathcal{R} t \hbar^2 k^2$$

so $D_{p,abs} = \frac{1}{2} \mathcal{R} \hbar^2 k^2$

Aside: some special and not entirely correct assumptions have been used:

We are assuming a 1-D random walk for both absorption and emission impulses.

For absorption, this is OK. For emission it is <u>not</u>. But let's ignore this for the moment and assume this 1-D world where emissions are along the 1-D axis.

We assumed a Poisson distribution to get this result — the fluctuations in N photons absorbed are $\sqrt{N}$ — the probability of absorb in any given interval is independent of what happened before — not true at high intensity.

If we have a standing wave, there will be fluctuations of the dipole force.

Note that fluctuations in the direction of absorption is equivalent to fluctuations in the number of photons absorbed per unit time — the fluctuations would be there even if the absorption was from a single beam!

Let's ignore all these problems and calculate the temperature under these assumptions.

$$D_{p,tot} = D_{p\,spont} + D_{p\,abs.} = \kappa \hbar^2 k^2$$

$$\kappa = \frac{\Gamma}{2} \frac{I/I_0}{1+(2\delta/\Gamma)^2} \times 2 \leftarrow \text{2 beams}$$

(we are assuming $I/I_0 \ll 1$ here)

$$D_p = \frac{\Gamma I/I_0 \hbar^2 k^2}{1+(2\delta/\Gamma)^2} \quad , \quad \alpha = \frac{-4\hbar k^2 (2\delta/\Gamma) I/I_0}{[1+(2\delta/\Gamma)^2]^2}$$

$$k_B T = \frac{D_p}{\alpha} = \frac{\hbar\Gamma}{4} \frac{1+(2\delta/\Gamma)^2}{(2\delta/\Gamma)}$$

this is the Doppler cooling temperature. It minimizes at $2\delta/\Gamma = 1$ where

$$k_B T = \frac{\hbar\Gamma}{2} \quad , \text{ the Doppler cooling limit}$$

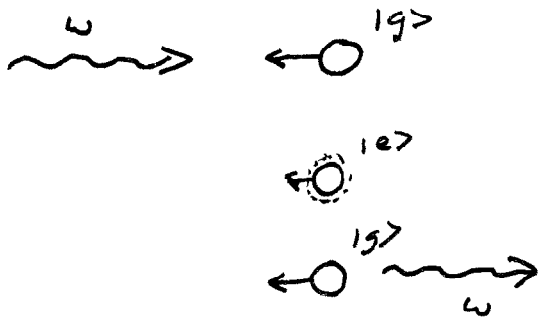
for Na $\Gamma/2\pi = 10$ MHz

$$T_{Dopp,\,min} = 240\,\mu K$$

for ^{23}Na the $\langle v^2 \rangle^{1/2} \simeq 30$ cm/s at this temperature (v_{rms} along one axis), compared to $v \approx 1000$ m/s at $T \approx 1000$ K.

Where does the energy go?

1) As the atoms are cooling to equilibrium, the red-detuned laser beam is scattered at high frequency on average than the input frequency



only if the radiated photon goes in the same direction as the incoming photon will the frequency be as low as the incoming frequency (for velocity of atom opposite $\hat{}$ incoming photon propagation ($\vec{k} \cdot \vec{v} < 0$). All other directions will slow the atom and radiate a photon of higher energy than the incoming photon.

2) at equilibrium, the incoming and outgoing frequencies are the same on average. Heating due to recoil and cooling are balanced.

Note that when the laser is on resonance, it heats the atoms, radiating a photon redder than input by, on average, 2 recoil energies. (It recoils on absorption and again, randomly, on emission, gaining $\rightarrow E_{rec}$ on average per absorption–emission cycle.)

Recall that we assumed in our calculation the $kv \ll \Gamma, \delta$. If we take $\delta = \Gamma/2$ for best cooling then these two conditions will be essentially the same.

Is it so?

for Na: $v_{rms} = 30$ cm/s

$\Gamma/k = 600$ cm/s

$kv/\Gamma \approx 1/20$

for Cs $v_{rms} \approx 9$ cm/s

$\Gamma/k \approx 460$ cm/s

$kv/\Gamma \approx 1/50$

so, it looks pretty good.

Is the velocity distribution a thermal (Maxwell - Boltzmann) one? — is the a temperature?

for $v_{rec} \to 0$ we can write a differential equation for the damping and the heating (with a velocity-dependent damping term and a velocity-independent fluctuating force term) this is called a Fokker-Planck equation.

The F-P equation is the same as one would write for a gas in equilibrium due to collisions between the gas atoms, and it leads to a M-B distribution.

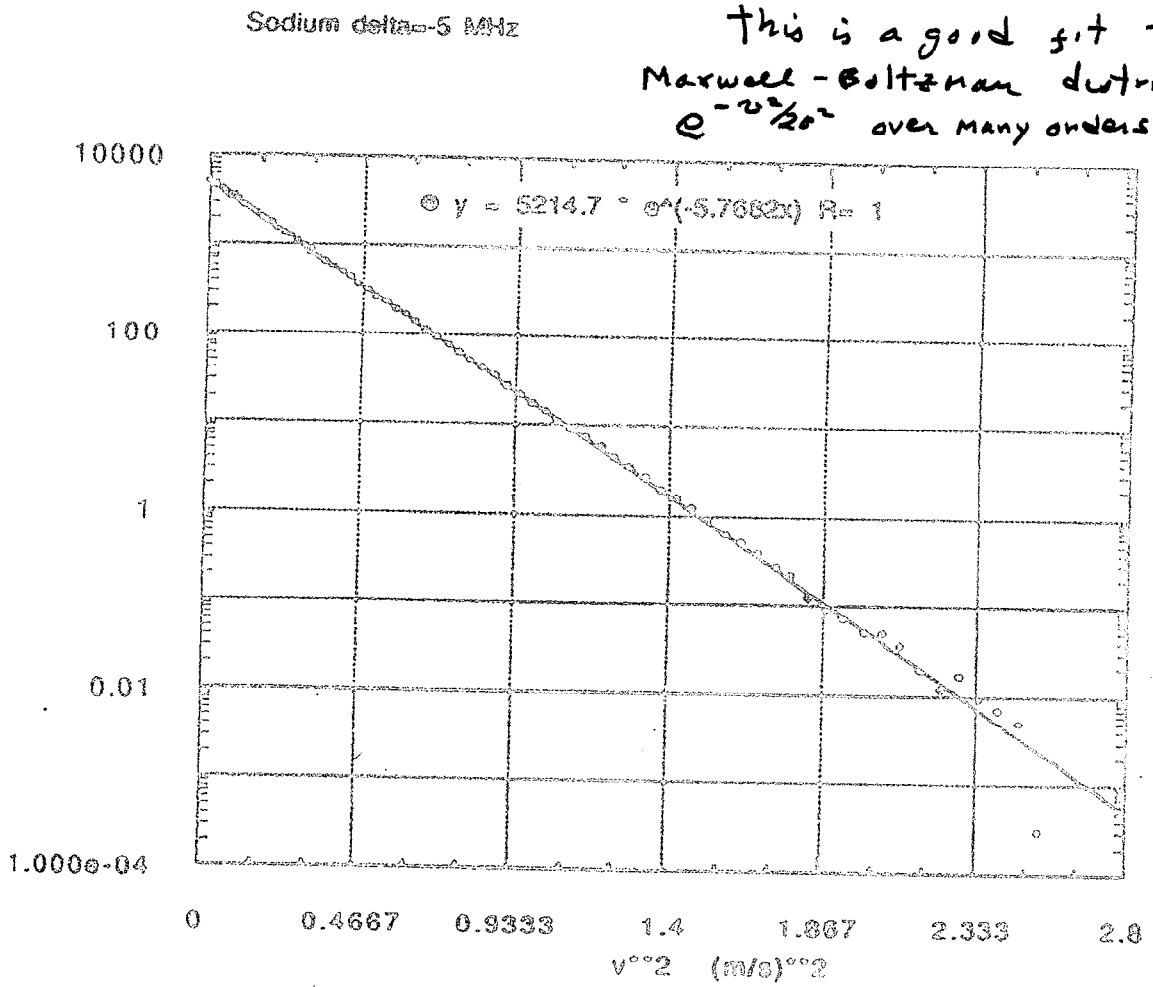
Is $v_{rec} \approx 0$ a good approximation?

for Na $\quad \frac{v_{rec}}{v_{rms}} \approx \frac{1}{10}$

for Cs $\quad \frac{v_{rec}}{v_{rms}} \approx \frac{1}{25}$

How good is the approximation that predicts Maxwell Boltzman?

A numerical simulation in which a Na atom absorbs and emits randomly with a detuning $\delta = \Gamma/2$ leads to a velocity distribution, after many cycles, recording the frequency of occurrence of a given velocity:



this is a good fit to a Maxwell-Boltzman distribution $e^{-v^2/2\sigma^2}$ over many orders of magnitude

What happens in reality?

If we do 1-D cooling, measuring the temperature along the 1-D axis, we expect a slightly lower temperature — because some of the emissions are not along the 1-D axis

But in 3-D, the emissions in directions other than those for absorption are in the directions of other beams. — for a symmetric situation we expect the calculated Doppler limit at low intensity

What about non-negligible intensity. Using the uniform beam model, following exactly the same procedure as in the preceding lecture we get

$$\alpha = \frac{-4\hbar k^2 (I/I_0)(2\delta/\Gamma)}{\left[1 + 2I/I_0 + \left(\frac{2\delta}{\Gamma}\right)^2\right]^2}$$

and $$D_p = \hbar k^2 \frac{\Gamma}{2} \frac{2 I/I_0}{1 + 2I/I_0 + \left(\frac{2\delta}{\Gamma}\right)^2}$$

We can maximize the expression for α with respect to both I/I_0 and $2\delta/\Gamma$ by taking the partial derivatives

$$\frac{\partial \alpha}{\partial (I/I_0)} \quad \text{and} \quad \frac{\partial \alpha}{\partial (2\delta/\Gamma)}$$

and setting both to zero

giving $\alpha = \frac{\hbar k^2}{4}$ for $\frac{2\delta}{\Gamma} = -1$ and $I/I_0 = 1$

for Na (in 1-D) this is a damping time for velocity $\gamma^{-1} = 13\,\mu s$.

$$\gamma = \alpha/M$$

$\gamma = \frac{\hbar k^2}{4M}$ for optimal damping

$= \frac{1}{2} \frac{E_{rec}}{\hbar}$

recall $F = -\alpha v$

$$a = F/M = -\frac{\alpha}{M} v = \frac{dv}{dt}$$

$$\dot{v} = -\gamma v$$

If we use the alternating beam model in 3D, we get 3 times less damping (for optimal damping) and 3 times longer damping time.

Still — the cooling happens fast — on a time scale ~ τ_{exc}

If we choose the parameter for maximum damping and calculate the temperature.

$$k_B T = \frac{D_p}{\alpha} = \frac{\hbar\Gamma}{4} \frac{1 + 2I/I_0 + (2\delta/\Gamma)^2}{2\delta/\Gamma} \qquad (\text{in } 1\text{-D})$$

we find, for $I/I_0 = 1$, $2\delta/\Gamma = -1$

$$k_B T = \hbar\Gamma \quad (\text{twice the low-intensity minimum temperature})$$

what is the actual result in the lab?

Looking at the velocity distribution of atoms released from optical molasses, we found in 1988

$$T_{observed} \ll T_{dopp,\,min}$$

eventually, when we understood what was happening

$$T_{obs} \approx \frac{1}{200} T_{dopp,\,min} \text{ for Cs.}$$

this is a story for another lecture.

Spatial diffusion in optical molasses

an atom with velocity v_{rms} has its velocity damped in a time γ^{-1} the distance traveled is $l \sim v_{rms}\gamma^{-1}$

taking this as a random walk step that is repeated $t_{diff}\gamma$ times (number of steps of time-duration γ^{-1} in a diffusion time) the distance r diffused is given by

$$\langle r^2\rangle \approx l^2 t_{diff}\cdot\gamma = \frac{\langle v^2\rangle}{\gamma}\cdot t_{diff}$$

recalling that $k_B T = D_p/\alpha = M\langle v^2\rangle$
and $\gamma = \alpha/M$

$$\frac{\langle v^2\rangle}{\gamma} t_{diff} \approx \frac{D_p}{\alpha^2} t_{diff} \approx \langle r^2\rangle$$

taking optimum damping for 1-D $\quad \alpha = \frac{\hbar k^2}{4}$

$$D_p = \frac{\hbar^2 k^2 \Gamma}{4}$$

$$\langle r^2\rangle \approx t_{diff}\cdot 4\frac{\Gamma^2}{k^2}$$

$$\langle r^2\rangle = t_{diff}\cdot 4 v_p\cdot\lambda \qquad \Gamma/k = v_p = 6\,cm/s \text{ for } Na$$

in 1 sec. $\langle r^2\rangle^{1/2} \approx \left(4\cdot 6\,cm/s\cdot 1\,sec\cdot 10^{-7}\,m\right)^{1/2} \approx$ 2 mm

by comparison in 1 sec. v_{rms} takes you 300 mm. — the molasses effect!

The fact that the spatial motion is diffusive and atoms diffuse characteristic distances much slower than ballistic times is characteristic of optical molasses.

Note that we only did an order of magnitude calculation — real 3-D 2-level molasses won't be quite as sticky, but multilevel molasses is even stickier!!

But — how do atoms get cold enough to feel the force of molasses?

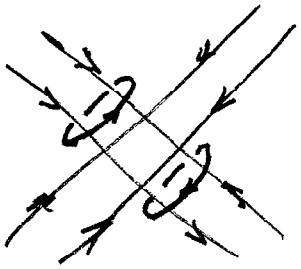
for optimum detuning $\delta = \Gamma/2$, the force is strong only for $|kv| \lesssim \Gamma$

for Na $\Gamma/k = 6$ m/s but v_{rms} at "typical" temperatures is 500 m/s
so, few atoms have "coolable" velocities

There are several approaches that are used in practice

- deceleration of a beam: chirp or Zeeman
- collection from vapor

collection from vapor.



magneto-optical trap

atoms with $v \leq v_{capture}$ are slowed + trapped in the MOT - they collect there.

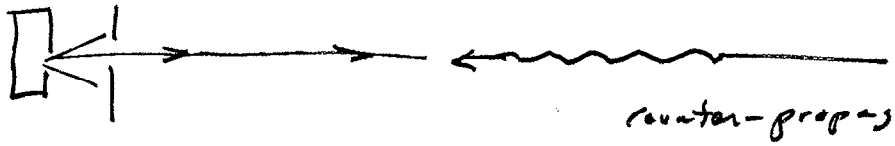
speed distribution ~ $v^2 e^{-v^2}$ integrates to available atoms ~ $v_{capture}^3$, so a larger volume, giving larger capture velocity increases the number of atoms rapidly.

balance of rate of capture ~ density of vapor and loss due to background ~ density of vapor.

so # of atoms is roughly independent of vapor density, but speed of filling goes up.

sometimes, the pressure is reduced after filling.

beam deceleration



beam source

counter-propagating laser beam.

wide range of velocity $F(\vec{v})dv \sim v^3 e^{-v^2}$ for flux

width ~ ave velocity. – small fraction is resonant, small deceleration possible before going out of resonance.

2 solutions: • chirp frequency of laser up as velocity decreases

• change magnetic field along length of beam to compensate changing Doppler shift.