The Dipole Force - continued

Leftovers from lecture #15

At the end of lecture #15 we had written

\[ \text{Force} = \nabla \vec{E}_0 \cdot \text{Im-phase} \quad \text{which was then written as} \quad -\nabla W \quad \text{with} \quad W = \langle \vec{E} \cdot \vec{d} \rangle \]

This appears to be wrong because both \( \langle \vec{E} \rangle \) and \( \langle \vec{d} \rangle \) depend on \( \vec{E}_0 \) and the correct expression \( \nabla \vec{E}_0 \cdot \frac{\vec{d}}{\text{Im-phase}} \) only has the derivative of \( \vec{E}_0 \).

Indeed, the expression \(-\nabla W\) is incorrect, for exactly that reason. The force is not the gradient of a potential derived from the work done on the atom alone, but on the entire system, atom + field + interaction. The calculation we did was correct, but trying to identify it as the gradient of the energy \( \nabla \vec{E} \) is incomplete.

The field energy involves the fact that in considering moving from one location to another, some photons will be radiated as fluorescence at a frequency different from the laser frequency. This difference goes to a constant as the motion becomes quasi-static.
For a complete description of this issue, see


In particular see sect. 5 on pp. 1711-1712 in which it is
made clear that the "total" energy change
(called $\Delta W$ there) is not equal to the mean potential
energy of the atom (called $U_a$ there), and essentially
like what we were calling $V = \langle V \rangle$). Note particularly
Remark 7 on p. 1712 which underscores the quantization
with position change of the populations of the dressed states, which
is related to the change of the dipole moment with position.

So, the correct expression for the dipole force is

$$ F_{\text{dip}} \text{ (or } F_{\text{rest}}) = -\nabla V \frac{\hbar \mathbf{S}}{\hbar^2} \frac{1}{\mathbf{S}^2 + \mathbf{S}_2 + \mathbf{P}_2^2} $$

as we
derived, and let's forget about interpreting it too globally
as the gradient of a dipole energy, although it is
certainly related to that.

Also: the reference to photoelectric effect without photons:

W.E. Lamb and M.O. Scully, in *Polarization, Matière, et Rayonnement*,
ed. Société Française de Physique, Presses Universitaires de
Returning now to the induced dipole:

the in-phase component of the induced dipole is

\[ \vec{\mu} = \frac{s}{r^3} \left( \vec{S} - \vec{E} \right) \]

remember that \( \vec{E} = -\nabla \times \vec{B} \), so for

\( s < 0 \), the dipole is in the same direction as \( \vec{E} \)

(that is, the in-phase component is in the direction of \( \vec{E} \))

so, the dipole energy \( -\vec{D} \cdot \vec{E} \) is negative, and the force is toward higher field

for \( s > 0 \) the opposite is true

(this change of sign of the force can be seen classically)

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Now consider another special case – a standing wave

\[ \vec{E}(\vec{r},t) = \frac{2E_0}{\kappa^2} \cos(\omega t - \kappa z) + \cos(\omega t + \kappa z) \]

\[ \uparrow + \text{ travelling wave} \]

\[ \uparrow - \text{ travelling wave} \]

\[ \vec{E}(\vec{r},t) = \frac{2E_0}{\kappa^2} \cos(\omega t) \]

\( \text{spherically modulated, } \)

\( \text{oscillating, but with no spatial dependence of the phase} \)
\[ \nabla \Phi = 0 \Rightarrow F_{\text{scat}} = 0 \]

but \( \nabla \Phi \neq 0 \):

\[ -\nabla \Phi E(\mathbf{r}) = \Psi(\mathbf{r}) = -x_0 \cos k\mathbf{e} \quad \text{when} \quad x_0 = -\nabla \Phi \]

\[ \nabla \Phi = -k x_0 \sin k\mathbf{e} \]

\[ F_{\text{scat}} = F_{\text{dipole}} = -\frac{k e}{\Omega} \frac{\Psi}{2} \frac{\nabla \omega}{r^{3/2}} + \frac{\delta \sin k \mathbf{e} \cos k\mathbf{e}}{r^{3/2}} \]

\[ F_{\text{dip}} = \frac{\hbar k x_0^2}{2} \frac{\delta \sin k \mathbf{e} \cos k\mathbf{e}}{r^{3/2}} \]

\[ = \frac{\hbar k x_0^2}{4} \frac{\delta \sin 2k\mathbf{e}}{r^{3/2}} + \frac{\delta \sin k \mathbf{e} \cos k\mathbf{e}}{r^{3/2}} \]

Note that the dipole force can be arbitrarily large as \( x_0 \) is increased.

This force, like the intensity, is modulated at a spatial frequency \( 2k \) (period of \( \pi/2 \))

Now plot \( S^2 \) (proportional to the intensity), the potential \( U \), and the force \( F \) as a function of position.
\[ \frac{\delta^2}{\lambda} \]

\[ \delta > 0 \]

blue detuning

\[ \delta < 0 \]

red detuning
We can interpret the dipole force as arising from (absorption + stimulate emission). The scattering force arises from (absorption + spontaneous emission).

The force must come from stimulated processes since it is not dissipative

Consider $\delta < 0$, so the dipole moment (the in-phase $u$-term part, is along $\vec{E}$). What happens at $\varphi = \pi/8$?

The field is the sum of waves travelling to the right ($+\hat{z}$) and left ($-\hat{z}$). There can be a force due to absorption from one wave and stimulation into the other. (Stimulation into the same beam, which does happen, does not lead to a force)

\[
\vec{E} = \vec{E}_0 \cos(wt - \varphi) + \vec{E}_L \cos(wt + \varphi)
\]

\[
\begin{align*}
\vec{E}_R & = \vec{E}_0 \cos(wt - \varphi) \\
\vec{E}_L & = \vec{E}_L \cos(wt + \varphi)
\end{align*}
\]

Since the dipole is along $\vec{E}$, the dipole induced by $\vec{E}_R$ is

\[
d_R = \cos(wt - \pi/4) \quad \text{at } \varphi = \pi/8
\]

The work done on this dipole by $\vec{E}_L$ is

\[
W_L \propto \vec{E}_L \cdot \vec{dR} \propto \cos(wt + \pi/4) [-\sin(wt - \pi/4)]
\]
\[ W_L = -\sin \omega t \cos (\omega t + \pi/2) \]
\[ = -\sin \omega t \left[ -\sin \omega t \right] \]
\[ W_R = \sin^2 \omega t \quad \text{work done on right-induced dipole by left beam} \]

i.e., positive work done by left-going beam,
so force is to the left (negative), consistent with the derivative of the potential predicted in the dotted curve of \( \text{p.} \text{12} \).

Now consider the dipole induced by \( \text{E}_L \) and the work done on it by \( \text{E}_R \).

\[ d_L = \cos (\omega t + \pi/4) \quad \text{at } t = \frac{3}{8} \]

\[ W_R = \text{E}_R \cdot d_L = \cos (\omega t - \pi/4) \left[ -\sin (\omega t + \pi/4) \right] \]
\[ = -\sin (\omega t) \cos (\omega t - \pi/2) \]
\[ W_L = -\sin^2 \omega t \quad \text{work done on left-induced dipole by right-going beam} \]

i.e., negative work done by right-going beam;
the right-going beam stimulates the atom to emit,
so the force is to the left as for the left-going beam, which is absorbed by the atom. The force at \( \frac{3}{8} \) is negative because the atom absorbs from one beam and is stimulated to emit into the other. The opposite will happen at \( \frac{3}{8} \).
Let us also consider the dipole force from the dressed atom perspective. (We will do this in more detail in the homework.)

Consider a laser field with only a $\nabla E_0$ and no $\nabla \phi$, e.g., a focused laser beam, where we consider motion $\perp$ to the optic axis.

In the focal plane, the wavefront is flat and we can neglect the scattering force in the $x$ direction.

The intensity along $x$ looks like:

\[ I \]
\[ x \]

Typically a Gaussian distribution of intensity.

Now consider the dressed states as a function of distance along $x$. 
\[ \delta > 0 \] (blue detuning)

cascading spontaneous emission from \( |11, N+1\rangle, |12, N+1\rangle \) to \( |11, N\rangle, |12, N\rangle \) establishes steady state populations of the dressed levels.

The dressed level connected to \( |19\rangle \) is always more populated, as it determines the direction of the force.

For \( \delta > 0 \) the force expels atoms from high laser fields.
For $\delta < 0$, red detuning, the force draws the atoms into the strongest field.

We can also think of the dipole force in terms of refraction of light by a material with an index of refraction.

The index of refraction of a gas as a function of $\omega$, near a resonance at $\omega_0$:

below resonance, $\omega < \omega_0$, the index is greater than one (as for visible light in air)
Consider a laser beam, with a Gaussian intensity profile, incident on a spherical sample of gas. The laser frequency is \( \omega \). The deflection of stronger ray to the right draws the gas to the left — toward the strong part of the field.

This is the usual treatment for optical tweezers — manipulation of \( > \lambda \)-size transparent objects with focussed beams. Works also along axis — objects are drawn toward focus.