lecture #16, 25 Oct. 2005
The Dipole Force - continued

Leftovers from lecture #15

at the end of lecture #15 we had written

$F_{react} = \nabla E_0 \cdot \vec{d}_{in\text{-}phase}$ which was then written as $-\nabla W$ with $W = \langle \vec{E} \cdot \vec{d} \rangle$

this appears to be wrong be cause both $\langle E \rangle$ <u>and</u> $\langle d \rangle$ depend on $\vec{R}$, and the correct expression, $\vec{\nabla} E_0 \cdot \hat{d}_{in\text{-}phase}$, only has the derivative of E_0.

Indeed the expression $-\nabla W$ is <u>incorrect</u>, for exactly that reason. The force is not the gradient of a potential derived from the work done on the atom alone, but on the entire system, atom + field + interaction. The calculation we did was correct, but trying to identify it as the gradient of the energy $\vec{d} \cdot \vec{E}$ is incomplete.

The field energy involves the fact that in considering moving from one location to another, some photons will be radiated as flourescence at a frequency different from the laser frequency. this difference goes to a constant as the motion becomes quasi static

For a complete description of this issue, see

Dalibard + Cohen-Tannoudji, J. Opt. Soc. B Vol. 2, no. 11, p. 1707 – 1720.

In particular see sect. 3 on pp. 1711 – 1712 in which it is made clear that the "total" energy change (called dW there) is not equal to the mean potential energy of the atom (called U_A there, and essentially like what we were calling $U = \langle E \cdot d \rangle$). Note particularly Remark #1 on p. 1712 which discusses the quasi-static change of the populations of the dressed states with position which is related to the change of the dipole moment with position.

so, the correct expression for the dipole force is

$$F_{dip} \text{ (or } F_{react}) = -\nabla \Omega^2 \frac{\hbar \delta}{4} \frac{1}{\delta^2 + \Omega^2/2 + \Gamma^2/4}$$

as we derived, and let's forget about interpreting it too glibbly as the gradient of a dipole energy, although it is certainly related to that.

Also: the reference to photoelectric effect without photons: W.E. Lamb and M.O. Scully, in Polarization, Matière, et Rayonnement, ed. Société Francaise de Physique, Presses Universitaires de France, 1969, pp. 363 – 369.

Returning now to the induced dipole:

the in-phase component of the induced dipole is

$$d_{eg}\, u \;=\; d_{eg}\frac{\Omega}{2}\;\frac{\delta}{\Gamma^2/4+\delta^2+\Omega^2/2}$$

remember that $\hbar\Omega \equiv -d_{eg}E$, so for

$\delta < 0$, the dipole is in the same direction as $\vec{E}$ (that is, the in-phase component is in the direction of $\vec{E}$) so, the dipole energy $-\vec{d}\cdot\vec{E}$ is negative, and the force is toward higher field

for $\delta > 0$ the opposite is true

(this change of sign of the force can be seen classically)

Now consider another special case - a standing wave

$$\vec{E}(\vec{R},t) = \vec{E}_0\left[\underbrace{\cos(\omega t - kz)}_{+\hat{z}\text{ travelling wave}} + \underbrace{\cos(\omega t + kz)}_{-\hat{z}\text{ travelling wave}}\right]$$

$$\vec{E}(\vec{R},t) = \underbrace{2\vec{E}_0\cos kz}_{\text{spatially modulated amplitude}}\;\underbrace{\cos\omega t}_{\text{oscillating, but with no spatial dependence of the phase}}$$

so, $\nabla\phi = 0 \Rightarrow F_{scatt} = 0$

but $\nabla\Omega \neq 0$:

$$-\vec{d}_{eg}\cdot\vec{E}(\vec{R}) = \Omega(\vec{R}) = \Omega_0 \cos kz \qquad \text{where } \Omega_0 = -d_{eg}\, 2E_0$$

$$\nabla\Omega = -k\,\Omega_0 \sin kz$$

$$F_{react} = F_{dipole} = -\hbar u \nabla\Omega = \frac{-\hbar\delta}{2}\,\Omega\,\frac{\nabla\Omega}{\Gamma^2/4+\delta^2+\Omega^2/2}$$

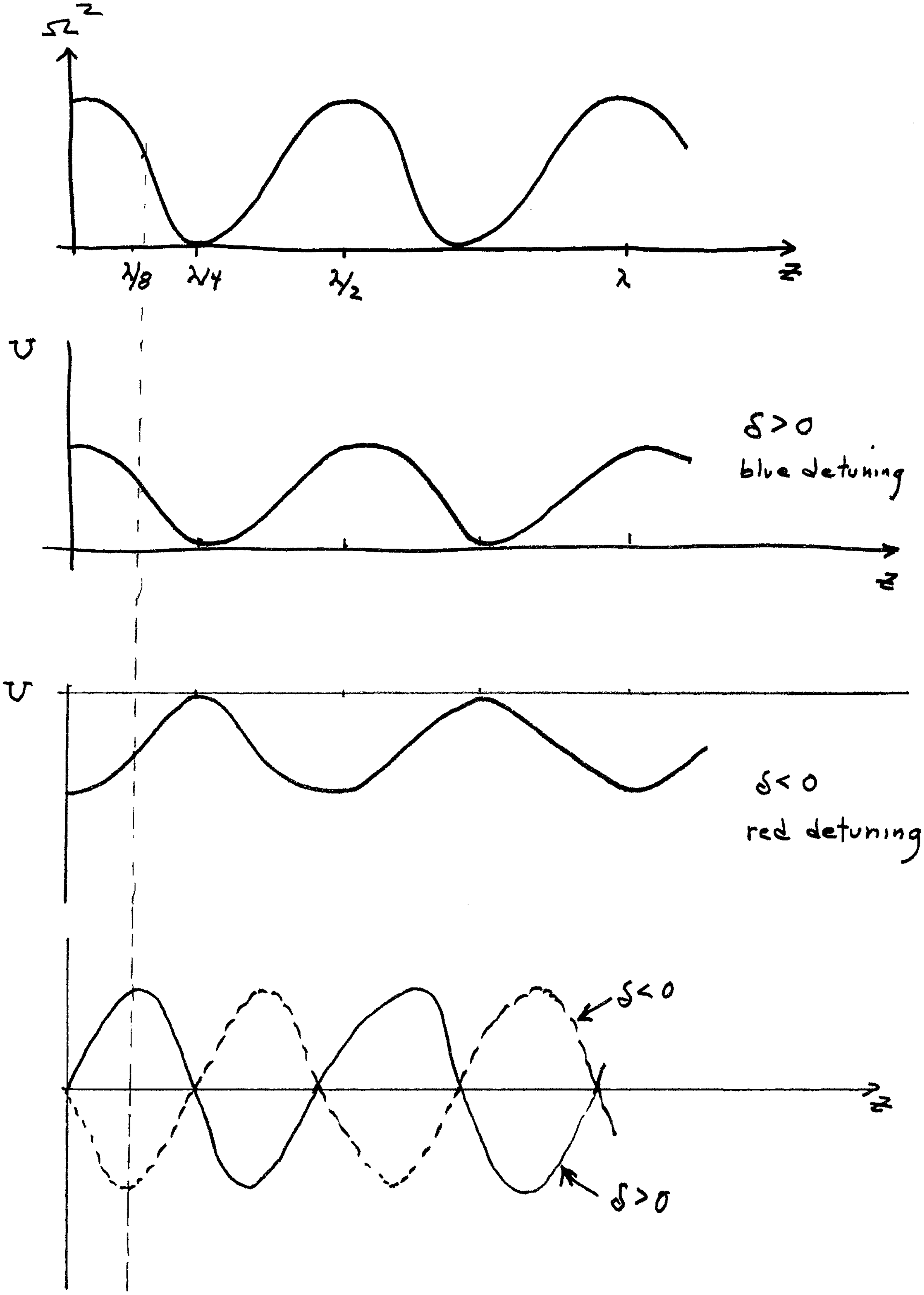
$$F_{dip} = \frac{\hbar k \Omega_0^2}{2}\;\frac{\delta \sin kz \cos kz}{\Gamma^2/4+\delta^2+\Omega^2/2}$$

$$= \frac{\hbar k \Omega_0^2}{4}\;\frac{\delta \sin 2kz}{\Gamma^2/4+\delta^2+\Omega^2/2}$$

Note that the dipole force can be arbitrarily large as Ω, δ increase.

This force, like the intensity, is modulated at a spatial frequency $2k$ (period of $\lambda/2$)

Now plot Ω^2 (proportional to the intensity), the potential U, and the force F as a function of position.



Ω^2
$\lambda/8$
$\lambda/4$
$\lambda/2$
λ
z
U
$\delta > 0$
blue detuning
z
U
$\delta < 0$
red detuning
$\delta < 0$
z
$\delta > 0$

We can interpret the dipole force as arising from (absorption + stimulate emission). The scattering force arises from (absorption + spontaneous emission).

The force <u>must</u> come from stimulated processes since it is not dissipative.

consider $\delta < 0$, so the dipole moment (the in-phase u-term part, is along $\vec{E}$). What happens at $z = \lambda/8$?

The field is the sum of waves travelling to the right $(+\hat{z})$ and left $(-\hat{z})$. There can be a force due to absorption from one wave and stimulation into the other. (stimulation into the same beam, which does happen, does not lead to a force)

$$\vec{E} = \underbrace{\vec{E}_0 \cos(\omega t - kz)}_{E_R} + \underbrace{\vec{E}_0 \cos(\omega t + kz)}_{E_L}$$

since the dipole is along $\vec{E}$, the dipole <u>induced by</u> <u>E_R</u> is

$$d_R \propto \cos(\omega t - \pi/4) \quad \text{at } z = \lambda/8$$

the work done on this dipole by E_L is

$$W_L \propto E_L \dot{d}_R \propto \cos(\omega t + \pi/4)[-\sin(\omega t - \pi/4)]$$

$$W_L \propto -\sin\omega t \cos(\omega t + \pi/2)$$
$$\propto -\sin\omega t\,[-\sin\omega t]$$
$$W_L \propto \sin^2\omega t$$ work done on right-induced dipole by left beam

i.e., positive work done by left-going beam, so force is to the <u>left</u> (negative), consistent with the derivative of the potential predicted in the dotted curve of p. 12.

Now consider the dipole induced by E_L and the work done on it by E_R.

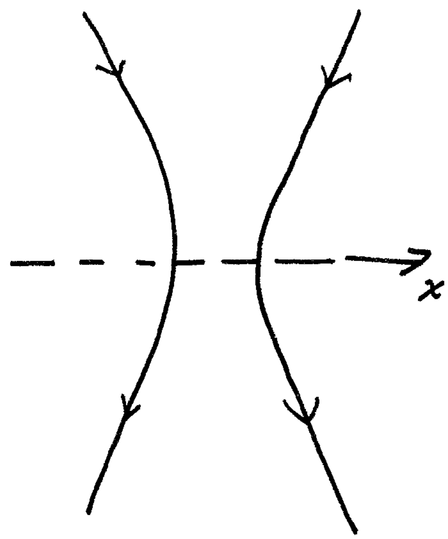
$$d_L \propto \cos(\omega t + \pi/4) \qquad \text{at } z = \lambda/8$$

$$W_R \propto E_R \dot{d}_L \propto \cos(\omega t - \pi/4)\,[-\sin(\omega t + \pi/4)]$$
$$\propto -\sin(\omega t)\cos(\omega t - \pi/2)$$
$$W_L \propto -\sin^2\omega t$$ work don on left-induced dipole by right-going beam.

i.e. negative work done by right-going beam: the right going beam stimulates the atom to emit, so the force is to the <u>left</u> as for the left-going beam, which is absorbed by the atom. The force at $\lambda/8$ is negative because the atom absorbs from one beam and is stimulated to emit into the other. The opposite will happen at $3\lambda/8$.

Let us also consider the dipole force from the dressed atom perspective
(we will do this in more detail in the homework)

consider a laser field with only a ∇E_0 and no $\nabla \phi$
e.g. a focussed laser beam, where we consider motion $\perp$ to the optic axis

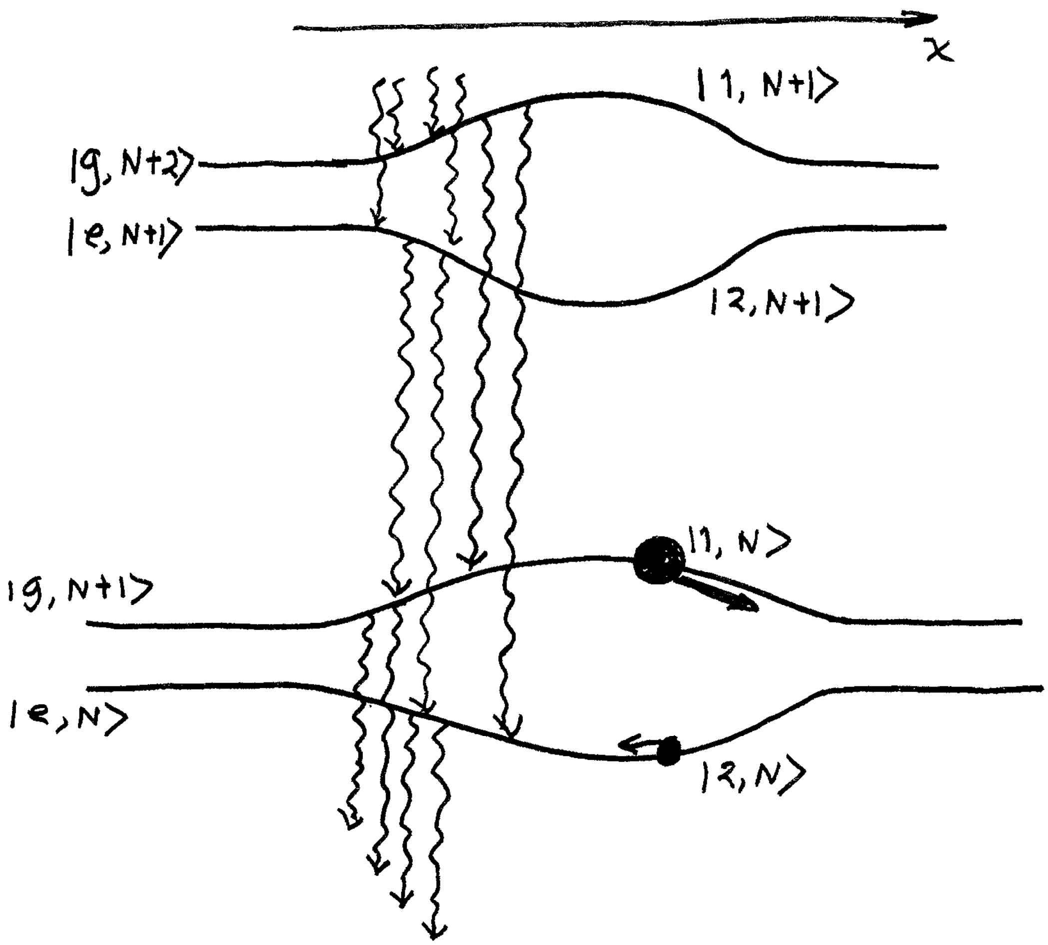


In the focal plane the wavefront is flat and we can neglect the scattering force in the $\hat{x}$ direction

The intensity along x looks like:

typically a Gaussian distribution of intensity.

Now consider the dressed states as a function of distance along x

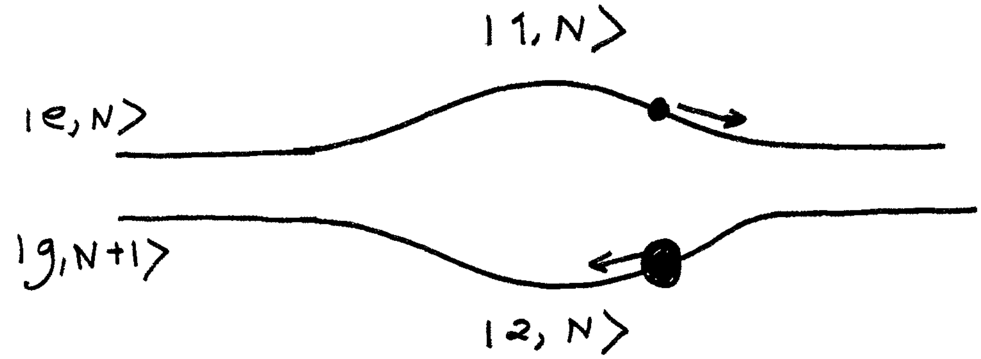


$\delta > 0$
(blue detuning)

cascading spontaneous emission from $\{|1, N+1\rangle, |2, N+1\rangle\}$ to $\{|1, N\rangle, |2, N\rangle\}$ establishes steady state populations of the dressed levels

The dressed level connected to $|g\rangle$ is always more populated, at it determines the direction of the force.

For $\delta > 0$ the force expels atoms from high laser fields

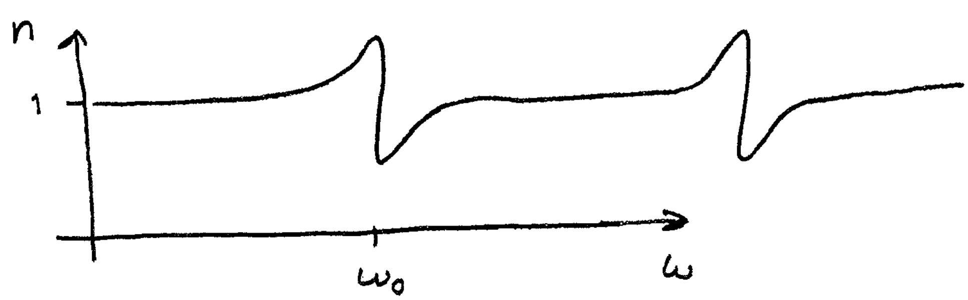


$\delta < 0$

For $\delta < 0$, red detuning, the force draws the atoms into the strongest field.

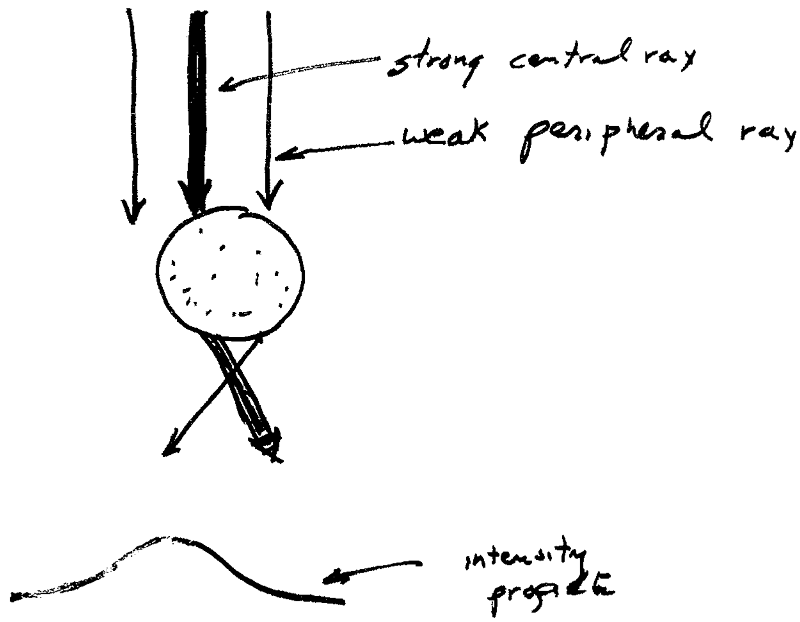
We can also think of the dipole force in terms of refraction of light by a material with an index of refraction

The index of refraction of a gas as a function of ω, near a resonance at ω_0:



below resonace, $\omega < \omega_0$, the index is ~~[illegible]~~ greater than one (as for visible light in air)

Consider a laser beam, with a Gaussian intensity profile, incident on a spherical sample of gas. the laser frequency $\omega < \omega_0$



deflection of stronger ray to the right draws the gas to the left — toward the strong part of the field.

This is the usual treatment for optical tweezers — manipulation of $> \lambda$-size transparent objects with focussed beams. Works also along axis — objects are drawn toward focus.