Lecture #15   20 October 2005

Optical forces on atoms – continued

at the end of the last lecture we had found

$$\langle \hat{d} \rangle = 2d_{eg}\,(u\cos\omega t - v\sin\omega t)$$

where u + v are the in-plane components of the Bloch vector and the two terms represent the part of the induced dipole moment in phase and out of phase with the driving field

Note that the vector character of $\hat{\vec{d}}$ is not contained in the Bloch vector. $\langle \hat{\vec{d}} \rangle$ is along $\vec{E}$ in this approximation (scalar polarizability).

The Bloch vector is in an abstract space representing the state of the 2-level atom.

Leftovers: photoelectric effect w/o photons

Mandel's role in quantum optics after Glauber

$$\langle \hat{\mathcal{F}} \rangle = 2 d_{eg} \left(u \cos\omega t - v \sin\omega t \right) \times$$
$$\left(\nabla E_0(R) \cos\omega t - \nabla\phi(R) E_0 \sin\omega t \right)$$

$$= 2 d_{eg} \left(u \cos^2\omega t \, \nabla E_0(R) + v \sin^2 E_0 \nabla\phi(R) \right)$$
$$+ (\text{something}) \cos\omega t \sin\omega t$$

averaging over a cycle of the optical field
$\cos^2, \sin^2 \to 1/2$ $\cos\sin \to 0$ so

$$\langle \hat{\mathcal{F}} \rangle = d_{eg} \left(u \nabla E_0(R) + v E_0 \nabla\phi(R) \right)$$

recall that $-d_{eg} E_0 \equiv \hbar\Omega_{Rabi}$

so the force has two distinct terms:

$$\langle \hat{\mathcal{F}} \rangle = F = \underbrace{-u \nabla \hbar\Omega(\vec{R})}_{1} \underbrace{- v \hbar\Omega \nabla\phi(\vec{R})}_{2}$$

These two components of the force have very different character:

1. This is associated with "reactive" (non-dissipative) interaction (like the inductive & capacitive parts of an electrical circuit). Often called the "dipole" or "stimulated" force.

2. Associated with dissipative interaction (like the resistive parts of a circuit) Often called the "scattering" or "spontaneous" force. It is analogous to classical radiation pressure.

We will get these two terms by evaluating the steady-state u and v.

recall the optical Bloch equations:

$$\dot{u} = \delta v - \Gamma/2\, u$$
$$\dot{v} = -\delta u - \Omega w - \Gamma/2\, v$$
$$\dot{w} = \Omega v - \Gamma w - \Gamma/2$$

see lecture #8 p. 24

The steady state solutions (you did this for homework) are

$$u = \frac{\Omega}{2} \frac{\delta}{\Gamma^2/4 + \delta^2 + \Omega^2/2}$$

$$v = \frac{\Omega}{2} \frac{\Gamma/2}{\Gamma^2/4 + \delta^2 + \Omega^2/2}$$

Note that v has absorptive Lorentzian shape while u has dispersive Lorentz shape

$$F_{reactive} = -u \nabla \hbar \Omega(R)$$

$$= -\hbar \nabla \Omega \frac{\Omega}{2} \frac{\delta}{\Gamma^2/4 + \delta^2 + \Omega^2/2}$$

$$\boxed{F_{react} = -\nabla \Omega^2 \frac{\hbar \delta/4}{\Gamma^2/4 + \delta^2 + \Omega^2/2}}$$

$$F_{dissipative} = -v \hbar \Omega \nabla \phi$$

$$= -\frac{\hbar \Omega^2}{2} \nabla \phi \frac{\Gamma/2}{\Gamma^2/4 + \delta^2 + \Omega^2/2}$$

$$\boxed{F_{diss} = -\hbar \Gamma \nabla \phi \frac{\Omega^2/4}{\Gamma^2/4 + \delta^2 + \Omega^2/2}} = -\hbar \Gamma \nabla \phi \, \rho_{ee}$$

Let us re-write

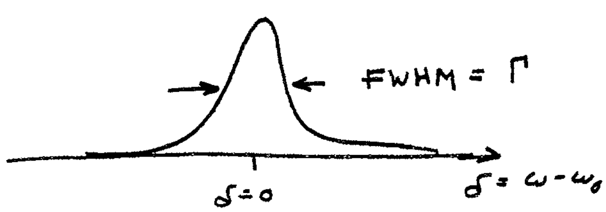
$$\rho_{ee} = \frac{\Omega^2/4}{\Gamma^2/4 + \delta^2 + \Omega^2/2}$$

$$= \frac{\Omega^2/\Gamma^2}{1 + 2\Omega^2/\Gamma^2 + (2\delta/\Gamma)^2}$$

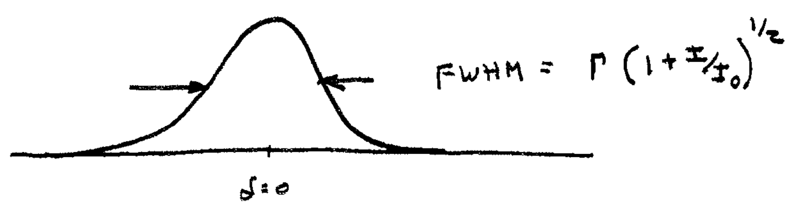
define $2\Omega^2/\Gamma^2 \equiv I/I_0$

$$\rho_{ee} = \frac{1}{2} \frac{I/I_0}{1 + I/I_0 + (2\delta/\Gamma)^2}$$

for $I/I_0 \ll 1$



in general



$I/I_0 \to \infty$, $\rho_{ee} \to \frac{1}{2}$

Consider

$$F_{diss} = -\hbar \Omega v \nabla\phi$$

it is associated with v, the out-of-phase component of the Bloch vector, and the out-of-phase component of the induced dipole $\langle d \rangle$

this component is related to the absorption of light (or scattering of light) why?

the electric field does work on the induced dipole

$$\langle \vec{d} \rangle = e \langle \vec{r} \rangle$$

$$\text{Power} = \vec{F} \cdot \underset{\substack{\uparrow \\ \text{velocity}}}{\vec{v}} = \vec{E} \cdot e \dot{\vec{r}} = \vec{E} \cdot \dot{\vec{d}}$$

$$\langle d \rangle = 2 d_{eg} \left(u \cos\omega t - \underset{\substack{\uparrow \\ \text{Bloch vector component}}}{v} \sin\omega t \right)$$

$$\vec{E} \cdot \dot{\vec{d}} = E_0 \cos\omega t \cdot 2 d_{eg} \left(-u \sin\omega t - v \cos\omega t \right) \omega$$

$$\langle \vec{E} \cdot \dot{\vec{d}} \rangle_{\text{time ave.}} = -d_{eg} E_0 v \omega = \hbar \omega \Omega \frac{\Omega}{2} \frac{\Gamma/2}{\Gamma^2/4 + \delta^2 + \Omega^2/2}$$

$$\text{Power} = \underbrace{\hbar\omega}_{\text{energy per photon}} \underbrace{\Gamma \rho_{ee}}_{\text{rate of scattering (emitting) photons}}$$

So, the v term is the one on which the field does work, at a rate equal to the rate of scattering photon energy.

Now consider the special case of an atom in a plane wave:

$\vec{E} = \vec{E}_0 \cos(\omega t - kz)$, travelling in $+z$ direction

$\nabla\Omega = 0$, since E_0 does not depend on $\vec{R}$ (that is, z)

but $\phi = -kz$ so $\nabla\phi = k\hat{z}$

so only $F_{scatt} = F_{diss} \neq 0$

$$F_{scatt} = -\hbar\Gamma\nabla\phi\,\rho_{ee}$$

$$= \underbrace{\hbar k}_{\text{momentum per photon}}\ \underbrace{\Gamma\rho_{ee}}_{\text{scattering rate}}\ \hat{z}$$

direction of propagation

so the scattering force is the rate of scattering momentum

spontaneous emission is symmetric, and doesn't contribute to the average force.

How big is the scattering force?

If $\Omega^2 \to \infty$ $\quad \rho_{ee} \to 1/2$

$$F_{scatt,max} = \frac{\hbar k \Gamma}{2}$$

let's evaluate this for a real atom

Na $\quad \Gamma^{-1} = \tau = 16\ ns$

$\lambda \approx 0.6\ \mu m$

$$\frac{F_{max}}{m} = a_{max} \qquad M = 23\ amu$$

$$= \frac{h}{\lambda m} \frac{1}{2\tau}$$

$$= \frac{6.6 \times 10^{-34}\ J \cdot s}{0.6 \times 10^{-6} m\ (23)\ 1.66 \times 10^{-27}\ Kg} \frac{1}{32 \times 10^{-9} s}$$

$\approx 10^6\ m/s^2 \approx 10^5 \times$ gravity

Na atoms with a velocity of 10^3 m/s will come to rest in $t = v/a = 10^{-3}$ sec

in a distance of $\frac{1}{2} a t^2 = 0.5\ m$

Now examine the force

$$F_{react} = -u \nabla \hbar \Omega(\vec{R})$$

It relates to the u term, which does not result in any work being done on the induced dipole

$$F_{react} = -\nabla \Omega^2 \frac{\hbar\delta}{4} \frac{1}{\Gamma^2/4 + \delta^2 + \Omega^2/2}$$

this force can be derived from a "potential"

$$F_{react} = -\nabla U \quad \text{with}$$

$$U = \frac{\hbar\delta}{2} \ln\left(1 + \frac{\Omega^2/2}{\delta^2 + \Gamma^2/4}\right)$$

$$-\nabla U = -\frac{\hbar\delta}{2} \frac{\frac{1}{2}\nabla\Omega^2}{1 + \frac{\Omega^2/2}{\delta^2 + \Gamma^2/4}} = -\nabla\Omega^2 \frac{\hbar\delta}{4} \frac{1}{\Gamma^2/4 + \delta^2 + \Omega^2/2}$$

If we write $F_{react} = \underbrace{d_{eg}\, u}_{\substack{\text{in-phase} \\ \text{component of} \\ \text{dipole moment}}} \nabla E_0(R)$

this is wrong!! see Lecture # 16

$$F_{react} = \nabla \vec{E}_0 \cdot \vec{d}_{in\text{-}phase} = -\nabla W \qquad W = \langle -\vec{E} \cdot \vec{d} \rangle$$

(the out-of-phase component averages to zero)