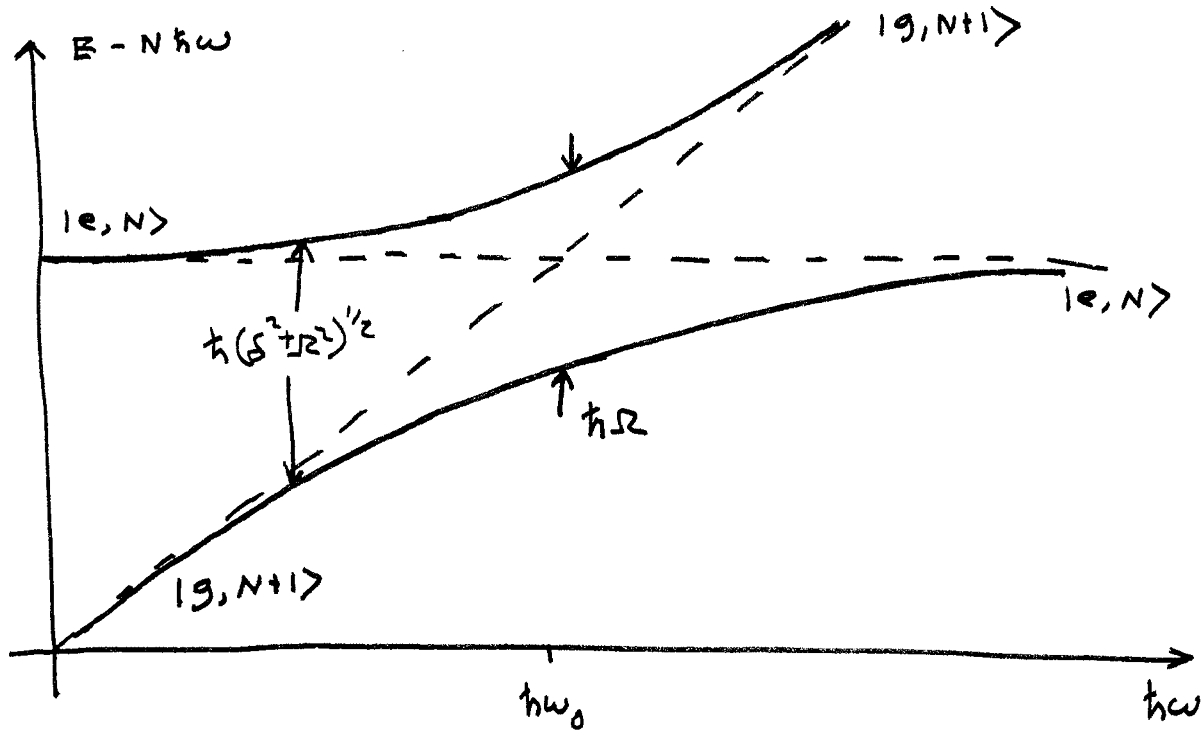
Lecture # 14 18 Oct. 2005

Optical forces on atoms — density matrix approach

But first, some left-overs from dressed atoms

recall the 2-level dressed atom anticrossing



- Rabi flopping
- adiabatic passage
- adiabatic turn on/off
- adiabatic criterion; $d\omega/dt \ll \Omega^2$ for passage
- Landau-Zener theory

level-crossing and adiabaticity are both common themes in QM

Force on an atom. Use a force operator $\hat{\mathcal{F}}$

$$F = \langle \hat{\mathcal{F}} \rangle = T_r\,(\hat{\rho}\,\hat{\mathcal{F}})$$

recalling that classically $\vec{F} = \dot{\vec{P}}$ we write

$$\hat{\mathcal{F}} = \dot{\hat{P}} \qquad \text{where} \quad \vec{P} = i\hbar \frac{\partial}{\partial \hat{\vec{R}}}$$

$\hat{R}$ is the atomic center-of-mass coordinate operator

from the Heisenberg equation of motion

$$\dot{\hat{\vec{P}}} = \frac{i}{\hbar}\left[\hat{\mathcal{H}}, \hat{\vec{P}}\right] \qquad \text{can be shown to yield}$$

$$\dot{\hat{\vec{P}}} = -\frac{\partial}{\partial \hat{R}}\hat{\mathcal{H}} \,,$$

which is analogous to the classical Hamiltonian equation of motion.

for the atom-field system the Hamiltonian is

$$\mathcal{H} = \underbrace{\frac{\hat{P}^2}{2M}}_{\text{atomic center-of-mass motion}} + \underbrace{H_A}_{\text{atom internal energy}} + \underbrace{H_L}_{\text{Laser field energy}} - \underbrace{\hat{d}\cdot \mathbf{E}(\hat{\vec{R}})}_{\text{atom-field interaction}}$$

Of these terms, only the interaction $\hat{\vec{d}} \cdot E(\hat{\vec{R}})$ does not commute with $\hat{P}$, so only it contributes to $\mathring{\hat{P}}$, and the force.

so

$$\hat{\mathcal{F}} = \mathring{\hat{p}} = -\partial/\partial\hat{R}\,\left(-\hat{d}\cdot E(\hat{R})\right) = \nabla_R\, \vec{d}\cdot\vec{E}(\hat{R})$$

For a 2-level atom we will assume $\vec{d}$ to be ~~wtu~~ along $\vec{E}$. This is equivalent to making the polarizability a scalar. In general, for a multi-level atom, this is not the case, but we would simply adjust the Rabi frequency to account for it.

In general we can write an electric field at frequency ω as:

$$\vec{E}(\hat{\vec{R}}) = \vec{E}_0(\hat{\vec{R}})\left[\cos(\omega t + \phi(\hat{\vec{R}})\right)$$

(this is like our ability to write a general Q.M. wavefunction as $\psi(R) = A(R)\,e^{i\phi(R)}$)

thus, the force is

$$\hat{\mathcal{F}} = \hat{d}\left[\nabla_R E_0(\hat{\vec{R}})\cos[\omega t + \phi(\hat{\vec{R}})] - E_0(\hat{\vec{R}})\sin[\omega t + \phi(\hat{\vec{R}})]\,\nabla_R\phi(\hat{\vec{R}})\right]$$

We now make the semi-classical approximation that the atom has a definite location. (This is not always a good approximation. It might fail if the wavepacket representing the atom were bigger than λ.) In this approximation $\hat{\vec{R}}$ is replaced with the numerical value of the COM position of the atom. Matrix elements are diagonal in an R-basis; we treat $\vec{R}$ as a classical variable.

For simplicity put the atom at $\vec{R}=0$ and choose the origin of time so $\phi(\vec{R}=0)=0$

then $$\hat{\vec{\mathcal{F}}} = \hat{d}\left[\nabla_R E_0(\vec{R})\cos\omega t - \nabla_R \phi(\vec{R}) E_0 \sin\omega t\right]$$

where we evaluate the derivatives at $\vec{R}=0$.

We want to calculate

$$\langle \hat{\vec{\mathcal{F}}} \rangle = \mathrm{Tr}(\hat{\rho}\, \hat{\vec{\mathcal{F}}})$$

In our semiclassical ~~ ~~ approximation only $\hat{\vec{d}}$, the atomic dipole moment operator has to be traced with the density operator — the rest is just a number.

<u>Caution:</u> "semi classical" can have different meanings. Here it means treating the COM motion classically. It can also mean treating the field classically and the atom quantum mechanically. (we are, in fact, doing that, too.)

We need $\langle \hat{d} \rangle = Tr(\hat{\rho}\hat{d})$

$$\hat{\rho}\hat{d} = \begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix} \begin{pmatrix} 0 & d_{eg} \\ d_{ge} & 0 \end{pmatrix}$$

taking $d_{eg} = d_{ge}$ = a real number, we have

$$\hat{\rho}\hat{d} = \begin{pmatrix} \rho_{eg} d_{eg} & \times \\ \times & \rho_{ge} d_{eg} \end{pmatrix}$$

we don't care about these off-diagonal terms for the trace

$$Tr(\hat{\rho}\hat{d}) = d_{eg}(\rho_{eg} + \rho_{ge}) = \langle \hat{d} \rangle$$

Note: the terms ρ_{eg}, ρ_{ge} are matrix elements of $\hat{\rho}$, <u>not</u> $\hat{\tilde{\rho}}$, the density matrix in the rotating frame.

$$\rho_{eg} = \tilde{\rho}_{eg} e^{-i\omega t}$$

$$\rho_{ge} = \tilde{\rho}_{ge} e^{i\omega t}$$

note also the this is the reduced density matrix — what we called $\tilde{\sigma}$ in lecture #8 — it has had the field traced over already.

For the sake of physical insight and for convenience, we will express $\langle d \rangle$ in terms of the components of the Bloch vector

$$u = \frac{1}{2}\left(\tilde{\rho}_{ge} + \tilde{\rho}_{eg}\right)$$

see lecture #8 p. 23

$$v = \frac{1}{2i}\left(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}\right)$$

$$\langle \hat{d} \rangle = d_{eg}\left(\rho_{eg} + \rho_{ge}\right) = d_{eg}\left(\tilde{\rho}_{ge}\, e^{i\omega t} + \tilde{\rho}_{eg}\, e^{-i\omega t}\right)$$

giving (do the algebra yourself)

$$\langle \hat{d} \rangle = 2 d_{eg}\left(u \cos\omega t - v \sin\omega t\right)$$

this term is in-phase with the driving field

this term is out-of-phase: $-\sin\omega t = \cos(\omega t + \pi/2)$

Note: the true spatial vector character of $\vec{d}$ is given by $\vec{E}$. The Bloch vector is in an abstract space.