Light Shifts and Dressed Atoms

Electric and magnetic fields produce "Stark shifts" and "Zeeman shifts".

Light field produce "light shifts" also known as "ac Stark shifts" (and humorously called "lamp shifts" when first discovered in the days before lasers).

Let us calculate this shift in 2nd order perturbation theory.

\[
\text{perturbation } \Delta \omega = \frac{1}{2} g \left( |e\rangle \langle e| + |g\rangle \langle g| \right) \nabla \cdot \vec{E}
\]

for our 2-level atom.

as usual, we take \( \vec{E} = E_0 \cos(\omega t) = \frac{E_0}{2} \left( e^{i\omega t} + e^{-i\omega t} \right) \),

only one term of which will couple \( |e\rangle \) and \( |g\rangle \) -- one term going up and one going down.

with \( t_{ij} = -g E_0 \),

\[
\langle i | \frac{\partial}{\partial t} | g \rangle = \langle i | \nabla \times \vec{E} | g \rangle = \frac{\hbar g E_0}{2}
\]

since \( H' \) has no diagonal matrix elements we go to 2nd order to get the lowest 2nd order energy shift:

\[
\Delta E = \frac{\langle g | \frac{\partial}{\partial t} | e \rangle \langle e | \frac{\partial}{\partial t} | g \rangle}{E_0 + \hbar \omega - E_0} \text{ shift of ground state}
\]

note that if \( \omega = 0 \), i.e., for a static field, this would be the ordinary Stark shift and the ground state would go down.
\[ \Delta E_2 = \left( \frac{m_e}{\hbar} \right) \left( \frac{1}{E_2} \right) \frac{\hbar \omega}{\hbar (\omega - \omega_0)} = \frac{\hbar \omega}{\sqrt{\hbar \omega}} \text{ where } \Delta E = \omega - \omega_0 \]

\[ \hbar \omega = \frac{\hbar \omega}{\sqrt{\hbar \omega}} \]

Note that the shift is down for \( \Delta < 0 \) – which puts the field closer to \( \omega_0 \).

Note also that if the field really were near \( \omega_0 \), we would not be able to make the resonant approximation and would keep both positive and negative frequency terms.

Finally note that if we had calculated the shift of the excited state, we would have gotten the same magnitude shift with opposite sign.

Now, we will take a different approach that is \underline{not} perturbative, but reduces to the result above in the \( \Delta \to 0 \), \( \Delta \to \Delta_2 F \) limit, but gives much more information about the atomic behavior.

The Dressed Atom

The major architect of this approach was Claude Cohen–Tsarovdji (he discovered the light shift in his thesis work). It was first applied to atoms in rf fields, later to optical fields (when lasers began to provide strong, coherent optical fields just as rf and microwave oscillators had provided coherent fields at lower frequencies).
Assume a 2-level atom and a single mode of the E-M field, near-resonant.

The dressed atom picture considers atom + field as a single system.

First consider the systems separately — that is, not the dressed atom, but the bare atom basis:

(We take the field to be quantized, with a large number $N$ of photons, and use a Fock-number state - basis for the field)

\[
\begin{align*}
\ldots & \quad |N+3\rangle \\
\ldots & \quad |N+2\rangle \\
\ldots & \quad |N+1\rangle \\
1\rangle & \quad \uparrow w_0 \\
1\rangle & \quad \downarrow w \\
1\rangle & \quad \downarrow \\
\ldots & \quad |N-1\rangle \\
\ldots & \quad |N-2\rangle \\
\ldots & \quad |N-3\rangle \\
\ldots & \quad \ldots
\end{align*}
\]

atom energy \hspace{1cm} field energy
The atom spectrum is the usual 2-levels; the field spectrum is a (nearly infinite) ladder of states, labeled by the number of photons in the single mode, separated in energy by $\hbar \omega$.

Now consider the combined system, but without any interaction between the atom and field — we just add the energies.

for $\delta < 0$

\[
\begin{align*}
1e, N+1 \rangle \quad & \quad w_0 - w = -\delta \\
1g, N+2 \rangle \quad & \quad w_0 \\
1e, N \rangle \quad & \quad w \\
1g, N+1 \rangle \quad & \\
1e, N-1 \rangle \quad & \quad w \\
1g, N \rangle \quad & \\
1e, N-2 \rangle \quad & \quad w_0 \\
1g, N-1 \rangle \quad & 
\end{align*}
\]
This is the dressed basis, uncoupled --- this corresponds to the actual energy eigenstates when, for example, the atom and field are in different places and therefore do not interact.

for $\delta > 0$
In this spectrum pairs of states like \( |e, N \rangle \) and \( |g, N+1 \rangle \) are nearly degenerate for \( \delta \ll \omega, \omega_0 \) (and are indeed degenerate for \( \delta = 0 \)).

But for an atom in a laser field there is generally a coupling between these states, \( \vec{J} \cdot \vec{E} \), that mixes them and lifts the degeneracy.

Explicitly, there are matrix elements

\[
|e, N \rangle \langle e' | \vec{J} \cdot \vec{E} | g, N+1 \rangle \text{ and h.c.}
\]

that couple the joint atom and field states through the atom operator \( \hat{d} \) and the field operator \( \vec{E} \)

\[
\hat{d} = \text{deg} \left( |1e\rangle \langle 1g| + |1g\rangle \langle 1e| \right)
\]

\[
\vec{E} = E_0 \left( \vec{E} \hat{a} e^{-i\omega t} + \hat{a}^* e^{i\omega t} \right)
\]

(leaving out the spatial dependence of the field is like making a semiclassical approximation — atom is at a point — with the atom at \( x = 0 \) so \( e^{i\hbar k x} = 1 \))

recall that this leads to 4 terms in \( \vec{J} \cdot \vec{E} \), of the form

\[
|1e\rangle \langle 1g| \hat{a} \hat{a}^* + |1g\rangle \langle 1e| \hat{a} \hat{a}^* + |1g\rangle \langle 1e| \hat{a}^* \hat{a} + |1e\rangle \langle 1g| \hat{a}^* \hat{a}
\]

\( |1e\rangle \langle 1g| \hat{a} \hat{a}^* \) and \( |1g\rangle \langle 1e| \hat{a}^* \hat{a} \) are "resonant"

we ignore the non-resonant terms for \( \delta \ll \omega_0, \omega \)
Having made the dual atom + field operators and matrix elements explicit in the quantized field notation, we will nevertheless proceed, treating the field classically. This is OK for $N \geq 1$. Then, we can take a single classical value of $E_0$ for $N$, $N \geq 1$, $N \geq 2$.

\[ E = \frac{\hbar}{2} \omega \left( e^{-i\omega t} + e^{i\omega t} \right) \]

\[ V_{eg} = -\langle e | \hat{\mathbf{E}} | g \rangle = \frac{\hbar n}{2} \]

Taking

\[ \hat{H} = \hat{H}_{\text{atom}} + \hat{H}_{\text{field}} + \hat{H}_{\text{int}} \]

the eigenstates of this part are the dressed, uncoupled basis.

Focusing on the $N$-photon manifold as an isolated two-level system (good in the approx. that $\omega \ll w_0$, and $\Gamma$ is neglected), $\hat{H}$ in the uncoupled dressed basis is:

\[ \left( \begin{array}{c} 1e, N \rangle \\ 1g, N+1 \rangle \end{array} \right) \]

\[ \frac{\hbar}{\Delta} \left( \begin{array}{cc} -\Delta/2 & \Delta/2 \\ -\Delta/2 & \Delta/2 \end{array} \right) \]

(for $\Delta > 0$), where we choose the zero of energy between the uncoupled states:

\[ \left( \begin{array}{c} 1g, N+1 \rangle \\ 1e, N \rangle \end{array} \right) \]

\[ \frac{\hbar}{\Delta} \frac{\Delta/2}{\Delta/2} \]

$\Delta > 0$.
This 2×2 Hamiltonian can be diagonalized to get eigenvalues and eigenvectors.

Secular equation

\[
\det \begin{pmatrix}
  -\frac{1}{2} - E & -\frac{1}{2} \\
  -\frac{1}{2} & \frac{1}{2} - E
\end{pmatrix} = 0
\]

\[E^2 - \frac{\delta^2}{4} - \frac{\alpha^2}{4} = 0\]

\[E^2 = \frac{1}{4} (\delta^2 + \alpha^2)\]

\[E = \pm \frac{1}{2} (\delta^2 + \alpha^2)^{1/2}\] are the two eigenvalues for \(\delta > 0\).

Uncoupled \[|1, N+1\rangle\] Coupled

| 1g, N+1\rangle
| 1e, N+1\rangle

\[|2, N+1\rangle\]

\[|1g, N+1\rangle \downarrow \delta \uparrow (\delta^2 + \alpha^2)^{1/2} \downarrow |1, N\rangle\]

\[|1e, N\rangle \downarrow \uparrow |2, N\rangle\]

\[|1g, N\rangle \downarrow \uparrow |1, N-1\rangle\]

\[|1e, N-1\rangle \downarrow \uparrow |2, N-1\rangle\]
Note that the levels are split by the "generalized"
Rabi frequency

\[ S_{\text{eff}} = \sqrt{s^2 + \delta^2} \]

 Derived for 2-level Rabi flopping, which is also
the frequency of precession about the effective
field in the Bloch vector picture.

If we take an analogous vector, \( \hat{S}_{\text{eff}} \), the
Bloch vector precesses around it.

The Bloch vector corresponds to some state of the
2-level system.

Eigenstates of the uncoupled \( \hat{H} \) are along \( \hat{z} \). Eigenstates
of the full, coupled \( \hat{H} \) are along \( \hat{S}_{\text{eff}} \). Any state that is
not an eigenstate of the full \( \hat{H} \) evolves as \( \hat{S}_{\text{eff}} \).

The dressed levels are the states parallel and antiparallel
to \( \hat{S}_{\text{eff}} \).
We can calculate the eigenvectors of the dressed, coupled atom. They satisfy

$$H |\psi\rangle = E |\psi\rangle$$

using the uncoupled states as a basis, we can write

$$|\psi\rangle = \cos \theta |1e, N\rangle + \sin \theta |1g, N+1\rangle$$

for one eigenvector, and the other will be orthogonal to it.

The solutions can be expressed as (do this yourself)

$$\sin 2\theta = \frac{\delta}{(x^2+E^2)^{1/2}} \quad \cos 2\theta = \frac{-x}{(x^2+E^2)^{1/2}}$$

Consider some limiting cases:

$$S \rightarrow 0 \rightarrow \sin \theta = 0 \text{ so the eigenvectors}$$

$$1, N \rightarrow 1g, N+1 \quad \text{(i.e., the uncoupled vectors)}$$

$$12, N \rightarrow 1e, N$$

$$S \rightarrow 0 \quad \text{for} \quad \cos 2\theta = 0 \quad 2\theta = \pm \frac{\pi}{2} \quad \theta = \pm \frac{\pi}{4} \quad \sin 2\theta = -1$$

$$\cos \theta = \frac{1}{\sqrt{2}} \quad \sin \theta = -\frac{1}{\sqrt{2}}$$

$$1, N \rightarrow \frac{1}{\sqrt{2}} (1g, N+1 - 1e, N)$$

$$12, N \rightarrow \frac{1}{\sqrt{2}} (1g, N+1 + 1e, N)$$

(i.e., on resonance the dressed states are equal admixture of \(1e\) + \(1g\))
The light shift:

\[ \Delta = \frac{1}{\sqrt{\lambda}} \Delta_{\text{inertial}} \]

For \( \delta > 0 \)

\[ |g, N+1\rangle \quad \uparrow \quad \frac{\delta}{\sqrt{\lambda}} \quad \downarrow \quad |1, N\rangle \]

\[ |e, N\rangle \quad \downarrow \quad \delta (\frac{\delta^2 + \omega^2}{2}) \quad \uparrow \quad |2, N\rangle \]

For \( \delta > \frac{5\omega}{2\lambda} \)

\[ \sin \theta = -\frac{5\omega}{\delta} \quad \sin \phi = -\frac{\pi}{25} \quad \phi/| \]

so \( |1, N\rangle = |g, N+1\rangle - \frac{\pi}{25} |e, N\rangle \]

i.e., the upper dressed level is almost all ground state

(The admixture is as would have been obtained in perturbation theory: \( \Delta V / \Delta \epsilon \)

The shift \( \Delta = \frac{(\delta^2 + \omega^2)^{1/2} - \delta}{\sqrt{2}} \approx \delta (1 + \frac{1}{2} \frac{\delta^2}{\omega^2} \rangle - \delta \)

\( \Delta = \frac{\delta^2 - \omega^2}{4\delta} \)

exactly as we calculated for perturbation theory

For \( \delta < 0 \)

\[ |e, N\rangle \quad \uparrow \quad \delta < 0 \]

\[ |g, N+1\rangle \quad \downarrow \]
In the dressed atom picture it is easy to see why the excited light shift is opposite to the ground state, whichever sign it has.

Let's draw the dressed levels as a function of the laser frequency \( \omega \) (i.e., as a function).

For simplicity, we subtract the energy of \( N \) photons.
The dotted lines are the uncoupled states & energies.

The interaction produces an avoided crossing—levels always "repel" producing opposite light shifts.

At the crossing there is equal admixture and entanglement between atom and field states.

\[ |N\psi = \frac{1}{\sqrt{2}} (|g, N+1\rangle - |e, N\rangle) \]

is maximally entangled for the state of the atom and the "extra" photon.

Starting in \(|e\rangle\) or \(|g\rangle\) at some detuning \(|e\rangle \gg |g\rangle\) and going through the crossing will invert the population:

this is adiabatic inversion or adiabatic fast passage.

adiabatic in the Landau-Zener sense, fast in the sense of not allowing relaxation.
the dressed atom picture allows an analysis of interesting strong-field spectral features

consider a 3-level system with weak excitation

now consider a strong field at $\omega = \omega_0$

Autler-Townes splitting
\( \left| \psi \right\rangle \)

\( \left| e, N \right\rangle - e \left| g, N+1 \right\rangle \)

\( \left| g, N+1 \right\rangle + e \left| e, N \right\rangle \)

\( \omega < \omega_0 \)

\( P_x \)

"virtual" level, light shifted
2-photon resonance.

\( s=0 \)

\( \left| g, N+1 \right\rangle + \left| e, N \right\rangle \)

\( \left| g, N+1 \right\rangle - \left| e, N \right\rangle \)

\( \left| g, N \right\rangle + \left| e, N-1 \right\rangle \)

\( \left| g, N \right\rangle - \left| e, N-1 \right\rangle \)

Fluorescence spectrum

\( P(\omega) \)

Mollow triplet