1) Anomalous Zeeman effect solution:

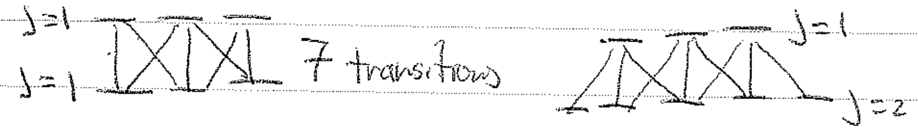
There are 9 total transitions, and the transition is allowed.

^3P can have $J=0,1,2$, but $J=0$ will not have 9 transitions.

$\longrightarrow$ initial state $^3P_1$ or $^3P_2$

case 1: $^3P_1$ has allowed transitions to

$^3S_1$, $^3D_1$, $^3D_2$ $\rightarrow$ only $^3D_2$ has 9 transitions



possible final states

case 2: $^3P_2$ has allowed transitions to

$^3S_1$, $^3D_1$, $^3D_2$, $^3D_3$

$\rightarrow$ only $J'=1$ has 9 transitions $\rightarrow$ $^3S_1$ $^3D_1$

(using similar arguments.)

calculate g factors

$$g_J = \frac{3}{2} + \frac{S(S+1)-L(L+1)}{2J(J+1)}$$

$$\boxed{g_J(^3P_1) = 3/2 = 9/6 \qquad g_J(^3D_2) = 7/6}$$

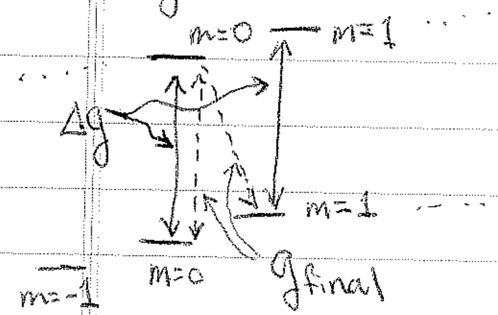
$$\boxed{g_J(^3P_2) = 3/2 \qquad g_J(^3S_1) = 2 \qquad g_J(^3D_1) = 3/4}$$

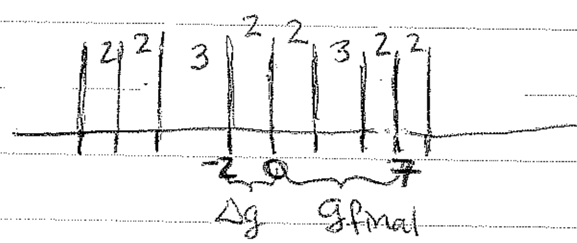
g_{final}

$\Delta g(^3P_1 \rightarrow {}^3D_2) = 2/6$

$\Delta g(^3P_2 \rightarrow {}^3S_1) = -1/2$

$\Delta g(^3P_2 \rightarrow {}^3D_1) = 3/4$





$\Delta g / g_{final} = 2/7$

so only $^3P_1 \rightarrow {}^3D_2$

2) Soln.

$$E_{HF} = \frac{1}{2} A_J K + \frac{B_J \left(3K(K+1) - 4I(I+1)J(J+1) \right)}{8I(2I-1)J(2J-1)}$$

where $K_F = F(F+1) - I(I+1) - J(J+1)$

$$\Delta E = E(F) - E(F-1) = A_J F + \frac{3 B_J}{8I(2I-1)J(2J-1)} \underbrace{\left(K_F(K_F+1) - K_{F-1}(K_{F-1}+1) \right)}_{\equiv \beta_F}$$

e.g. for ^{87}Rb, $\beta_F = 4F(F^2-7)$ $\left(= 2F(F^2+1-2I(I+1)-2J(J+1)) \right)$

for ^{85}Rb, $\beta_F = 4F(F^2-12)$

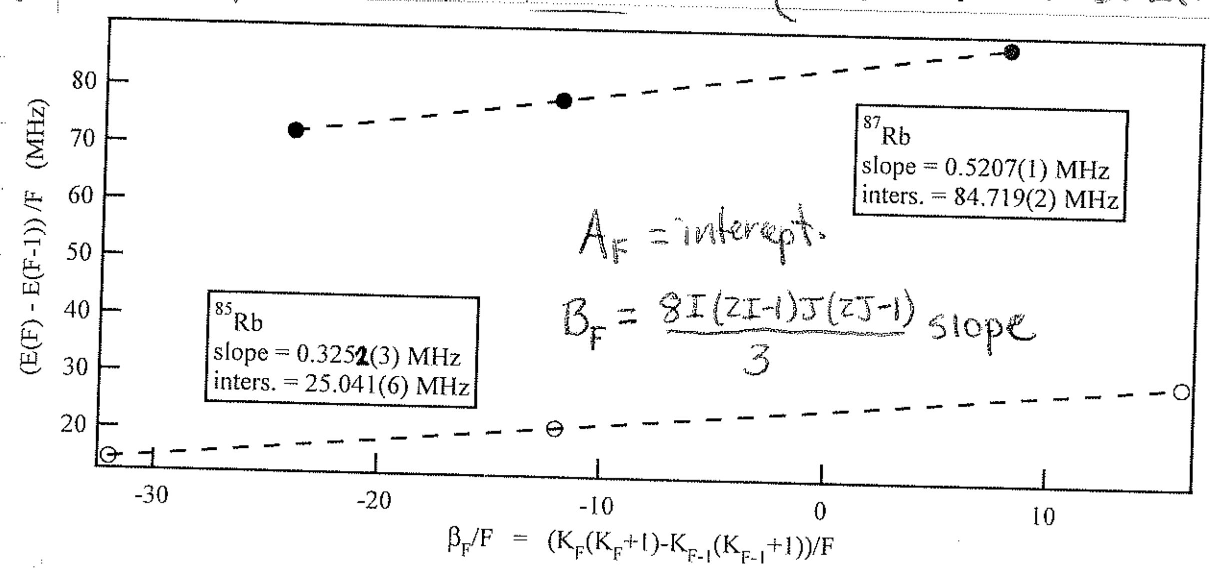
Plot & fit $\Delta E/F$ vs. β_F/F

^{87}Rb

ΔE	$\Delta E/F$	β_F/F
266.650(9)	88.883(3)	8
156.947(7)	78.4735(35)	-12
72.218(4)	72.218(4)	-24

^{85}Rb

ΔE	$\Delta E/F$	β_F/F
120.99(3)	30.246(7)	16
63.38(3)	21.127(10)	-12
29.34(3)	14.652(15)	-32



MHz $\left\{ \begin{array}{l} ^{87}Rb \quad A_J = 84.719(2) \\ \qquad\quad B_J = 12.497(4) \end{array} \right.$

^{85}Rb: $A_J = 25.041(6)$

$B_J = 26.013(24)$

2)

The electronic wave functions for ^{87}Rb & ^{85}Rb are almost identical $\Rightarrow$ tells us information about the nucleus.

Namely, the additional neutrons for ^{87}Rb make the nuclear charge distribution rounder (less oblate), but the total dipole is less well cancelled $\Rightarrow$ larger nuclear dipole.

^{85}Rb is more oblate.

3) Vector light shift

Solution:

$$H_{AC} = -\frac{1}{2}\alpha_0(\omega)\left|E_{AC}\right|^2 + \frac{i}{2}\alpha_1(\omega)\left(\vec{E}_{AC}\times\vec{E}_{AC}^{*}\right)\cdot\vec{J}$$

$$H_{DC} = -\frac{1}{2}\alpha_0(0)\left|E_{DC}\right|^2 + \frac{1}{2}\alpha_2(0)\,E_{DC}^2\left(\frac{3J_z^2 - J(J+1)}{J(2J-1)}\right)$$

H_{DC} assumes $\vec{E}_{DC} = E_{DC}\hat{z}$, so let $\vec{E}_{DC}$ define $\hat{z}$ axis.

a) circularly polarized light, traveling in the $\hat{z}$ direction:

$$\vec{E}_{AC} = E_{AC}\left(\frac{\hat{x}+i\hat{y}}{\sqrt{2}}\right) \qquad J_z = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left(\vec{E}_{AC}\times\vec{E}_{AC}^{*}\right) = -i E_{AC}^2\,\hat{z}$$

$$H_{AC} = -\frac{1}{2}\alpha_0(\omega)\,E_{AC}^2 + \frac{1}{2}\alpha_1(\omega)\,E_{AC}^2\,J_z$$

$$H_{AC} + H_{DC} = -\frac{1}{2}\left(\alpha_0(0)E_{DC}^2 + \alpha_0(\omega)E_{AC}^2\right)$$

$$+ \frac{1}{2}\alpha_1(\omega)\,E_{AC}^2\,J_z + \frac{1}{2}\alpha_2(0)\,E_{DC}^2\,\frac{3J_z^2 - J(J+1)}{J(2J-1)}$$

$\rightarrow$ diagonal in the $J = |1,1\rangle, |1,0\rangle, |1,-1\rangle$ basis

$$\Delta E = -\frac{1}{2}\left(\alpha_0(0) + 2\alpha_2(0)\right)E_{DC}^2 - \frac{1}{2}\alpha_0(\omega)E_{AC}^2$$

$$+ \frac{1}{2}\alpha_1(\omega)\,E_{AC}^2\,m - \frac{3}{2}\alpha_2(0)\,E_{DC}^2\,m^2$$

$$m = 0, \pm 1$$

3) Cont

b) Circularly polarized light, traveling in the $\hat{x}$ direction.

One could work out from $\vec{E}_{AC} = E_{AC} \frac{\hat{y} + i\hat{z}}{\sqrt{2}}$,

but it is obvious that if

$$\frac{H_{AC}(\hat{z})}{i\alpha_1(\omega)/2} = -i E_{AC}^2 \hat{z} \cdot \vec{J} = -i E_{AC}^2 J_z$$

then

$$\frac{H_{AC}(\hat{x})}{i\alpha_1(\omega)/2} = -i E_{AC}^2 J_x \qquad J_x = \begin{pmatrix} 0 & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 0 \end{pmatrix}$$

The scalar parts are the same, but the vector shift has $J_z \to J_x$

$$H = -\tfrac{1}{2}\left(\alpha_0(0) - 2\alpha_2(0)\right)E_{DC}^2 - \tfrac{1}{2}\alpha_0(\omega) E_{AC}^2$$

$$+ \tfrac{1}{2}\left(aJ_x - bJ_z^2\right)$$

$$a = \alpha_1(\omega) E_{AC}^2 \qquad b = 3\alpha_2(0) E_{DC}^2$$

$$aJ_x - bJ_z^2 = \begin{pmatrix} -b & -a/\sqrt{2} & 0 \\ -a/\sqrt{2} & 0 & -a/\sqrt{2} \\ 0 & -a/\sqrt{2} & -b \end{pmatrix}$$

We need to determine the Eigenvalues of this Hamiltonian, which is done finding the roots of the determinant of

$$\left| aJ_x - bJ_z^2 - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right|$$

3) Cont'd

$$\begin{vmatrix} -b-\lambda & -a/\sqrt{2} & 0 \\ -a/\sqrt{2} & \cancel{-\lambda} & -a/\sqrt{2} \\ 0 & -a/\sqrt{2} & -b-\lambda \end{vmatrix} =$$

$$\underbrace{0 \times \left(\frac{a^2}{2} + 0 \times \lambda \right)}_{0} - \left(\frac{-a}{\sqrt{2}} \right) \left((+b+\lambda) \frac{a}{\sqrt{2}} - 0 \times \frac{a}{\sqrt{2}} \right) + (b+\lambda)\left((b+\lambda)\lambda - \frac{a^2}{2} \right)$$

$$= \frac{a^2}{2}(b+\lambda) - (b+\lambda)\left[(b+\lambda)\lambda - \frac{a^2}{2} \right]$$

$$= (b+\lambda)\left[\frac{a^2}{2} - (b+\lambda)\lambda + \frac{a^2}{2} \right] = (b+\lambda)(-\lambda^2 - b\lambda + a^2)$$

So the solutions are

$$\lambda = -b, \quad \lambda = \frac{-b}{2} \pm \sqrt{\frac{b^2}{4} + a^2}$$

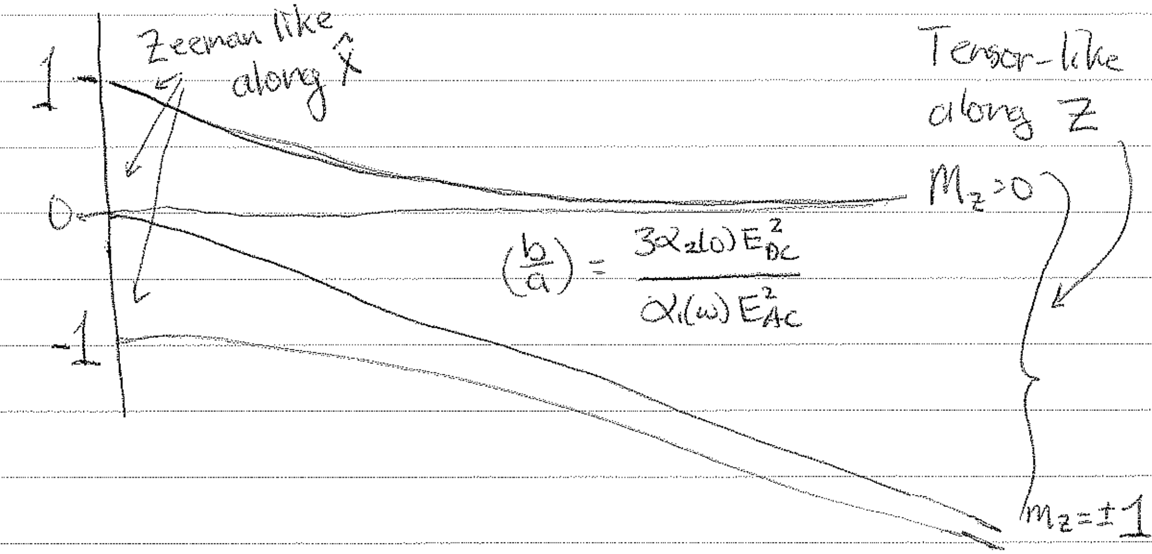
$$= -\frac{1}{2}\left(b \pm \sqrt{b^2 + 4a^2} \right)$$

$$\text{So } \Delta U = -\tfrac{1}{2}\left(\alpha_0(0) - 2\alpha_2(0) \right) E_{DC}^2 - \tfrac{1}{2}\alpha_0(\omega) E_{AC}^2$$

$$+ \begin{cases} -3\alpha_2(0) E_{DC}^2 \\ \quad \text{or} \\ -\frac{1}{2}\left(3\alpha_2(0) E_{DC}^2 \pm \sqrt{9\alpha_2^2(0) E_{DC}^4 + 4\alpha_1^2(\omega) E_{AC}^4} \right) \end{cases}$$

3) cont.

Sketch energies:



c) This is essentially identical to a situation with a static E-field & static B-field.

4)

Solution: Optical Dipole Trap

a) $I(r) = I(0)\, e^{-2r^2/w^2}$

$$P = \int I(r)\, d^2r = I(0)\, 2\pi \int_0^\infty r\,dr\, e^{-2r^2/w^2}$$

using $\int_0^\infty x e^{-x^2} dx = \frac{1}{2}$, changing variables to

$$x^2 = 2r^2/w^2$$
$$2x\,dx = 4r\,dr/w^2$$
$$r\,dr = \frac{w^2}{2} x\,dx$$

then $P = I(0)\, 2\pi \int_0^\infty \frac{w^2}{2} x e^{-x^2} dx = \frac{2\pi w^2}{4} I(0)$

$$\boxed{I(0) = \frac{2P}{\pi w^2}}$$

b) the dipole potential is

$$U = \frac{\hbar\delta}{2} \ln\left[1 + \frac{\Omega^2/2}{\delta^2 + \Gamma^2/4}\right] \qquad \text{where } 2\Omega^2/\Gamma^2 = I/I_0$$

$$U = \frac{\hbar\delta}{2} \ln\left[1 + \frac{2\Omega^2/\Gamma^2}{1 + (2\delta/\Gamma)^2}\right] = \frac{\hbar\delta}{2} \ln\left[1 + \frac{I/I_0}{1 + (2\delta/\Gamma)^2}\right]$$

using the expression from a) for I at $z=0$, $r=0$:

$$\boxed{U = \frac{\hbar\delta}{2} \ln\left[1 + \frac{\frac{2P}{\pi w_0^2 I_0}}{1 + (2\delta/\Gamma)^2}\right]}$$

4)

c) Taking Na to be a 2-level atom with $I_0 =$ 6 mW/cm^2 (this is true for the transition $3S_{1/2}$ (F = 2, m_F = 2) $\rightarrow$ $^3P_{3/2}$ (F = 3, m_F = 3) with σ^+ light)

$$I/I_0 = \frac{2P}{\pi w_0^2 I_0} = \frac{2 \cdot 1W}{\pi (10^{-3} cm)^2 \, 6 \times 10^{-3} W/cm^2} = 1.06 \times 10^8$$

$$(2\delta/\Gamma)^2 = \frac{2 \cdot 10^5 \, MHz}{10 \, MHz} = (2 \times 10^4)^2 = 4 \times 10^8$$

$$U = \frac{\hbar\delta}{2} \ln\left(1 + \frac{I/I_0}{1 + (2\delta/\Gamma)^2}\right) = \frac{\hbar\delta}{2} \ln\left(1 + \frac{1.06}{4}\right)$$

$$\frac{U}{h} = \frac{\delta/2\pi}{2} \ln\left(1 + \frac{1.06}{4}\right) = \frac{\delta/2\pi}{2} \, 0.235$$

for $\delta/2\pi$ = 100 GHz = 10^5 MHz

$$\frac{U}{h} = 1.2 \times 10^4 \text{ MHz}$$

$$\frac{U}{k_B} = \frac{6.6 \times 10^{-34} \, J \cdot s \cdot 1.2 \times 10^{10} \, Hz}{1.38 \times 10^{-23} \, J/K} = 0.56 \, K$$

this indicates why only very low temperature atoms can be trapped easily.

4)

2d) $$F = -\nabla U = \frac{-\hbar\delta}{2}\left(\frac{\nabla I/I_0}{1+I/I_0+(2\delta/\Gamma)^2}\right)$$

where $I(r) = I(0)\,e^{-2r^2/w_0^2}$ in the focal plane

$$\frac{dI}{dr} = \frac{-4r}{w_0^2} I(r)$$

so $$F = -\frac{\hbar\delta}{2}\,\frac{\left(\frac{4r}{w_0^2}\cdot I(r)/I_0\right)}{1+\frac{I(r)}{I_0}+\left(2\delta/\Gamma\right)^2}$$

$$\left.\frac{dF}{dr}\right|_{r=0} = \frac{2\hbar\delta}{w_0^2}\cdot\frac{I(0)/I_0}{1+I(0)/I_0+(2\delta/\Gamma)^2} = K,$$ the spring constant

$$\omega_r = \sqrt{\frac{K}{M}} = \left[\frac{2\cdot 6.6\times10^{-34}\,J\cdot s\cdot 10^{11}\,Hz}{(10^{-5}m)^2\cdot 23\cdot 1.67\times10^{-27}\,kg}\;\frac{1.06\times10^{8}}{1+10^{8}+4\times10^{8}}\right]$$

$$= 2.7\times10^{6}\,s^{-1}$$

$$\frac{\omega_r}{2\pi} = 430\ kHz$$

in the longitudinal direction we take $I(z) = \dfrac{I(0)}{1+(z/z_0)^2}$

and, proceeding as before get

$\omega_{long} = 3.48\times10^{4}\,s^{-1}$

$\dfrac{\omega_{long}}{2\pi} = 5.5\ kHz$

note how much weaker the trap is in the longitudinal direction than in the radial direction.

4)

a) What is the scattering rate for an atom at the focus. What is it for the given parameters?
What is the heating rate for such an atom ~~in K/s~~. Calculate this in K/s for the parameters given.

sol: $R = \frac{\Gamma}{2} \frac{I/I_0}{1 + I/I_0 + (2\delta/\Gamma)^2}$ with $I/I_0 = \frac{2P}{\pi w_0^2 I_0}$

$I/I_0 \sim 10^8$

$(2\delta/\Gamma)^2 \sim 4\times10^8$ $\quad R = \frac{\Gamma}{2}\cdot\frac{10^8}{1+5\times10^8} = \frac{\Gamma}{10} = \frac{6.3\times10^7\ s^{-1}}{10} =$

6.3×10^6/sec

$R = 6.3\times10^6\ s^{-1}$

heating rate $= 2R\cdot\frac{(\hbar k)^2}{2M k_B} = 2R\cdot\frac{T_{rec}}{k_B} = 12.6\times10^6\ s^{-1}\cdot\frac{T_{rec}}{2} =$

T_{rec} ≈ ½ recoil with T_{rec} $\quad 6.3\times10^6\ s^{-1}\cdot 2.4\times10^{-6}\ K =$

15 K/s.

as δ increases $R \sim 1/\delta^2$ but $U \sim 1/\delta$ so the ratio U/R improves.

solutions

5)

$\pi/2$ + Free induction decay

a i. $T \ll \Gamma^{-1}$

ii. Bloch equations are:
$$\dot{u} = \delta v - \Gamma/2\, u$$
$$\dot{v} = -\delta u + \Omega w - \Gamma/2\, v$$
$$\dot{w} = -\Omega v - \Gamma/2 - \Gamma w$$

for $\delta = 0$, $\Gamma = 0$ we have

$$\dot{u} = 0$$
$$\dot{v} = \Omega w$$
$$\dot{w} = -\Omega v$$

by inspection the solution for $w(0) = -1/2$ is

$$w = -\tfrac{1}{2} \cos \Omega t$$
$$v = -\tfrac{1}{2} \sin \Omega t$$

differentiation w.r.t. t gives the original diff. eqns.

condition for a $\pi/2$ pulse is $\int_0^T \Omega(t)\, dt = \pi/2$

with $\frac{E_0 \cdot d_{eg}}{\hbar} = \Omega$ $\quad \frac{d_{eg}}{\hbar} \int_0^T E_0(t)\, dt = \pi/2$

5)

a. iii. $\Omega T = \pi/2$

$$\Omega^2 T^2 = \pi^2/4$$

$$2\Omega^2/\Gamma^2 = I/I_0$$

$$\Omega^2 = \frac{\Gamma^2 I/I_0}{2}$$

$$(I/I_0)\cdot\frac{\Gamma^2 T^2}{2} = \pi^2/4$$

$$I/I_0 = \frac{\pi^2}{2}\frac{1}{(\Gamma T)^2}$$

let $\Gamma^{-1} = 16\text{ ns}$

take $T = 100\text{ps}$

$$I/I_0 = \frac{\pi^2}{2}\cdot\left(\frac{16\times10^3}{100}\right)^2 = \frac{\pi^2}{2}\cdot 2.56\times10^4 = 12.6\times10^4$$

$I_0 = 6\times10^{-3}\ W/cm^2 \Rightarrow I = 76\times10 = 760\ W/cm^2 \quad 760\ W/cm^2$

5)

After the pulse is finished, the light is turned off, $\Omega = 0$, $\delta = 0$, initial conditions

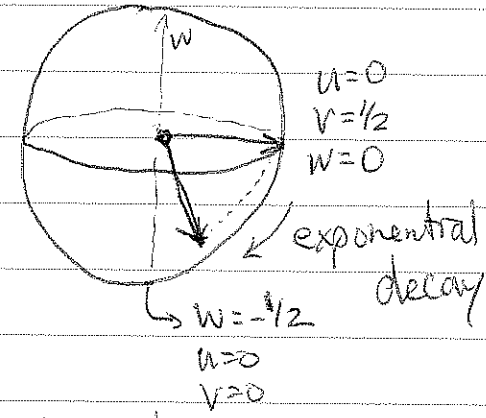
$$\dot{u} = -\Gamma/2\, u \qquad u(0) = 0$$
$$\dot{v} = -\Gamma/2\, v \qquad v(0) = -1/2$$
$$\dot{w} = -\Gamma/2 - \Gamma w \qquad w(0) = 0$$

$u = 0$

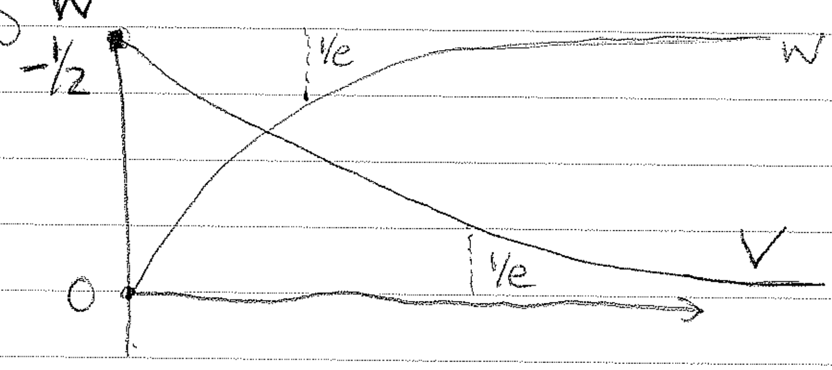
$v = -\frac{1}{2} e^{-\Gamma/2\, t}$

$w = \frac{1}{2}\left(e^{-\Gamma t} - 1\right)$

$\rho_{ee} =$



Note that V decays at half the rate of W

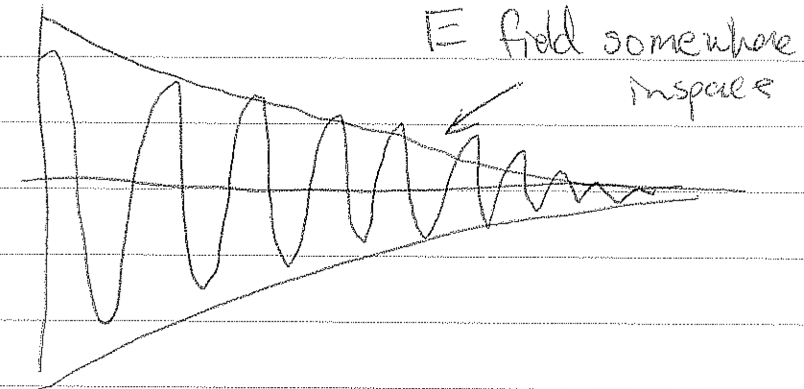


The dipole moment $\propto \mathrm{Tr}(\hat{\rho}\, \vec{d})$

but due to parity, only off diagonal terms will contribute: ρ_{eg} and ρ_{ge}.

5)

Qualitatively, the dipole oscilates and decays as it oscillates:



More quanditatively, the dipole operator will only have off-diagonal components (generally complex)

$$d = \begin{pmatrix} 0 & d_1 + i d_2 \\ d_1 - i d_2 & 0 \end{pmatrix} \rightarrow \mathrm{Tr}(\hat{\rho}\hat{d}) = i d_2(\rho_{ge} - \rho_{eg}) + d_2(\rho_{ge} + \rho_{eg})$$

$$V \equiv \frac{1}{2i}\overbrace{(\tilde{\rho}_{ge} - \tilde{\rho}_{eg})}^{\text{rotating frame}} = \frac{1}{2i}(\rho_{ge} e^{i\omega t} - \rho_{eg} e^{-i\omega t}) = |\rho_{eg}| \sin(\omega t)$$

$$|\rho_{eg}| \propto e^{-\Gamma/2\, t}$$

(6) solution:

a. $$\vec{B} = \frac{\mu_0}{2}\,\frac{I_c r^2\,\hat{z}}{\left[r^2+(z-z_0)^2\right]^{3/2}}$$ for the coil with positive helicity current

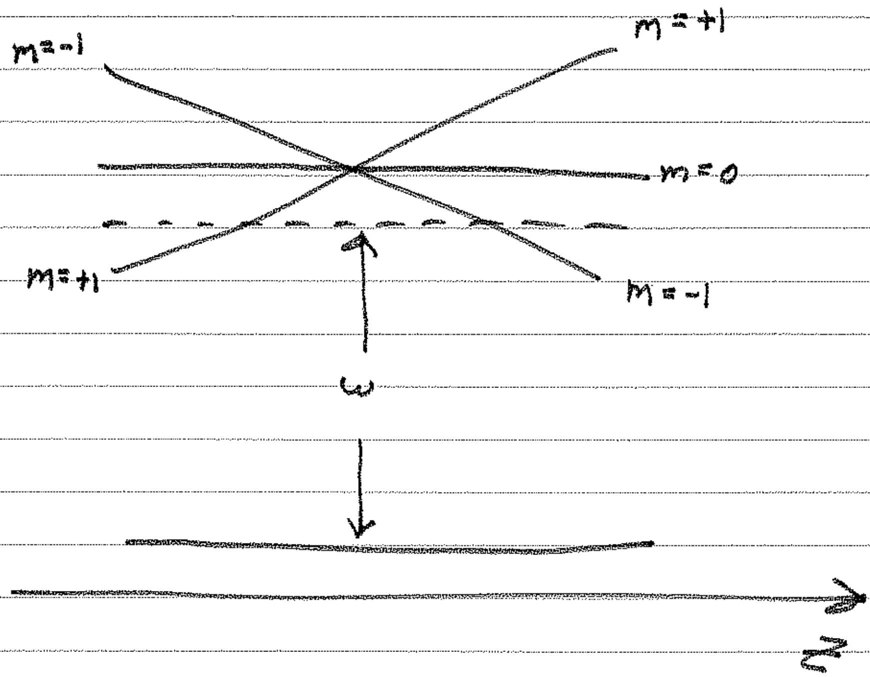
$$B_{tot} = \frac{\mu_0 I r^2 \hat{z}}{2}\left[\frac{1}{\left[r^2+(z-z_0)^2\right]^{3/2}} - \frac{1}{\left[r^2+(z+z_0)^2\right]^{3/2}}\right]$$

expanding about $z=0$ to first order:

$$\vec{B}_{tot} = \frac{\mu_0 I_c r^2 \hat{z}}{2}\cdot 2\cdot\frac{3 z_0 z}{(r^2+z_0^2)^{5/2}} = \frac{3\mu_0 I_c r^2 z_0 z}{(r^2+z_0^2)^{5/2}}\,\hat{z}$$

so $$b = \frac{3\mu_0 I_c r^2 z_0}{(r^2+z_0^2)^{5/2}}$$

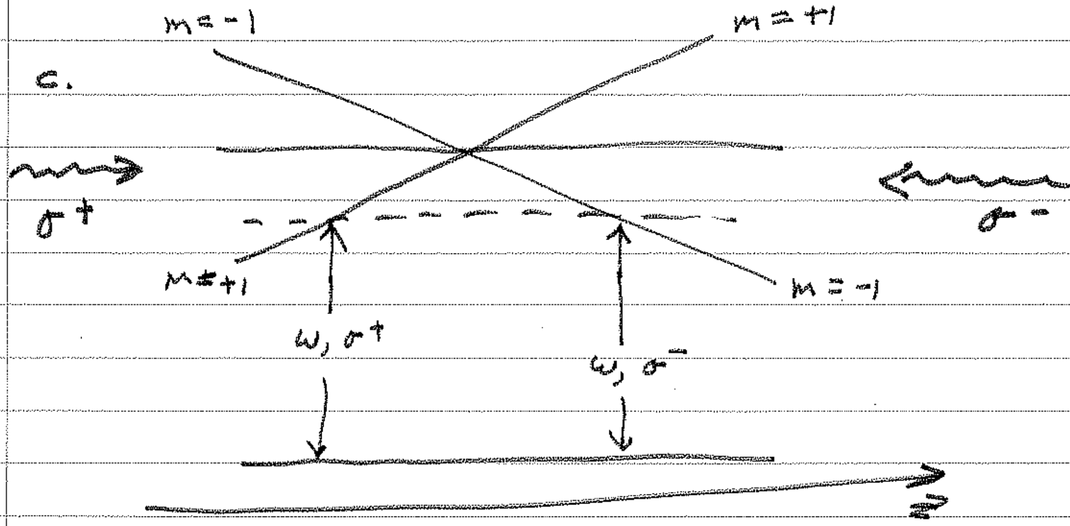
b.



Solution

(6)

c.



the σ^- should propagate along $-\hat{z}$ and σ^+ along $+z$.

d. $\beta = \dfrac{b \cdot \mu_B}{\hbar}$

e. $T \ll$ cooling time, time to traverse a Zeeman shift gradient equivalent to Γ

$T \gg 1/\Gamma$

f. $$F = \frac{\hbar k \Gamma}{2}\left[\frac{I/I_0}{1+\frac{2I}{I_0}+4\left(\frac{\delta - kv - \beta z}{\Gamma}\right)^2} - \frac{I/I_0}{1+\frac{2I}{I_0}+4\left(\frac{\delta + kv + \beta z}{\Gamma}\right)^2}\right] \quad \text{(exact)}$$

$$F = \frac{8\hbar k\left(\delta/\Gamma\right)\left(I/I_0\right)\left[kv + \beta z\right]}{\left[1 + 2I/I_0 + 4\delta^2/\Gamma^2\right]^2} \quad \text{for } kv, \beta z \ll \Gamma, \delta$$

solution

6)

g. a damped H.O. follows

$$\ddot{z} + \gamma \dot{z} + \omega_{trap}^2 z = 0$$

here $$\gamma = \frac{8\hbar k^2 (I/I_0)(\delta/\Gamma)}{M[1 + 2I/I_0 + 4\delta^2/\Gamma^2]^2}$$

and $$\omega_{trap}^2 = \frac{8\hbar k (I/I_0)(\delta/\Gamma)\beta}{M[1 + 2I/I_0 + 4\delta^2/\Gamma^2]^2}$$

where M = atomic mass

critical damping is $$\frac{\gamma^2}{4\omega_{trap}^2} = 1 = \frac{\hbar k^3 (I/I_0)(2\delta/\Gamma)}{\beta M[1 + 2I/I_0 + 4\delta^2/\Gamma^2]^2}$$

h. the damping γ and the stiffness ω_{trap}^2 have the same dependence on I/I_0 and δ/Γ, so they maximize together.

let $x = 2\delta/\Gamma$ and $y = 2I/I_0$

then γ (and ω_t^2) go like $$\gamma \sim \frac{xy}{(1+y+x^2)^2}$$

we want to simultaneously set

$$\frac{\partial \gamma}{\partial x} = 0 \quad \text{and} \quad \frac{\partial \gamma}{\partial y} = 0$$ these give $$1 = \frac{4x^2}{1+y+x^2} \quad \text{and} \quad 1 = \frac{2y}{1+y+x^2}$$

which lead to $x = 1$ and $y = 2$

that is $\delta = \Gamma/2$ and $I/I_0 = 1$

soln.

(e)

i.

$$\gamma = \frac{4\hbar k^2 (I/I_0)(2\delta/\Gamma)}{M\{1 + 2I/I_0 + (2\delta/\Gamma)^2\}^2}$$ take $2\delta/\Gamma = 1$ $I/I_0 = 1$

$$\gamma = \frac{4\hbar k^2}{16M} = \frac{1}{2}\frac{\hbar k^2}{2M}$$ but $\frac{\hbar k^2}{2M} = 2\pi \times 25\,\text{kHz}$

so $\gamma = \pi \times 25\,\text{kHz}$

$T_{damp} = \gamma^{-1} = 13\,\mu s$

the motion is strongly overdamped for $\beta = 14\,\text{MHz/cm}$, and could be made underdamped by increasing the detuning.