lecture #20 8 Nov. 2005

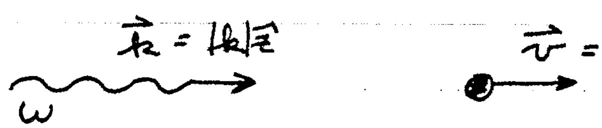
Laser Cooling and Optical Molasses
(Doppler Cooling)

recall the force on a 2-level atom in a plane wave

$$\langle \vec{F} \rangle_{scatt} = \underbrace{\hbar \vec{k}}_{\text{photon momentum}} \underbrace{\frac{\Gamma}{2} \frac{I/I_0}{1 + I/I_0 + (2\delta/\Gamma)^2}}_{\text{scattering rate}}$$

this is purely the scattering (dissipative, spontaneous ...) force because $\nabla \Omega = 0$. (This is the term that arises from $\nabla \phi$ in the O.B.E. derivation of the force.)

Now consider the Doppler effect for a wave in the $+z$ direction:



the apparent frequency in the atom's frame is $\omega - \vec{k}\cdot\vec{v}$.

The $\langle$force$\rangle$ is along $+\hat{z}$

The $\langle F \rangle$ for this situation is

$$\langle \vec{F} \rangle_{scatt} = \hbar \vec{k} \frac{\Gamma}{2} \frac{I/I_0}{1 + I/I_0 + \left[\frac{2(\delta - \vec{k}\cdot\vec{v})}{\Gamma}\right]^2}$$

Note: recall from the homework that in addition to to a Doppler shift $\vec{k}\cdot\vec{v}$, there is also a recoil shift $\Delta\omega_{rec} = \frac{\hbar k^2}{2M}$. Here we ignore that shift or imagine that it is included in a redefinition of the resonance frequency ω_0.

Let us assume that we have a velocity and detuning such that $\delta - \vec{k}\vec{v} = 0$, i.e., the atom is on resonance.

The acceleration is

$$\frac{F}{M} = \frac{\hbar k \Gamma}{2M} \frac{I/I_0}{1 + I/I_0} = v_{rec} \frac{\Gamma}{2} \frac{I/I_0}{1 + I/I_0}$$

with $v_{rec} = \hbar k / M$

$\left(\frac{F}{M}\right)_{max} = a_{max} = v_{rec} \, \Gamma/2$, and typically a will be smaller

Accelerating at this rate, the change in velocity

$$\Delta v(t) = \frac{v_{rec}\Gamma}{2} \cdot t$$

set $k\Delta v = \Gamma$:

$$k\Delta v(t_{ext}) = \frac{k v_{rec}\Gamma}{2} t_{ext} = \Gamma$$

$$t_{ext} = \frac{2}{k v_{rec}} = \frac{2M}{\hbar k^2} = \frac{\hbar}{E_{rec}}$$

$$\text{where } E_{rec} = \frac{\hbar^2 k^2}{2M}$$

t_{ext} is the characteristic time for the evolution of the center-of-mass (external variable) motion.

It is the characteristic time for changing the Doppler shift by a ~~#~~ linewidth.

If $\Gamma \gg \frac{E_{rec}}{\hbar}$ then many spontaneous emissions

happen before the atomic velocity changes very much, and it makes sense to write

$$\langle \vec{F} \rangle_{scatt} = \hbar \vec{k} \frac{\Gamma}{2} \frac{I/I_0}{1 + \frac{I}{I_0} + \left[\frac{2(\omega - k\cdot v)}{\Gamma}\right]^2}$$

if this were not true, the average over many absorption — spontaneous emission events would not make sense because a <u>single</u> event would change the velocity & hence the effective detuning.

Is this the case — is $\Gamma \gg E_{rec}/\hbar$?

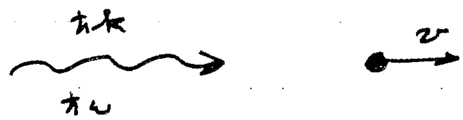
take Na: $\Gamma/2\pi = 10$ MHz

$E_{rec}/h = 25$ kHz so $\Gamma/E_{rec} \cong 400$

for most heavy atoms and allowed transitions, this ratio will be similarly large.

This also means that for a 2-level atom, with the O.B.E.s bringing steady state in times $\sim \Gamma^{-1}$, the internal state of the atom reaches steady state on a time scale short compared to the evolution of the C.O.M.

return again to

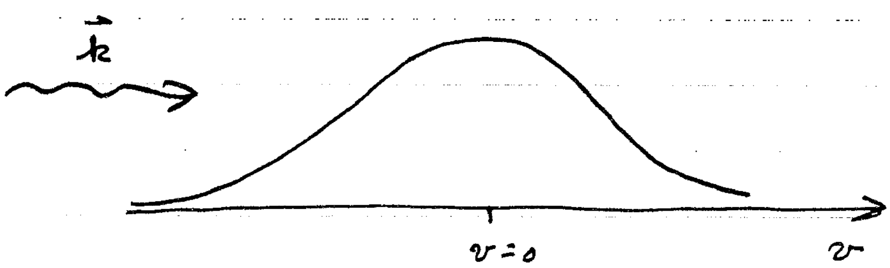


$$\langle F \rangle = \frac{\hbar k}{2} \Gamma \frac{I/I_0}{1 + I/I_0 + \left[\frac{2(\delta - kv)}{\Gamma}\right]^2}$$

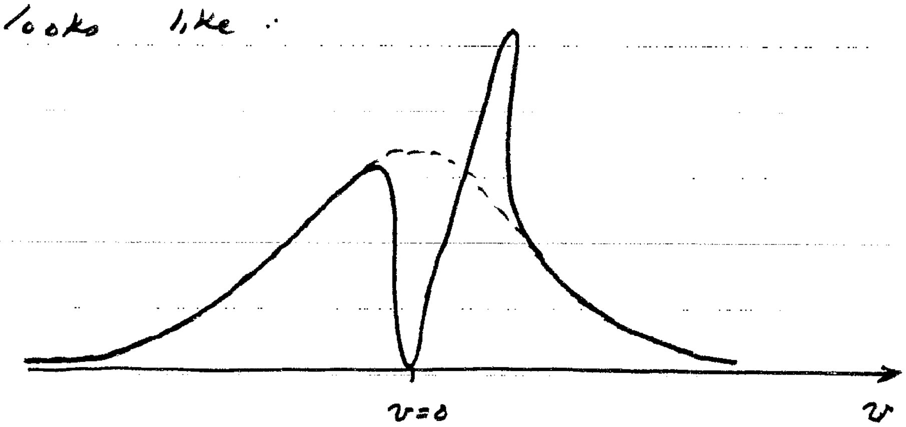
where we have dropped the vectors $\vec{k} \cdot \vec{v}$ in the scalar product, and take $\vec{v}$ along $+\hat{z}$ to be positive.

this is a velocity-dependent force, not derivable from a potential, not conservative. It can be used to damp the atomic velocity.

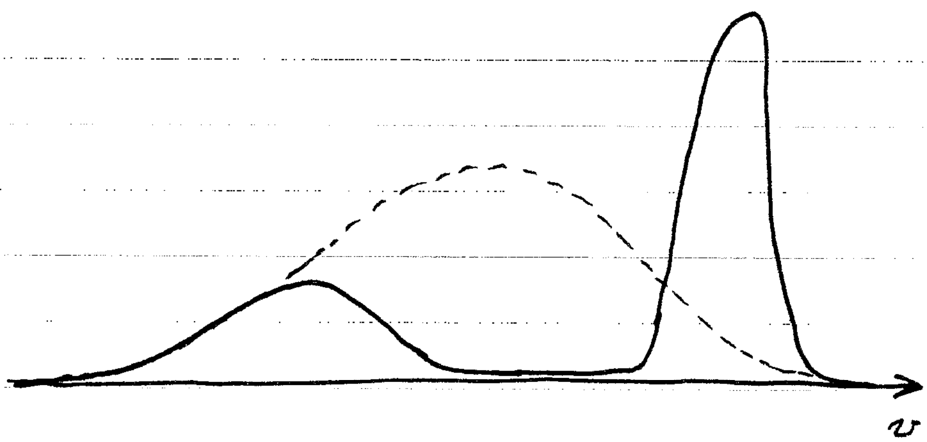
To see how this can work, consider a gas with a distribution of velocities, irradiated by a plane-wave tuned on resonance



atoms near $v=0$ will have a large positive acceleration, and after a while the distribution looks like:



after a longer time, we have something like:



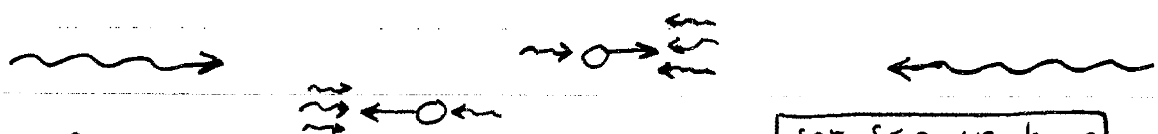
atoms are pushed to positive velocities, piling up in a peak that is generally narrower than the original distribution.

But the average velocity is increasing at all times (though more slowly as time goes on). If we want a conventional cold gas, we want a force that balances the radiation-pressure force.

Trapped ions have such a force applied with static or near-static electric + magnetic fields.

With neutral atoms this is possible, but harder (lower forces, field shift energy levels differently in contrast to shifts on charged particles.)

For neutral atoms the typical thing is to use counterpropagating laser beams



$\delta < 0$

$\boxed{\text{for } \delta < 0 \text{ we have damping}}$

$$F = F_+ + F_- = \frac{\hbar k \Gamma}{2} \frac{I/I_0}{1 + \left[\frac{2(\delta - kv)}{\Gamma}\right]^2} - \frac{\hbar k \Gamma}{2} \frac{I/I_0}{1 + \left[\frac{2(\delta + kv)}{\Gamma}\right]^2}$$

where we have take $I/I_0 \ll 1$

Note that we now have a standing wave, and strictly speaking $F_{Scatt} = 0$, leaving only the F_{dipole}.

But, for $I/I_0 \ll 1$ we can ignore the stimulated emission compared to spontaneous emission and treat the action of each beam separately, as if it produced an independent radiation pressure

Aside: We can indeed treat this weak standing wave as a dipole-force problem, and will then find that there is a non-adiabatic correction to the usual sinusoidally vary force (sinusoidal-like with position) this velocity-dependence of the dipole force will give the sum of the radiation pressure forces written above, and the sinusoidal dipole-force will average to zero.

This shows that there is a degree of ambiguity about what is to be considered a dipole force and what is a scattering force.

Now let us simplify the force expression for $I/I_0 \ll 1$. Take $|kv| \ll \Gamma, \delta$. We expand to 1st order in kv:

$$F(v) = F_+ + F_- = \underbrace{F_+(v=0) + F_-(v=0)}_{=\,0} + v \cdot \left(\frac{dF_+}{dv} + \frac{dF_-}{dv} \right)$$

$$\frac{dF_\pm}{dv} = \mp \frac{\hbar k \Gamma}{2} \, \frac{I/I_0 \left[\mp 4 \left(\frac{\delta \mp kv}{\Gamma} \right) \frac{2k}{\Gamma} \right]}{\left[1 + (2\delta/\Gamma)^2 \right]^2} = \frac{2\hbar k^2 (I/I_0)(2\delta/\Gamma)}{\left[1 + (2\delta/\Gamma)^2 \right]^2}$$

(Note that the expressions are simpler with normalized intensity (I/I_0) and detuning $(2\delta/\Gamma)$ kept intact.)

$$F(v) = F_+(v) + F_-(v) = \frac{4\hbar k^2 (I/I_0)(2\delta/\Gamma)}{[1 + (2\delta/\Gamma)^2]^2} v \equiv -\alpha v$$

$\alpha > 0$ (friction) for $\delta < 0$

$\vec{F} \cdot \vec{v} = -\alpha v^2 = \dot{E}_{cool}$, the energy cooling rate

note that for $\delta/\Gamma \to 0$ and $\delta/\Gamma \to \infty$ $\alpha \to 0$

so there is a value of δ/Γ that optimizes the friction coefficient, the damping α.

note that $\alpha = \hbar k^2 \cdot$ (dimensionless ratios)

$$a = \frac{F}{M} = \frac{\hbar k^2 v}{M} (\cdot \cdot \cdot \cdot) = -\frac{\alpha}{M} v \equiv -\gamma v$$

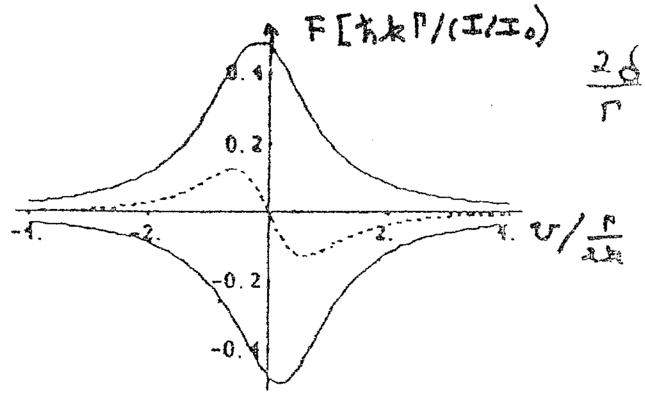
or $$\frac{dv}{dt} = -\gamma v$$

γ is the damping rate of the velocity $\langle v(t) \rangle = v(0) e^{-\gamma t}$

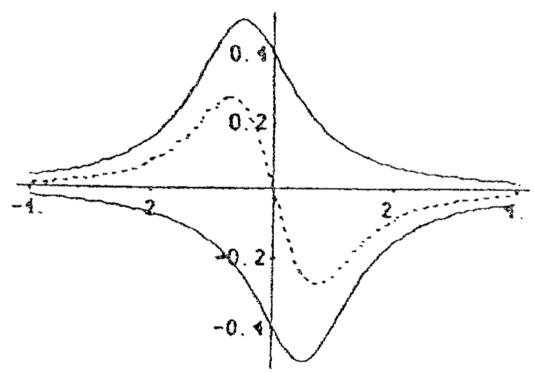
$\gamma^{-1} = \frac{M}{\hbar k^2} (\cdot \cdot \cdot)$ recall that $\frac{2M}{\hbar k^2} = \tau_{ext}$, the characteristic time, $\hbar / E_{rec}$ for changing the velocity by an amount sufficient to Doppler shift by Γ.

The velocity damping time has the same form. Note that it does not depend on Γ, but only on M and k.

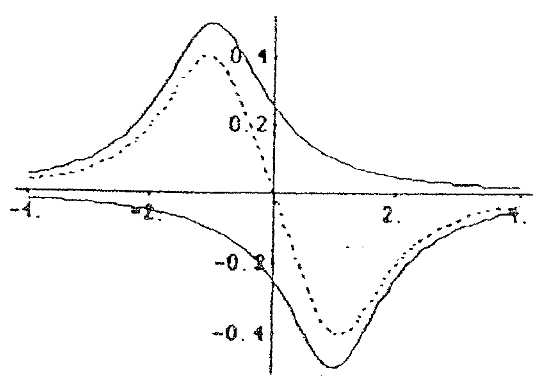
Here are F_+, F_- and F_{tot} for various detunings



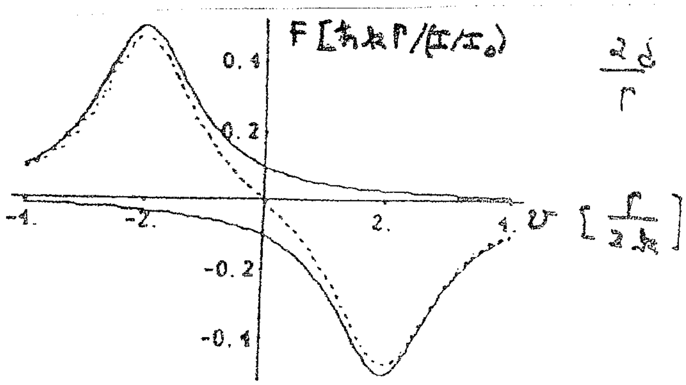
$\frac{2\delta}{\Gamma} = 0.2$



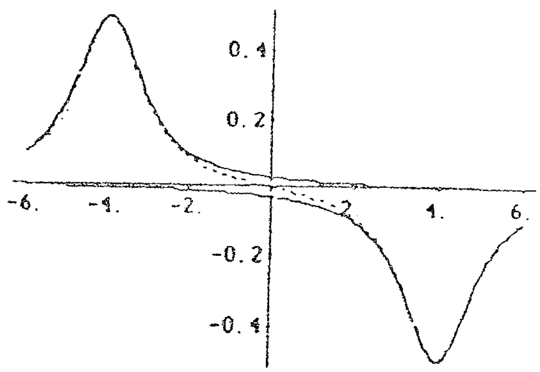
$\frac{2\delta}{\Gamma} = 0.5$



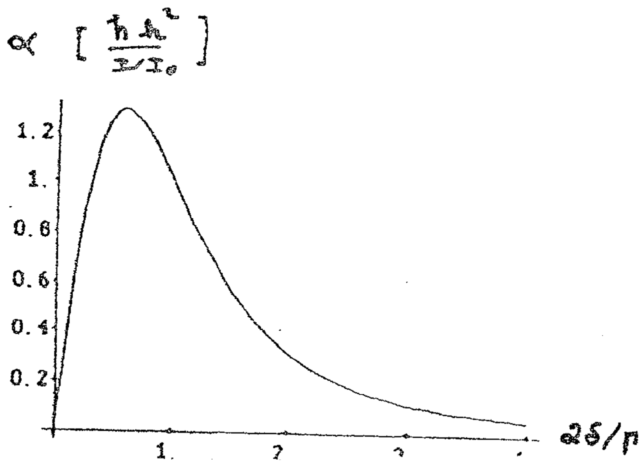
$\frac{2\delta}{\Gamma} = 1$



$\frac{2\delta}{\Gamma} = 2$



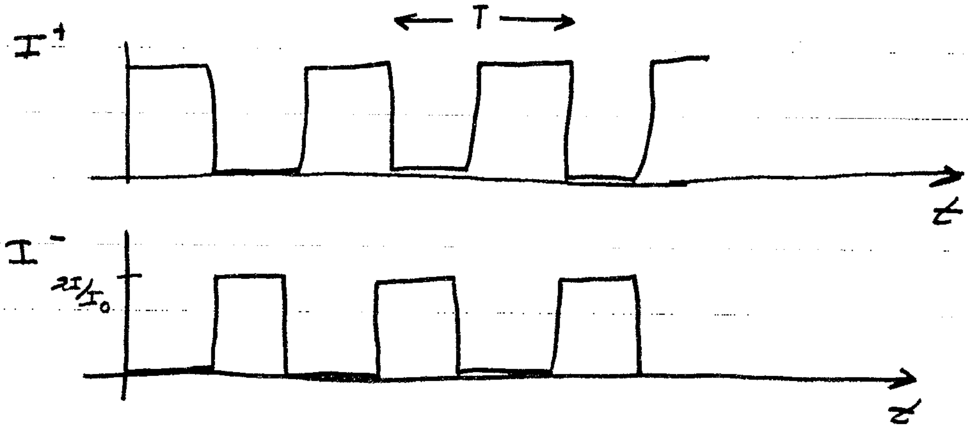
$\frac{2\delta}{\Gamma} = 4$



the friction maximizes at δ slightly less than $2\delta/\Gamma$.

Now, let us go beyond the $I/I_0 \ll 1$ limit.

In general, this is difficult because the standing wave effects are rather complicated. But we can get a good idea of moderate $I/I_0 \approx 1$ with an exact model of alternating the two beams.



We assume only one beam on at a time, 50% duty factor. Take the peak intensity of each beam to be $2I/I_0$ so the ave intensity is I/I_0

In this model, the force is:

duty factor

$$\langle F\rangle = \langle F_+\rangle + \langle F_-\rangle = \frac{\hbar k\Gamma}{2}\left[\frac{\frac{1}{2}\cdot 2I/I_0}{1+2I/I_0+\left[\frac{2(\delta-kv)}{\Gamma}\right]^2} - \frac{\frac{1}{2}\cdot 2I/I_0}{1+\frac{2I}{I_0}+\left[\frac{2(\delta+kv)}{\Gamma}\right]^2}\right]$$

saturation due to instanteneous power

Using the same approach as before we find

$F = -\alpha v$ with

$$\alpha = \frac{-4\hbar k^2 (I/I_0)(2\delta/\Gamma)}{\left[1 + \underbrace{2I/I_0}_{} + \left(\frac{2\delta}{\Gamma}\right)^2\right]^2}$$

this is the extra term that is due to saturation.

This is a reasonable approximation for a continuous standing wave for $I/I_0 \lesssim 1$. It fails badly for $I/I_0 >> 1$. It is a good approximation when counterpropagating σ^+/σ^- waves are used on a $J=0 \rightarrow J=1$ transition because no standing wave effects occur.

(see P. Lett et. al J. Opt. Soc. Am B 6, 2084 (1989).)

Both velocity and kinetic energy damp exponentially to zero with damping constants γ and 2γ.

But this is only the effect of the *average* force. In fact

$$F = \langle F \rangle + \delta F$$

where δF is a fluctuating force of zero mean.

The effect of this fluctuating force is to give fluctuating impulses to the atom, causing to momentum to random walk.

This diffusion of momentum is characterized by a momentum diffusion constant D_p :

$$\langle \mathring{p^2} \rangle \equiv 2 D_p$$

so $\frac{1}{2} \frac{\langle \mathring{p^2} \rangle}{M} = \mathring{E}_{heat} = D_p / M$

which increases the K.E. while the cooling decreases it. At equilibrium $\mathring{E}_{cool} + \mathring{E}_{heat} = 0$

$$D_p / M - \alpha \langle v^2 \rangle = 0 \quad \text{or}$$

$\frac{D_p}{2\alpha} = \frac{1}{2} M \langle v^2 \rangle = \frac{1}{2} k_B T$ (the K.E. per degree of freedom in stat. mech.)

so $$k_B T = \frac{D_p}{\alpha}$$

(This is from Einstein's 1905 theory of Brownian motion.