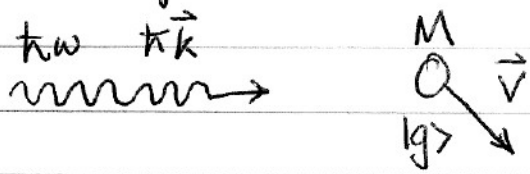
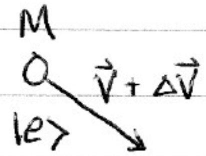
Homework # 6

1) Consider the resonant absorption of a photon by a moving atom:



resulting in an excited atom:

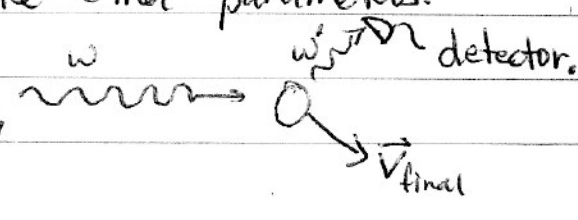


Assume the internal energy levels of the atom in the rest frame are E_g and E_e, and $\hbar\omega_0 = E_e - E_g$.

a) Calculate, using conservation of energy + momentum, $\Delta\vec{v}$ and $(\omega - \omega_0)$ for resonant absorption.

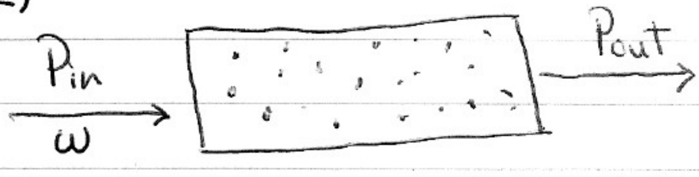
b) identify the term in $\omega - \omega_0$ that corresponds to the Doppler shift; discuss the other term. What happens when $\vec{v} = 0$? or when $\vec{v} \perp \vec{k}$

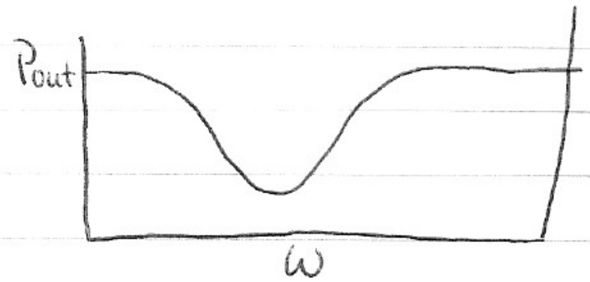
c) Consider that the excited atom radiates (ie. scatters the incident photon), and that there is a detector in the plane of $\vec{k}_{in}$ and $\vec{V}_{final}$. What is the frequency ω' of the detected photon as a function of θ and the other parameters? For what values of Γ would this be experimentally observable?



Homework #6 continued

2)





Consider a cell containing atoms at temperature T, with a density small enough that the 'absorption' is always small.

a) Write down the expression (involving an integral) for the lineshape of the absorption spectrum. (Ignore multiplicative constants.)

b) Evaluate this lineshape for the cases

i) $\Gamma \ll kV_{th}$

$$V_{th} = \sqrt{\frac{K_B T}{M}}$$

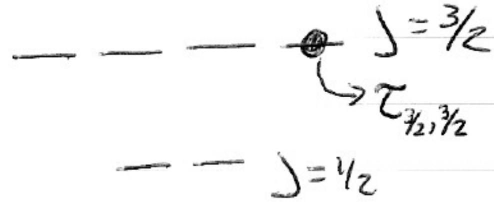
ii) $\Gamma \gg kV_{th}$

c) Discuss what this looks like for Na atoms for $T = 100^\circ C$, $\lambda = 589\,nm$ $\Gamma/2\pi = 10\,MHz$, assuming Na to be a 2-level atom.

Homework #6

3) Imagine an atom whose two lowest electronic levels have $J = 3/2$ and $J = 1/2$

a) Assume $E_{J=3/2} > E_{J=1/2}$, and that the excited state lifetime in the $|J = 3/2, m_J = 3/2\rangle$ state is $\tau_{3/2,3/2}$.



i) what is the lifetime of the $|J = 3/2, m_J = 1/2\rangle$ state?

ii) What are the branching ratios from $|J = 3/2, m_J = 1/2\rangle$ to the $J' = 1/2$ ground states?

b) Assume a fictitious different atom has <u>exactly</u> the same matrix elements $\langle J'M'|H_I|JM\rangle$ as the atom in a), but $E_{J=1/2} > E_{J=3/2}$

i) what is the lifetime $\tau'_{1/2,1/2}$ of this excited atom $|J' = 1/2, m_J' = 1/2\rangle$ in terms of $\tau_{3/2,3/2}$

ii) What are the branching ratios to various ground states?

Note: even if you use your intuition to answer these questions, use the Wigner Eckart theorem and Clebsch Gordan coefficients to prove your answer.