Homework #4 due: Thurs. Oct. 6

1a. Using the optical Bloch equations, find the steady-state solution for u, v, w.

1b. Use the solution to 1a. to get the steady state value of ρ_{ee}, the excited state population. ($\tilde{\sigma}_{ee}$ in the lecture notes)

1c. What is the steady state rate, per atom, at which photons are spontaneously emitted by the driven, 2-level atom?

1d. What is the maximum rate at which photons can be spontaneously emitted in steady state?

2. In homework #3, you used Fermi's Golden rule and 2^{nd} order perturbation theory to calculate the scattering rate as a function of intensity (Ω^2) and detuning $\delta\omega$. Compare that solution with answer 1c above, discussing when they agree and don't, and why.

~~due~~

Homework #4 continued:

3. The spin-orbit interaction, between an electron spin and its own orbital motion around a nucleus, takes the form:

$$H_{SO} = \xi(r)\, \vec{l} \cdot \vec{s}$$

where $\vec{s}$ = spin angular momentum

$\vec{l}$ = orbital angular momentum

a) Use a classical argument to find an expression for $\xi(r)$ based on the interaction of a spin (with moment $\vec{\mu} = g/2 \; e/m \; \vec{S}$) with the magnetic field it sees when traveling through the electric field of the atom with velocity $\vec{v}$. Assume circular motion of the electron, and spherically symmetric atom potential $U(r)$, where $U(r)$ is the potential energy, not the electrostatic potential. (Here $g = 2.002319304.. \simeq 2$ is the gyromagnetic ratio of the electron)

Write the answer in terms of $U(r)$ and fundamental constants.

b) Unless you know special relativity very well, the simple argument you gave for a) is not right. (It does have the right spirit, though!) The error comes from the fact that we've neglected a relativistic kinematic precession (known as Thomas precession) that decreases the spin-orbit interaction.

→ cont.

Homework #4

3 b) cont.

If a particle moves in a straight line in an electric field, then the energy is $\vec{\mu}\cdot\vec{B}'$, where $\vec{B}'$ is the magnetic field in its rest frame. If a particle is accelerated $\perp$ to its velocity ($\vec{a}\perp\vec{v}$, as in circular motion) then there is a relativistic rotation of the rest frame relative to the lab frame,

see e.g. Jackson's E+M

$$\vec{\omega}_T \simeq \frac{\vec{a}\times\vec{v}}{c^2} \qquad \left(\frac{v}{c} \ll 1\right)$$

and the total energy will be shifted by $\vec{S}\cdot\vec{\omega}_T$

With $\vec{a}$ determined by circular motion within the potential $U(r)$, show that the corrected form for $\xi(r)$ is

$$\xi(r) = \left(\frac{g-1}{2}\right)\frac{1}{m^2c^2}\,\frac{1}{r}\,\frac{dU}{dr}$$

This description by Thomas had great historical significance, because experimentalists had known there was a factor of 2 discrepency in the Zeeman shift ($\vec{\mu}\cdot\vec{B}_{ext}$) and the spin-orbit effect.

3 c) With $\left\langle \frac{1}{r^3} \right\rangle = \left(\frac{1}{24}\right)\left(\frac{1}{a_0^3}\right)$ for the Hydrogen atom in the 2p state, estimate the spin orbit splitting of the 2p states in Hydrogen. (called fine structure.)

in real units like eV or Hz

$$\left\langle \frac{1}{r^3} \right\rangle = \left(\frac{1}{\ell(\ell+1/2)(\ell+1)\,n^3}\right)\left(\frac{1}{a_0^3}\right)$$