Home Work # 3 Due Thurs. 29 Sept.

1) Consider the process of scattering weak incident light from a 2-level atom, initially in the ground state, where

E_e is the energy of the excited state $|e\rangle$
E_g " " " " ground state $|g\rangle$
$\Big\}\ E_e - E_g = \hbar\omega_0$

$|n_{k\lambda}\rangle$ is the number of photons in the weak incident beam, with photon energy and polarization $\hbar\omega$, $\vec{\epsilon}_\lambda$.
(where weak means spontaneous emission >> stimulated emission)

i.e. consider

$$|g; n_{k\lambda}, 0\rangle \rightarrow |g; n_{k\lambda}-1, 1_{k'\lambda'}\rangle,$$

$|1_{k'\lambda'}\rangle$ is a single photon with polarization $\vec{\epsilon}_{\lambda'}$

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a) Considering the electric dipole interaction perturbatively, what is the lowest order in $\hat{\vec{d}}\cdot\hat{\vec{E}}$ that $|g; n_{k\lambda}, 0\rangle$ and $|g; n_{k\lambda}-1, 1_{k'\lambda'}\rangle$ are coupled?

b) To that order, calculate the total matrix element $V_{g\to g}$ for the scattering process. (define $\langle e|\hat{\vec{r}}|g\rangle \equiv \vec{r}_{eg}$.)
(remember, only the final state has to conserve energy)

c) Using Fermi's golden rule, what is the total scattering rate as a function of ω (parameterize the angle between $\vec{\epsilon}_\lambda$ and $\vec{r}_{eg}$ with a constant C_{eg}: $|\vec{r}_{eg}\cdot\vec{\epsilon}_\lambda|^2 = C_{eg}^2|\vec{r}_{eg}|^2$), for the following cases:

i) $\omega \ll \omega_0$
ii) $\omega \gg \omega_0$
iii) what happens as $\omega \rightarrow \omega_0$?

Homework #3 Cont.

A more careful treatment of the spread of $|e\rangle$ due to its higher order coupling to the vacuum resolves the problem in c) iii), resulting in a replacement of E_e by $E_e - i\hbar\frac{\Gamma}{2}$, where Γ is excited state lifetime (i.e. spontaneous emission A-coeff.)

d) calculate the scattering rate with E_b replaced with $E_e - i\hbar\frac{\Gamma}{2}$, assuming $\omega \simeq \omega_0$ ($|\omega-\omega_0| \ll \omega_0, \omega$). Write you answer in terms of Γ, ω, ω_0, the incident intensity I, and an appropriately defined saturation intensity I_s. (you can assume $n_{k\lambda} \gg 1$, $n_{k\lambda}-1 \simeq n_{k\lambda}$)

e) can you give a handwaving argument for the plausibility of replacing $E_e \to E_e - i\hbar\frac{\Gamma}{2}$?