Homework #2 $\boxed{\text{due Thurs. Sept. 22}}$

1) Given $H_A = \sum_i \frac{\hat{p}_i^2}{2m_i} + V_{coul}$

and $H_I = -\vec{d}\cdot\hat{\vec{E}}(\vec{R}) - \vec{\mu}_L\cdot\hat{\vec{B}}(\vec{R}) - \sum_{\alpha\beta} Q_{\alpha\beta}\frac{\partial}{\partial X_\beta}\hat{\vec{E}}_\alpha(\vec{R})$

$- \hat{\vec{\mu}}_s\cdot\hat{\vec{B}}(\vec{R}) + \frac{e^2}{2m}\sum_i\left(\hat{\vec{r}}_i \times \hat{\vec{B}}(\vec{R})\right)^2$

a) crudely estimate

$$\frac{H_{dE}}{H_A} \text{ in terms of } |\vec{E}|$$

b) crudely estimate

$$\frac{H_{\mu_L}}{H_{dE}},\ \frac{H_{\mu_s}}{H_{dE}},\ \frac{H_Q}{H_{dE}},\ \frac{H_{B^2}}{H_{dE}} \text{ in terms of } \frac{H_{dE}}{H_A}$$

assuming that all alectrons in the atoms are confined to a size $r_i < a_0$ and the radiation field has a long wavelength $a_0 k \ll 1$.

Also assume low intensity $dE \sim ea_0E \ll H_A$.

Take all fields to be free fields, ie in the far field limit, ie. the quantized fields discussed in class.

2) a) as operators, discuss the parity properties of: $\hat{\vec{d}}$, $\hat{\vec{\mu}}_L$, $\hat{\overleftrightarrow{Q}}$, $\sum_i (\vec{r}_i \times \hat{\vec{B}})^2$

on the atom space, ie under inversion of coordinates

$\hat{x} \to -\hat{x}$, $\hat{y} \to -\hat{y}$, $\hat{z} \to -\hat{z}$.

b) as operators, discuss the photon states that are coupled by

$\hat{\vec{E}}$, $\hat{\vec{B}}$ and $\sum_i (\vec{r}_i \times \hat{\vec{B}})^2$.

(ie. how many photons are involved?)

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3) Consider the $2p \to 1s$ transition in Hydrogen, ($n=2, l=1 \to n=1, l=0$) for an atom in free space, initially in $n=2, l=1$, with no photons. What is the lowest order coupling? $\to \vec{d} \cdot \vec{E}$

The $n=2$ $l=0$ state is 3-fold degenerate, $m_l = 1, 0, -1$

a) Give an argument for why the lifetime of the states

$|n, l, m_l\rangle = |2,1,1\rangle, |2,1,0\rangle\ |2,1,-1\rangle$

is the same.

(3) cont $\Rightarrow$

3) cont.

From Fermi's Golden Rule, the decay rate of a photon into a direction of solid angle $d\Omega_K$ and polarization λ is:

$$\frac{dR}{d\Omega_K} = \frac{2\pi}{\hbar} \left|\langle 1,0,0; 1_{k\lambda} | \vec{d}\cdot\hat{\vec{E}}_{k\lambda} | 2 \ell m_\ell ; 0\rangle\right|^2 \rho(\hbar\omega_0)$$

where $\hbar\omega_0 = E_{21}$ is the 2_p - $1s$ energy difference.

b) integrate this over solid angle and sum λ, and evaluate the matrix elements to get the decay rate (inverse lifetime) of the state.
Compare to the experimental value (Google?)
Values for $\hbar\omega_0 = E_{21}$ and forms for the wavefunctions $\psi_{n\ell m}(\vec{r})$ can be found in your favorite Quantum mechanics book, (or Googled).
- Ignore spin
- A useful step is to show

$$\int |\vec{M}_{21}\cdot\vec{E}_K|^2 d\Omega_K = \frac{1}{3}4\pi|\vec{M}_{21}|^2 \text{ for any fixed Vector } \vec{M}_{21}$$

in particular for $\vec{M}_{21} = \langle 2\ell m|\vec{r}|100\rangle$ ($\vec{r}$ is electron position)

$Y_{\ell m} = Y_{\ell m}(\theta,\phi)$

- Since all m-levels should give the same answer, choose the one that's easiest
- it may be useful to write x, y, z in terms of r and spherical harmonics: $x = \sqrt{\frac{8\pi}{3}}\, r\,\frac{(-Y_{11}+Y_{1-1})}{2}$, $y = \sqrt{\frac{8\pi}{3}}\, r\,(Y_{11}+Y_{1-1})/2i$ $\quad z = \sqrt{\frac{4\pi}{3}}\, Y_{10}\, r$
- it may be useful to note $\int Y_{\ell m}(\theta,\phi)\, Y^*_{\ell' m'}(\theta,\phi)\, d\Omega = \delta_{\ell\ell'}\delta_{mm'}$