Homework #1:

1) The classical energy of transverse EM fields (in a box) is given by

$$E_R = \frac{1}{2}\int_{\text{Volume}} dV\left(\epsilon_0 \vec{E}_\perp\cdot\vec{E}_\perp + \frac{1}{\mu_0}\vec{B}\cdot\vec{B}\right).$$

Using the quantized fields for $\hat{\vec{E}}$ + $\hat{\vec{B}}$,

$$\vec{E} = \sum_{k\lambda} i\vec{\epsilon}_{k\lambda} E^0_{k\lambda}\left(\hat{a}_{k\lambda} e^{i\vec{k}\cdot\vec{r}} - \hat{a}^\dagger_{k\lambda} e^{-i\vec{k}\cdot\vec{r}}\right)$$

$$\vec{B} = \sum_{k\lambda} i\,\frac{\vec{k}}{|\vec{k}|}\times\epsilon_{k\lambda} B^0_{k\lambda}\left(\hat{a}_{k\lambda}e^{i\vec{k}\cdot\vec{r}} - \hat{a}^\dagger_{k\lambda}e^{-i\vec{k}\cdot\vec{r}}\right)$$

and Maxwell's eqn's

a) show that $B^0_{k\lambda} = \dfrac{E^0_{k\lambda}}{c}$

b) determine the size of the quantum unit of field $E^0_{k\lambda}$ by requiring that the energy expression reduce to

$$H = \sum_{k\lambda}\hbar\omega_k\left(\hat{a}^\dagger_{k\lambda}\hat{a}_{k\lambda} + \tfrac{1}{2}\right)$$

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2) The momentum of transverse EM fields is

$$\vec{P}_R = \epsilon_0 \int_{\text{volume}} dV \; \vec{E}_\perp(\vec{r}) \times \vec{B}(\vec{r}) .$$

a) using the quantized field operators, simplify $\hat{P}_R$ and show that it is diagonal in the Fock basis, with the eigenvalues $\hbar k n$, where n is the number of photons in the mode.

This is <u>not</u> homework, just a reminder of coherent states that you can work on at your leisure:

Show that the following definitions of a coherent state $|\alpha\rangle$ are equivalent:

a) $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ b) $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} e^{\alpha \hat{a}^\dagger} |0\rangle$ where $e^{\alpha \hat{a}^\dagger} = \sum_n \frac{\alpha^n \hat{a}^{\dagger n}}{n!}$

c) $|\alpha\rangle = \sum_{n=0}^{\infty} f(n)|n\rangle$, where $|f(n)|^2$ is poissonian

d) a displaced ground state $e^{-i\hat{p}\ell/\hbar}|0\rangle$, where $e^{-i\hat{p}\ell/\hbar}$ is the displacement operator.

Also, show $\langle\alpha|\hat{x}|\alpha\rangle$ oscillates.