

~~Atomic Physics - 1~~

DRESSED ATOMS, continued.

— The Optical Stern-Gerlach Effect —

— ignore spontaneous emission. —

— consider an atom moving with velocity v along \hat{z} ,
that passes through a laser field with spatial variation
in intensity (e.g. standing wave) along \hat{x}

we will have dressed states with a varying coupling:

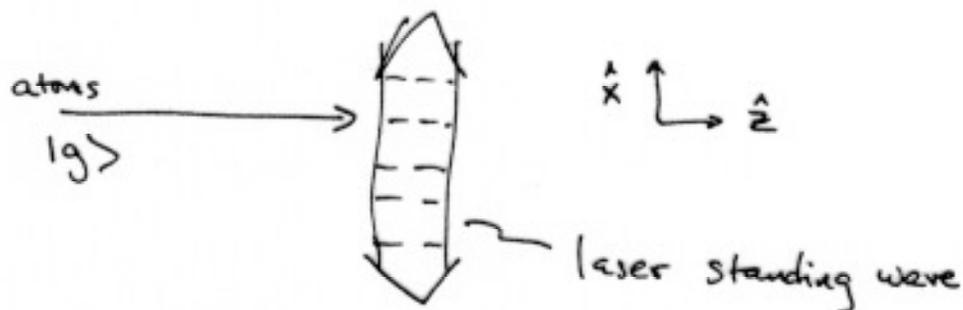
$$|1(N)\rangle = \sin\theta(x,t) |g, N+1\rangle + \cos\theta(x,t) |e, N\rangle$$

$$|2(N)\rangle = \cos\theta(x,t) |g, N+1\rangle - \sin\theta(x,t) |e, N\rangle$$

$$\text{where } \cos\theta = \frac{\delta}{[\delta^2 + \Omega_R^2(x,t)]^{1/2}} \quad ; \quad \sin\theta = \frac{\Omega_R(x,t)}{[\delta^2 + \Omega_R^2(x,t)]^{1/2}}$$

|| | |

we are considering a situation such as:



consider the case of $\delta=0$ (exact resonance)

then the 2 dressed states are:

$$|1(N)\rangle = \frac{1}{\sqrt{2}} (|g, N+1\rangle + |e, N\rangle)$$

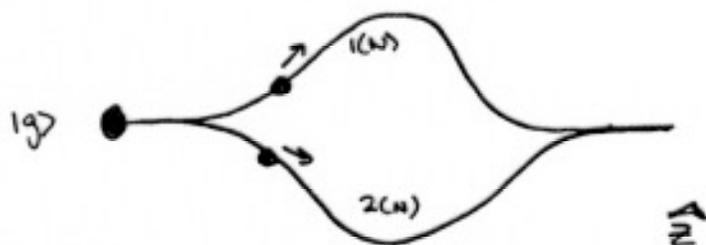
and

$$|2(N)\rangle = \frac{1}{\sqrt{2}} (|g\rangle - |e\rangle)$$

if the atoms enter the field non-adiabatically (suddenly),
then we can just project the wavefns. onto the new basis:

$$|g\rangle = \frac{1}{\sqrt{2}} |1(N)\rangle + |2(N)\rangle$$

if we assumed a gaussian laser cross-section we would draw something like:



the atom "splits" into 2 pieces travelling in the 2 dressed states.

But: recall that the laser has a varying Rabi frequency along \hat{x} , so the $1/2$ of the atom in $|1(N)\rangle$ experiences the potential $+\hbar\Omega_R(x,t)/2$, while the part in $|2(N)\rangle$ experiences $-\hbar\Omega_R(x,t)/2$

\Rightarrow this means the part of the atom in $|1(N)\rangle$ will experience a force $-\hbar\nabla\Omega_R(x,t)/2$, while the other half experiences the opposite force

\therefore after passage through this laser beam, the atom will be split into 2 components with transverse momenta that differ by $\Delta p = \hbar \int \nabla\Omega_R(x,t) !$

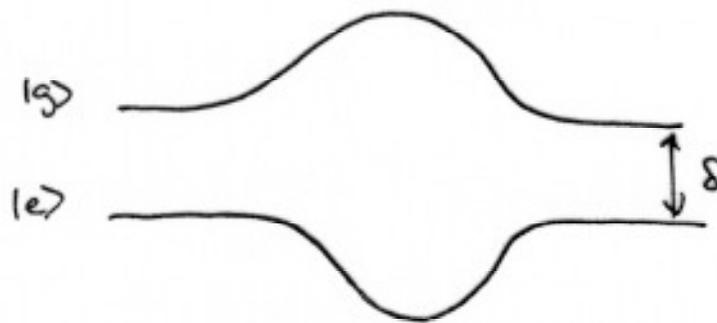
the atom splits in 2:



⇒ just like the classic Stern-Gerlach effect for the spin- $\frac{1}{2}$ system.

What happens for $\delta \neq 0$?

Our dressed atom picture looks like:



- does the atom "split", or does it follow one potential curve?

⇒ adiabatic vs. non-adiabatic ⇔

Adiabaticity:

write the time-dependent wave fn. of the system as:

$$|\psi(t)\rangle = \sum_j a_j(t) |\varphi_j(t)\rangle$$

where we will assume $|\psi(t)\rangle$ is slowly varying (the e^{iHt} term are contained in the a_j 's).

write Sch. Eq.:

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle$$

operate on this with $\langle \varphi_j(t) |$ to yield

$$\langle \varphi_j(t) | i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \langle \varphi_j(t) | H(t) |\psi(t)\rangle$$

expand $|\psi(t)\rangle$ in terms of φ_j 's:

$$\langle \varphi_j(t) | i\hbar \frac{\partial}{\partial t} \sum_i a_i |\varphi_i(t)\rangle = \langle \varphi_j(t) | H(t) | \sum_i a_i(t) \varphi_i(t) \rangle$$

$$\text{l.h.s.} = i\hbar \langle \varphi_j | \sum_i [\dot{a}_i(t) |\varphi_i(t)\rangle + a_i(t) |\dot{\varphi}_i(t)\rangle$$

$$= i\hbar [\dot{a}_j(t) + \sum_i a_i(t) \langle \varphi_j | \dot{\varphi}_i \rangle$$

(6)

$$\begin{aligned} \text{r.h.s.} &= \langle \varphi_j(t) \sum_i E_i(t) a_i(t) \varphi_i(t) \rangle \\ &= E_j(t) a_j(t) \end{aligned}$$

so we have arrived at the equation:

$$\dot{a}_j(t) = -\frac{i}{\hbar} E_j(t) a_j(t) - \sum_i a_i(t) \langle \varphi_j(t) | \dot{\varphi}_i(t) \rangle$$

let's extract the fast time dependence from the a 's, by writing

$$\tilde{a}_i(t) = a_i(t) e^{\frac{i}{\hbar} E_i t} \quad \text{or} \quad a_i(t) = \tilde{a}_i(t) e^{-\frac{i}{\hbar} E_i t}$$

we can rewrite the above equation as:

$$\begin{aligned} \left[-\frac{i}{\hbar} E_j(t) \tilde{a}_j(t) + \dot{\tilde{a}}_j(t) \right] e^{-\frac{i}{\hbar} E_j(t) t} &= -\frac{i}{\hbar} E_j(t) \tilde{a}_j(t) e^{-\frac{i}{\hbar} E_j(t) t} \\ &\quad - \sum_i \tilde{a}_i(t) e^{-\frac{i}{\hbar} E_i(t) t} \langle \varphi_j(t) | \dot{\varphi}_i(t) \rangle \end{aligned}$$

multiplying through by $e^{+\frac{i}{\hbar} E_j(t) t}$ we get:

$$\dot{\tilde{a}}_j(t) = - \sum_i \tilde{a}_i(t) e^{-\frac{i}{\hbar} (E_i - E_j) t} \langle \varphi_j(t) | \dot{\varphi}_i(t) \rangle$$

we want to determine how much of a transition there is from a state $m \rightarrow n$ due to the time dependent Hamiltonian

- let us assume that at time $t=0$ the atom is all in state $|\varphi_m\rangle$, so we can omit the summation and just integrate the equation:

$$\dot{a}_j(t) = e^{-\frac{i}{\hbar}(E_m(t)-E_j(t))t} \langle \varphi_j(t) | \dot{\varphi}_m(t) \rangle$$

we will further assume $E(t)$ & $\varphi(t)$ are slowly varying fns., so that they are essentially constant. We then solve for a_j :

$$a_j(t) = \frac{i\hbar}{E_m(t)-E_j(t)} \langle \varphi_j(t) | \dot{\varphi}_m(t) \rangle \left(e^{-\frac{i}{\hbar}(E_m-E_j)t} - 1 \right)$$

for this process to be adiabatic we want $a_j(t)$ to remain small, so the atom stays in the initial distribution of states. Since the term in $\langle \rangle$ is of order 1 and oscillatory, we find the system to behave adiabatically, if the coefficient out front remains small:

$$|\langle \varphi_j(t) | \dot{\varphi}_m(t) \rangle| \ll \frac{1}{\hbar} |E_j(t) - E_m(t)|$$

the l.h.s. is $\hat{=}$ speed of rotation of the eigenvectors, which must be small compared to the "Bohr" frequency of the $j \rightarrow m$ transition.

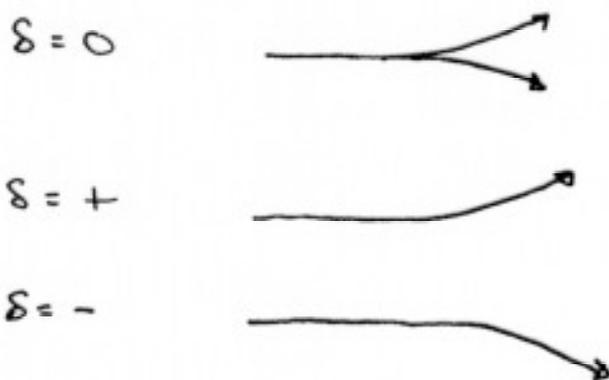
this condition is difficult to achieve on resonance, since

$\Delta E = 0$ at the start \Rightarrow expect the non-adiabatic result for $\delta = 0$.

For a large detuning, this is possible, and we expect the atom to adiabatically follow the dressed state that connects to $|g\rangle$.

In this case the atom should be deflected in 1 direction due to the dipole force acting on the atom.

So our predictions are:

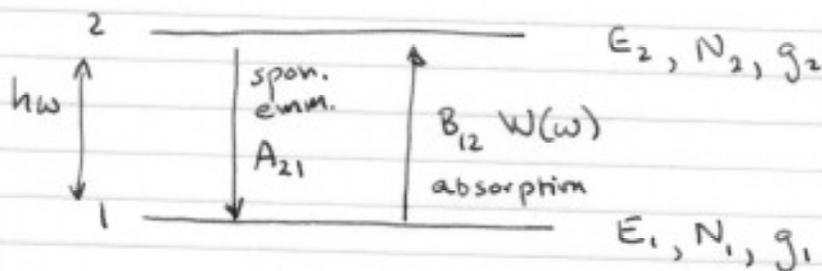


see T. Sleator, et. al. Phys. Rev. Lett. 68 1996 (1992)

EINSTEIN A & B COEFFICIENTS

The semiclassical approach to absorption & emission

consider two states of an atom



E_i = energy of state i

N_i = population of state i

g_i = degeneracy of state i (i.e. $2J_i+1$)

$A_{21} \equiv$ Einstein A-coefficient - rate of spontaneous emission (prob./unit time)

$B_{12} \equiv$ Einstein B coefficient describes rate of absorption

$W(\omega) \equiv$ energy density of radiation

consider a rate equation for states 1 & 2

$$\frac{dN_1}{dt} = -\frac{dN_2}{dt} = N_2 A_{21} - N_1 B_{12} W(\omega)$$

in equilibrium $\frac{dN_i}{dt} = 0$

so

$$N_2 A_{21} = N_1 B_{12} W(\omega)$$

consider thermal equilibrium (just black-body radiation?)

then we can solve for

$$W_T(\omega) = \frac{N_2 A_{21}}{N_1 B_{12}}$$

- but we know in thermal equilibrium the populations are given by the Boltzmann distribution

so
$$\frac{N_1}{N_2} = \frac{g_1 e^{-E_1/kT}}{g_2 e^{-E_2/kT}} = \frac{g_1}{g_2} e^{h\omega/kT}$$

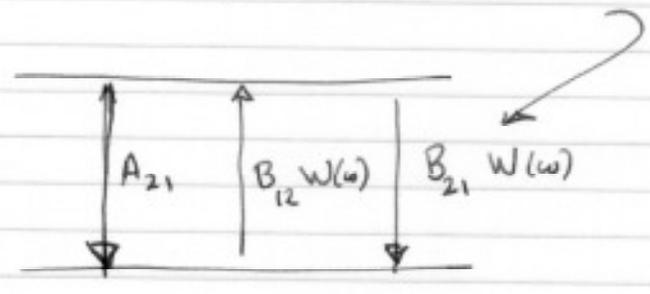
inserting this into eq. for W_T yields

$$W_T(\omega) = \frac{g_2 A_{21}}{g_1 e^{h\omega/kT} B_{12}}$$

We have just described a system that should obey Planck's law:

$$W_T(\omega) d\omega = \frac{h\omega^3}{\pi^2 c^3} \frac{d\omega}{e^{h\omega/kT} - 1}$$

⇒ we have a problem! There is no way to get the -1 in the denominator. We must have left something out: stimulated emission



redoing our rate eqns., we find

$$W_T(\omega) = \frac{A_{21}}{\frac{N_1}{N_2} B_{12} - B_{21}} = \frac{A_{21}}{\frac{g_1}{g_2} e^{h\omega/kT} B_{12} - B_{21}} = \frac{A_{21}}{B_{21} \left[\left(\frac{g_1}{g_2} \right) \frac{B_{12}}{B_{21}} e^{h\omega/kT} - 1 \right]}$$

this can be converted into the Planck's Law form if

$$B_{21} = \frac{g_1}{g_2} B_{12} \quad \text{and} \quad A_{21} = \frac{h\omega^3}{\pi^2 c^3} B_{21}$$

⇒ we find by using thermal equilibrium that all 3 coefficients are related (a very clever trick because coefficients are atomic properties).

— and we had to introduce stimulated emission

for a thermal radiation distribution, the average no. of photons for a mode is:

$$\bar{n} = \frac{1}{e^{h\omega/kT} - 1}$$

comparing to our above results:

$$W_T(\omega) d\omega = B_{21} \frac{A_{21}}{B_{21}} \frac{d\omega}{e^{h\omega/kT} - 1} = \frac{A_{21}}{B_{21}} \bar{n} d\omega$$

$$\text{so } B_{21} W_T(\omega) = \bar{n} A_{21}$$

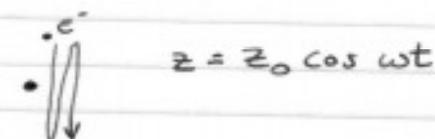
total rate of emission from state 2:

$$B_{21} \int W_T(\omega) d\omega = \bar{n} A_{21} \int d\omega$$

for a typical visible wavelength ($h\nu \sim 2\text{eV}$)
and room temperature ($300\text{K} \approx 1/40\text{eV}$)
 $\bar{n} \approx e^{-80}$, all emission is spontaneous.

We have relations now between A_{21} , B_{12} & B_{21} , but
we do not yet have expressions for them.

Let us consider classical emission of radiation

our "excited" atom  $z = z_0 \cos \omega t$

then dipole moment is

$$d(t) = d_0 \cos \omega t \hat{z} \quad \text{where } d_0 = ez_0$$

from E & M (such as Jackson), radiated fields

are:

$$E_{\theta} = - \frac{d_0 k^2 \sin \theta}{4\pi \epsilon_0 r} \cos(kr - \omega t)$$

$$H_{\phi} = - \frac{d_0 k^2 \sin \theta}{4\pi \epsilon_0 r (\epsilon_0 c)^2} \cos(kr - \omega t)$$

The Poynting vector is:

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^*$$
$$= \frac{\epsilon_0 c}{2} \left(\frac{d_0 k^2}{4\pi \epsilon_0} \right)^2 \frac{\sin^2 \theta}{r^2} \hat{r}$$

$\frac{1}{r^2}$ from a localized source
 $\sin^2 \theta$ dipole pattern.

this dipole is losing energy due to radiation as:

$$-\frac{dW}{dt} = \int_0^{2\pi} d\phi \int_0^\pi (\vec{S} \cdot \hat{r}) r^2 \sin \theta d\theta$$
$$= \frac{d_0^2 k^4 c}{12\pi \epsilon_0}$$

the classical oscillator has an energy

$$W = \frac{1}{2} m \omega^2 z_0^2$$

so the fractional energy lost / period is

$$\frac{1}{W} \left(\frac{2\pi}{\omega} \frac{dW}{dt} \right) = \frac{e^2 \omega}{3\epsilon_0 m c^3}$$

(which is quite small $\sim 10^{-8}$ for optical ν)

so we can write the diff. eq.

$$\frac{dW}{dt} = - \frac{e^2 \omega^2}{6\pi \epsilon_0 m c^3} W$$

which has the obvious soln. of $W = W_0 e^{-\gamma t}$

$$\text{where } \gamma = \frac{1}{\tau} = \frac{e^2 \omega^2}{6\pi \epsilon_0 m c^3}$$

how good is this crude estimate?

for Na	$\lambda = 589 \text{ nm}$	$\tau_{cl} = 15.6 \text{ ns}$	\uparrow	16.2
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Hg	$\lambda = 185 \text{ nm}$	$\tau_{cl} = 1.54 \text{ ns}$		1.3
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- seems amazingly good - but really only works for
1st resonance transition...

a "quantum" treatment:

$$\frac{dW}{dt} = - \frac{\omega^4 |\vec{d}_0|^2}{12\pi \epsilon_0 c^3} = \hbar \omega_{ki} A_{ki}$$

replace $|\vec{d}_0|^2$ with quantum expression

$$|\vec{d}_0|^2 \rightarrow 4\hbar^2 |d_{12}|^2 = \langle 2 | \hat{d} | 1 \rangle \quad \text{where } \hat{d} = e \sum_j \hat{r}_j$$

(8)

factor of 4 comes from

$$\vec{p}_0 \cos \omega t = \frac{1}{2} \vec{d}_0 (e^{i\omega t} + e^{-i\omega t})$$

$$\text{replace } \left(\frac{\vec{d}_0 e^{-i\omega t}}{2} \right)_{cl} \rightarrow \langle i | \hat{d} | k \rangle e^{-i\omega_{ik} t}$$

so we find

$$A_{21} = \frac{e^2 \omega^3}{3\pi \epsilon_0 \hbar c^3} |d_{12}|^2$$

(this result has left out degeneracy)

could also have found A_{21} by using Fermi's Golden rule for B_{12} (see Loudon)

and one finds

$$B_{12} = \frac{\pi e^2 |d_{12}|^2}{3 \epsilon_0 \hbar^2}$$

then using our previous relationships between A & B

$$A_{21} = \frac{\hbar \omega^3}{\pi^2 c^3} \frac{g_1}{g_2} B_{12} = \frac{g_1}{g_2} \frac{e^2 \omega^3}{3\pi \epsilon_0 \hbar c^3} |d_{12}|^2$$

note: these results agree with full QED calculation.

⑨

A convenient quantity is the oscillator strength f for $k \rightarrow i$

$$\text{define } A_{ki} \equiv -3f_{ki} \gamma$$

$$\text{and } g_i f_{ik} = -g_k f_{ki} \equiv gf$$

f describes what fraction of the energy of the classical oscillator belongs to a particular transition.

- Some rules:

for a strongly dipole allowed transition $\tau \sim ns$
 $f \sim \text{order } 1$

for forbidden transitions $\tau \sim \mu s$
 $f \sim 10^{-3} - 10^{-5}$

Sum rules:

for a 1-electron atom, $i = \text{gd. state}$

$$\sum f_{ik} = 1$$

from an excited state

$$\sum_{i < j} f_{ji} + \sum_{k > j} f_{jk} = 1$$

for more complicated atoms $1 \rightarrow Z = \text{no. of valence electrons.}$

(10)

define a saturation energy density where the
stimulated emission rate = spontaneous emission

$$W_s B_{21} = A_{21}$$

$$\text{then } W_s = \frac{A_{21}}{B_{21}}$$

turning this into an intensity density

$$I_s = c W_s = \frac{A_{21} c}{B_{21}}$$

using the rate eqs.

$$\frac{dN_1}{dt} = -\frac{dN_2}{dt} = -A_{12} N_1 - W B_{12} N_1 + N_2 A_{21} + N_2 W B_{21} = 0$$

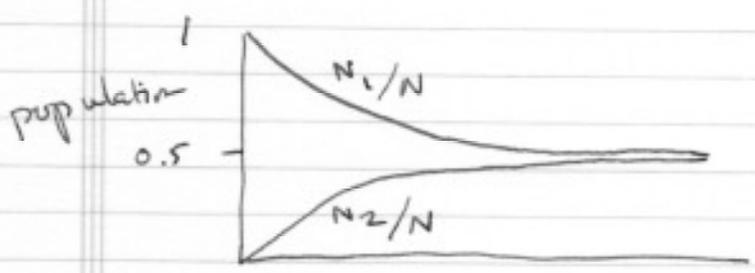
assume steady state

$$\text{and } g_1 = g_2 \Rightarrow B_{12} = B_{21} = B$$

$$\text{then } N_1 = \frac{A_{21} + W B}{W B} N_2$$

$$= \frac{W_s + W}{W} N_2 = \frac{W_s + W}{W} (N - N_1)$$

$$\Rightarrow N_1 = \frac{W_s + W}{W_s + 2W} N$$



if we look at rates:

absorption $N_1 B \cdot W = B \cdot W \frac{W_s + W}{W_s + 2W} N$

$$= N \cdot A \frac{W_s + W}{W_s + 2W} \cdot \frac{W}{W_s}$$

$\propto W$ for $W \ll W_s$ $\propto W, W \gg W_s$

stimulated emission

$$N_2 B \cdot W = N A \frac{W^2}{(W_s + 2W)W_s}$$

$\propto W^2$ for $W \ll W_s$ $\propto W \gg W_s$

spontaneous emission $N_2 \cdot A = N \cdot A \frac{W}{W_s + 2W}$

$\propto W$ for $W \ll W_s$ $\propto \text{const.}$ for $W \gg W_s$

