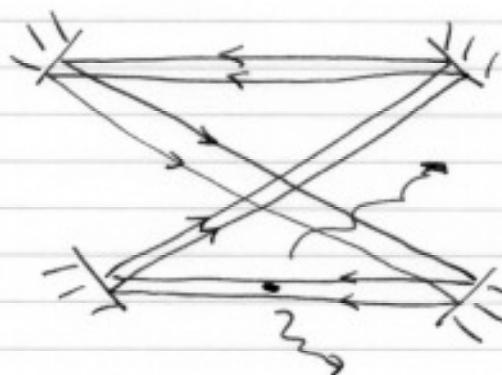


THE "DRESSED" ATOM

- Optical Block Egs. treated the radiation field classically.
- Total atom + field system is time independent, so full quantal treatment should yield eigenvalues of the system

ATOM + SINGLE-MODE QUANTUM FIELD =
DRESSED ATOM

Modeling a laser beam as a single mode:



consider an ideal (lossless) cavity
- travelling wave

- assume V (mode volume) is so large spontaneous emission is unmodified

$$\Sigma = \text{energy density} \sim \frac{\langle N \rangle}{V}^{\text{no. of photons in mod}}$$

$$\Sigma = \text{const. as } N \rightarrow \infty, V \rightarrow \infty$$

- a laser is a coherent state (we will learn about this later in semester)

- has Poissonian statistics

$$\Delta N = \sqrt{\langle N \rangle}$$

since $N \rightarrow \infty$

$\frac{\Delta N}{N} \ll 1$: photon no. distribution
is narrow, centered
around $\langle N \rangle$

- also assume spontaneous emission has negligible effect on laser field

$$N_s \approx T \cdot \frac{P}{2} \quad T = \text{interaction time}$$

$P = \text{linewidth}$

- since $\Delta N \rightarrow 0$ as $\langle N \rangle \rightarrow \infty$



the laser field Hamiltonian

$$1) H_L = \hbar\omega_L (\hat{n} + \frac{1}{2}) = \hbar\omega_L (a^\dagger a + \frac{1}{2})$$

atomic Hamiltonian

2)

$$H_A = \hbar\omega_0 |e\rangle\langle e|$$

$\uparrow \quad \langle e |$
 $\hbar\omega_0$
 $\downarrow \quad |g\rangle$

the uncoupled Hamiltonian $H_L + H_A$ has eigenstates labeled with 2 gu. nos., associated with the field (no. of photons) and the atom (g, e ,

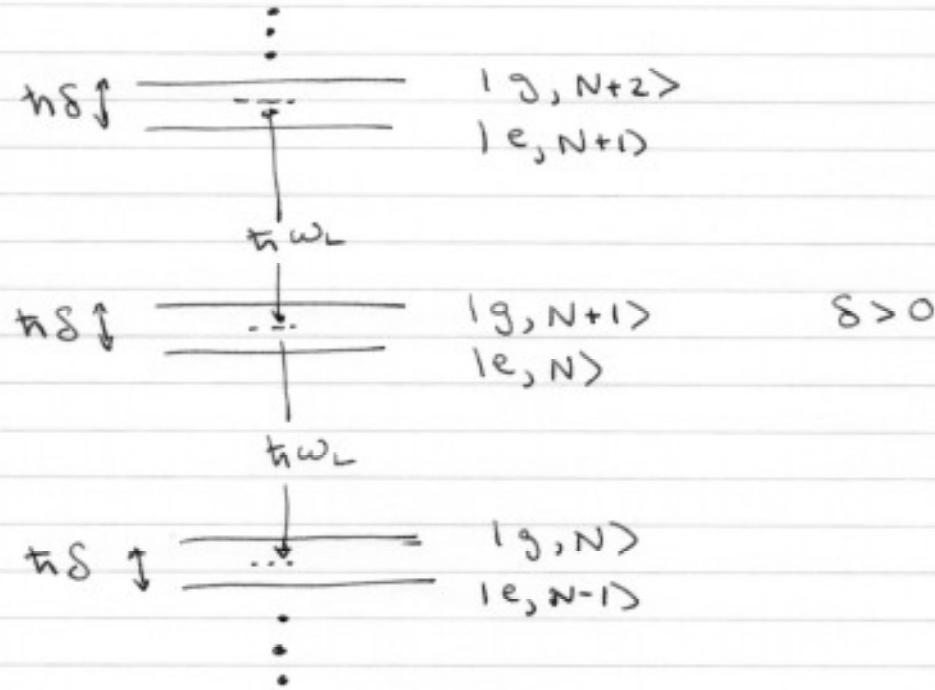
- assume $|\delta \equiv \omega_L - \omega_0| \ll \omega_0$

(usually good for optical, maybe not for RF,

then $|g, N+1\rangle$ and $|e, N\rangle$ have almost the same energy.

since N has no limit, there are an infinite number of states.

the dressed atom ladder:



- a ladder of pairs of states, separated by a laser photon energy $\hbar\omega_L$
- now consider the atom-laser coupling

$$3) V = -\vec{d} \cdot \vec{E} = g (|e\rangle\langle g| + |g\rangle\langle e|)(a^\dagger + a)$$

$$\text{where } g = -\tilde{\epsilon}_L \vec{d}_{eg} \sqrt{\frac{\hbar\omega_L}{2\epsilon_0 V}} \quad \vec{d}_{eg} = \langle e | \vec{d} | g \rangle$$

this interaction couples the 2 states of the closely spaced pairs of levels

$$4) \nu = \langle e, N | V | g, N+1 \rangle = g \sqrt{N+1}$$

- atoms go from $g \rightarrow e$, and a photon is removed from the laser field.

- there are also off-resonant ($2\omega_L$) couplings

between $|g, N\rangle \leftrightarrow |e, N+1\rangle$

\Rightarrow neglect

\hookrightarrow produce Bloch-Siegert shift
in RF

- variations in coupling due to ΔN

$$\frac{\Delta \nu}{\nu} = \frac{\Delta \sqrt{N+1}}{\sqrt{N+1}} \simeq \frac{1}{2} \frac{\Delta N}{\langle N \rangle} \ll 1$$

\Rightarrow neglect

$$\text{i.e. } \nu \simeq g \sqrt{N}$$

- so energy level diagram is periodic, at least over a range ΔN , for $N \sim \langle N \rangle$

a laser field = coherent state \simeq classical field

from OBEs

$$\hbar \Omega_R = -\mathbf{d} \cdot \mathbf{E}$$

$$E = 2\epsilon_L \sqrt{\frac{\hbar \omega_L}{2\epsilon_N}} \sqrt{\langle N \rangle}$$

$$\text{so } V = \frac{\hbar \Omega_R}{2}$$

to take this coupling into account, we diagonalize the matrix

$$\frac{\hbar}{2} \begin{pmatrix} -\delta & \Omega_R \\ \Omega_R & \delta \end{pmatrix}$$

which yields

$$\epsilon_{\pm} = \pm \frac{\hbar}{2} \sqrt{\delta^2 + \Omega_R^2}$$

\hookrightarrow generalized Rabi frequency

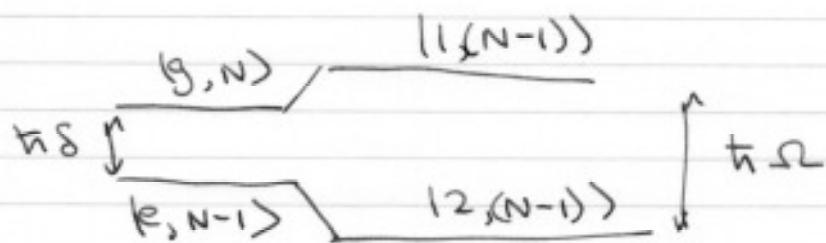
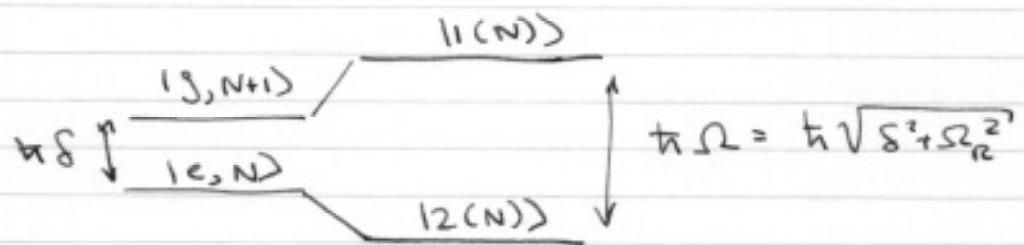
and eigenvectors:

$$|1(N)\rangle = \sin\Theta |g, N+1\rangle + \cos\Theta |e, N\rangle$$

$$|2(N)\rangle = \cos\Theta |g, N+1\rangle - \sin\Theta |e, N\rangle$$

$$\text{where } \tan 2\Theta = -\frac{\Omega_R}{\Gamma}$$

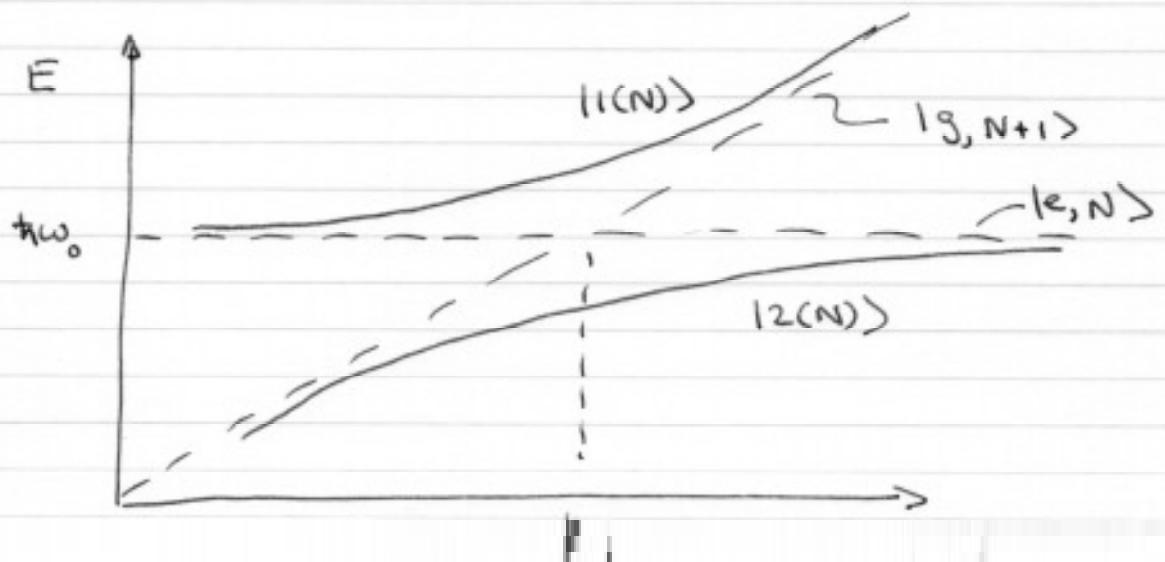
the pairs are split further apart



- the new eigenstates are mixtures of $|g\rangle$; $|e\rangle$

$$\text{for } \delta = 0 \quad |1(N)\rangle = \frac{1}{\sqrt{2}} (|g\rangle + |e\rangle)$$

$$|2(N)\rangle = \frac{1}{\sqrt{2}} (|g\rangle - |e\rangle)$$



(8)

for $|\delta| \gg \Omega_R$, the difference between the unperturbed & perturbed levels:

$$\Delta = \frac{1}{2} \sqrt{\delta^2 + \Omega_R^2} - \frac{\delta}{2}$$

$$= \frac{1}{2} \delta \sqrt{1 + \frac{\Omega_R^2}{\delta^2}} - \frac{\delta}{2} \approx \frac{\Omega_R^2}{48}$$

\Rightarrow this is Dynamic or AC Stark Shift
or the light shift

note: we could have found the light shift with 2nd-order perturbation thy:

$$E_i^{(2)} = \sum_{j \neq i} \frac{\langle i | H_1 | j \rangle \langle j | H_1 | i \rangle}{E_j^{(0)} - E_i^{(0)}}$$

$$\langle i | H_1 | j \rangle = \frac{\hbar \Omega_R}{2}$$

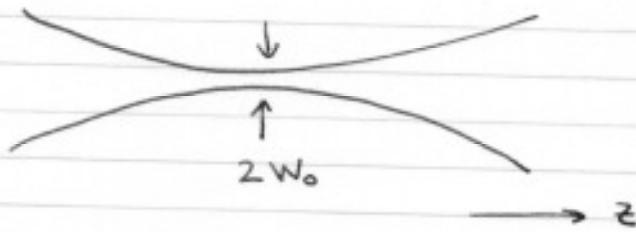
$$\text{and } \Delta E = \hbar \delta$$

$$\text{so } \Delta = E_i^{(2)} = \frac{\hbar \Omega_R^2}{48} \quad \checkmark$$

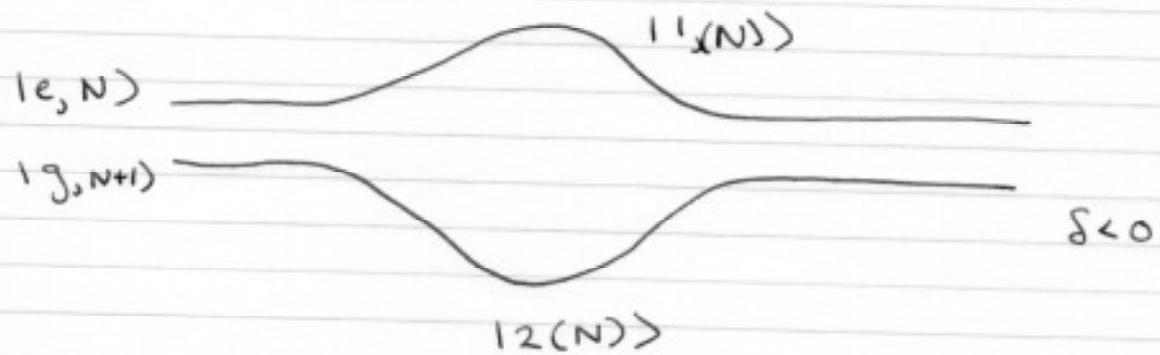
but diagonalization is correct to all orders!

- an example from laser cooling / trapping

consider a tightly focussed laser beam



the dressed states around the focus:



if atom is in $|2(N)\rangle$ it sees a trapping potential

note: need $\delta < 0$ so $|2(N)\rangle \sim$ mainly $|g\rangle$
is attractive

a real example

$\sim 1 \text{ Watt}$ laser focussed to $W_0 = 10 \mu\text{m}$

for a typical (alkali) atom

$$\frac{\Omega_R}{2\pi} \sim 10^5 \text{ MHz} \sim 10^4 \text{ rad/s}$$

$$\text{if } \delta \approx 10 \Omega_R \quad |2(N)\rangle \approx .996 |g_{N+1}\rangle \\ + .04 |e_N\rangle$$

depth

$$\Delta = \frac{\hbar \Omega_R^2}{48} \approx \frac{\hbar \Omega_R}{40} \approx 100 \text{ mK}$$

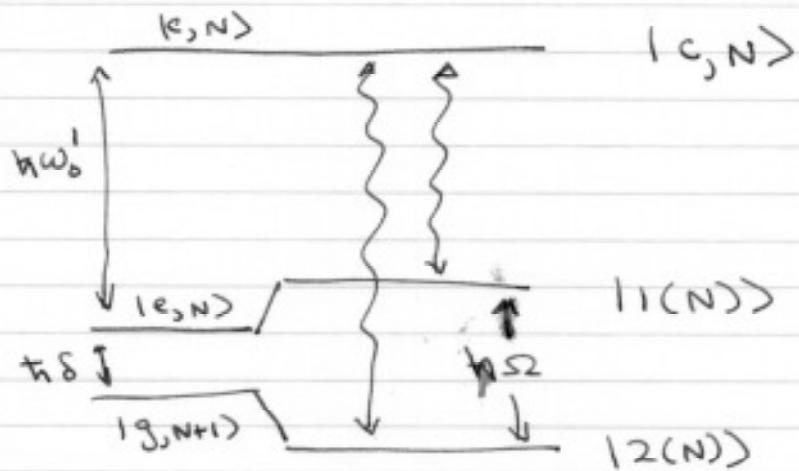
if atoms are cold enough, they can be trapped.

S. Chu, et. al. Opt. Lett. 11 73 (1986)

The Autler-Townes Effect:

Autler; Townes Phys. Rev. 100, 703 (1955).

use absorption to a third level to probe

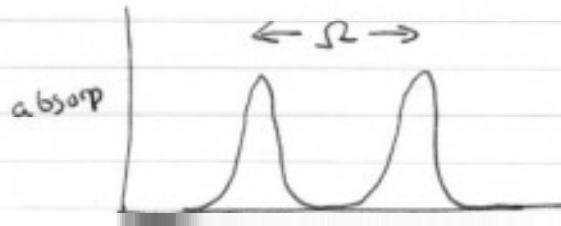


- assume $\omega_0' - \omega_0 \gg \Omega_R$ so dressing does not perturb $|c\rangle$

- use 2nd laser tuned around ω_0'

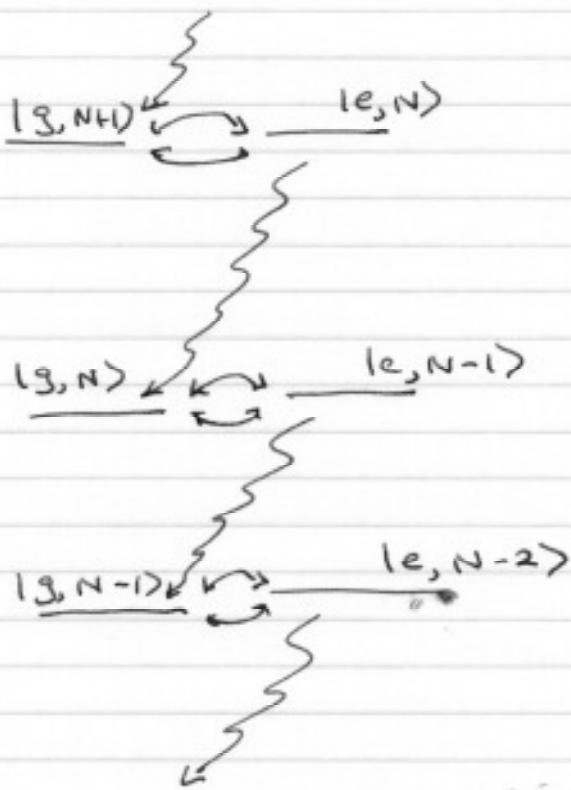
- find 2 transitions at $\omega_0' \pm \frac{1}{2}(\Omega_R + \delta)$

if $\delta = 0$



We have so far neglected spontaneous emission.

- look at fluorescence spectrum of spontaneously emitted photons
in the uncoupled basis



\rightsquigarrow - spontaneously emitted photon

does not change N (which is only no. of photons in laser mode)



absorption and stimulated emission

in the dressed basis, we must find which levels have non-zero dipole matrix elements,
i.e. taking $|e, N\rangle \rightarrow |g, N\rangle$

Recalling dressed states

$$|1(N)\rangle = \sin\theta |g, N+1\rangle + \cos\theta |e, N\rangle$$

$$|2(N)\rangle = \cos\theta |g, N+1\rangle + \sin\theta |e, N\rangle$$

note that $|1(N)\rangle$ & $|2(N)\rangle$ do not both have $|e, N\rangle$ and $|g, N\rangle \Rightarrow$ spont. emission will not couple dressed states of the same (N)

- So we must consider matrix elements proportional to $\langle i(N) | e \rangle \langle g | j(N-1) \rangle$

there are 4 non-zero elements:

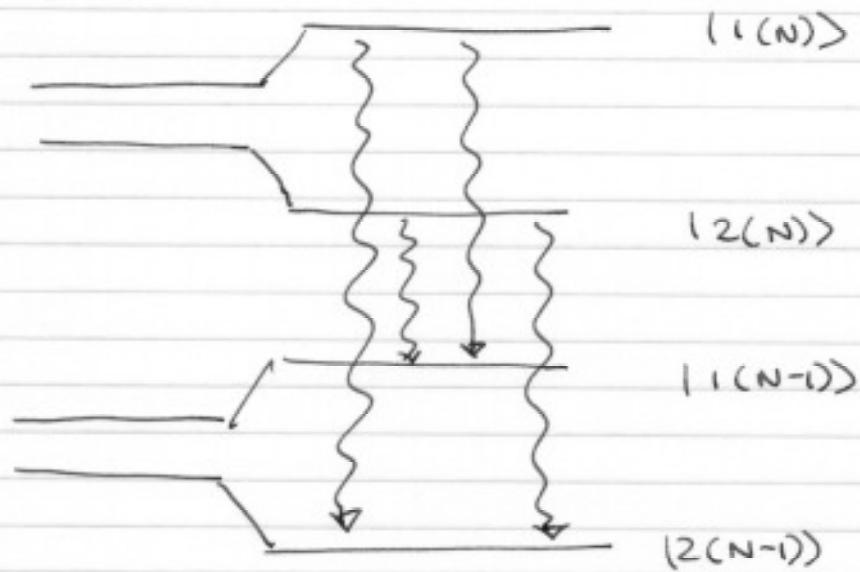
$$\langle 1(N) | e \rangle \langle g | 1(N-1) \rangle = \sin\theta \cos\theta$$

$$\langle 2(N) | e \rangle \langle g | 2(N-1) \rangle = -\sin\theta \cos\theta$$

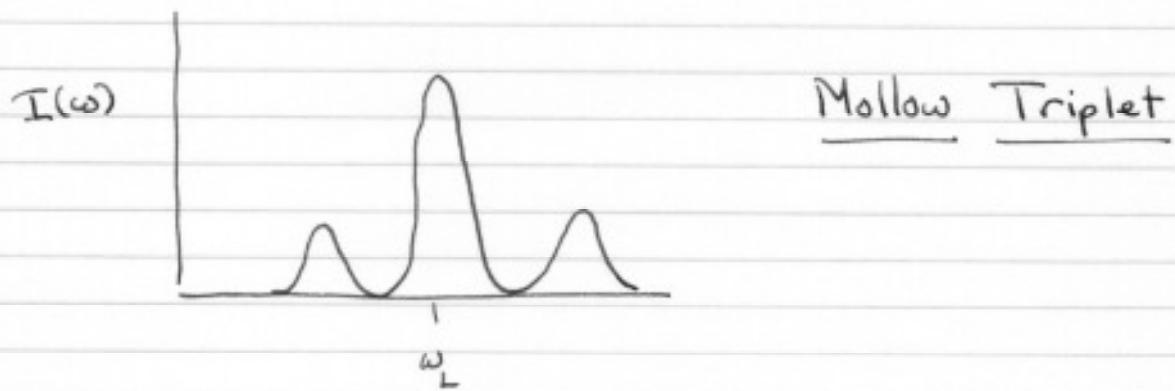
$$\langle 1(N) | e \rangle \langle g | 2(N-1) \rangle = \cos^2\theta$$

$$\langle 2(N) | e \rangle \langle g | 1(N-1) \rangle = -\sin^2\theta$$

so in the dressed basis:



the spectrum of fluorescence is a triplet
with frequencies ω_L , $\omega_L \pm \Delta$



Note: photon emissions are anti-correlated
red \rightarrow blue red \rightarrow red

- The actual calculation of the spectrum
is quite complicated.

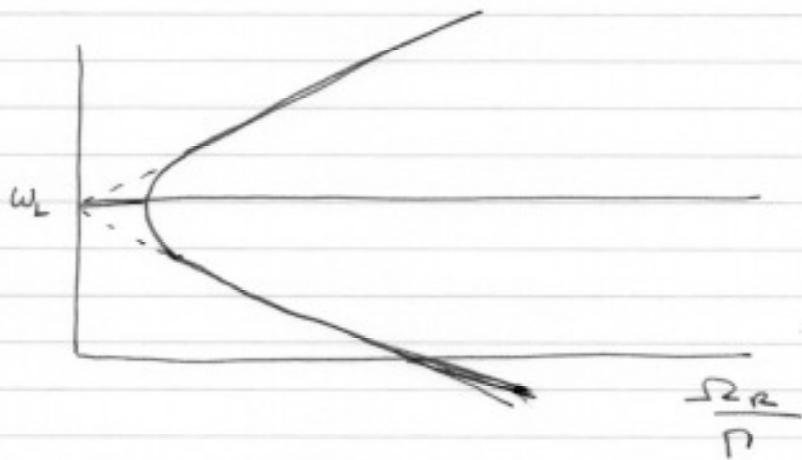
- see B. Mollow, Phys. Rev. 188, 1969 (1969)
for details.

The power spectrum is the Fourier transform
of the correlation fn. of the atomic dipole

$$S(\omega) = \frac{P}{2\pi} \int_{-\infty}^{\infty} \langle S_+(r) S_-(0) \rangle e^{-i\omega r} dr$$

$$\text{where } S_+ = |e\rangle\langle g| \quad S_- = |g\rangle\langle e|$$

one finds:



- triplet only appears for $\Omega_R > \Gamma/4$

- width of central peak is Γ , sidebands $\frac{3}{2}\Gamma$