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Atomic and Optical Physics.

This course is going to serve as an introduction and guide to experiments in A&O physics. The O part will be mostly Quantum Optics.

Very brief history of Q.O. (It is basically the history of Quantum Mechanics).

Experiments

Planck 1900 (Rutherford) Black Body Radiation.

Einstein 1905 (Hertz) Photo-electric effect.

G.I. Taylor 1909 double slit experiment with low flux. (not more than one photon).

Most physical optics experiments can be explained on the basis of classical E&M.

Shift to measuring fluctuations:

Hawthorne-Brown and Twiss 1956

Correlation of intensity fluctuations $g^{(2)}(\tau)$
Photon bunching (Cohen et al.)

1960 The laser \rightarrow same results for classical or quantum formulation \rightarrow Poissonian statistics

Birth of Quantum Optics.

1963 Glauber: Quantum theory of coherent
→ there exist fields with sub-Poissonian statistics
(very regular).

Ned negative probabilities! → non-classical field

The laser enables the study of other fields.

Resonance fluorescence:

one atom driven by resonance
Caves and Walls (1974) theory
Kimble Dagenais and Mandel (1977) experiment
observation of antibunching.

Now Q.O. is the testing ground for Quantum mechanics and a primary contender in the area of quantum information.

one polarization?

What we know of the EM field classically.

$$E = E(t) e^{i(\omega + \vec{k} \cdot \vec{z})} = E_0 e^{i(\omega t - \vec{k} \cdot \vec{z})} + \Delta E(t) e^{i(\omega t - \vec{k} \cdot \vec{z})}$$

We measure the intensity.

$$\underline{\underline{E_0 c / |E|^2}}$$

The detectors we have do not give us directly the field but the intensity

so what do we measure.

(Assume that E_0 and $\Delta E(t)$ are real so.

$$|E|^2 = E_0^2 + 2E_0 \Delta E(t) + \Delta E^2(t)$$

all the fast components are gone, but there is still some time dependence in the noise (if we assume it is small (first order))

$$E_0/E|^2 = I = I_0 + \Delta I(t)$$

$\Delta I(t)$ noise

~~$I = E_0^2 + 2E_0 \Delta E(t)$~~

$$I_0 + 3\sqrt{I_0}$$

the intensity noise scales as the intensity $^{1/2}$

It sounds like we are talking about Poisson distributions. The variance is equal to the mean.

Most of ~~most~~ present day Quantum optics has to do with fields where the fluctuations are large compared to the mean; the smallest fluctuation is a single photon!

The Electromagnetic wave is subject to Maxwell's equations. Do not forget that this includes diffraction! Light ~~from~~ from a laser propagates following diffraction it is a Gaussian beam.
(More about them see Siegman).

Quadratures of the electromagnetic field.

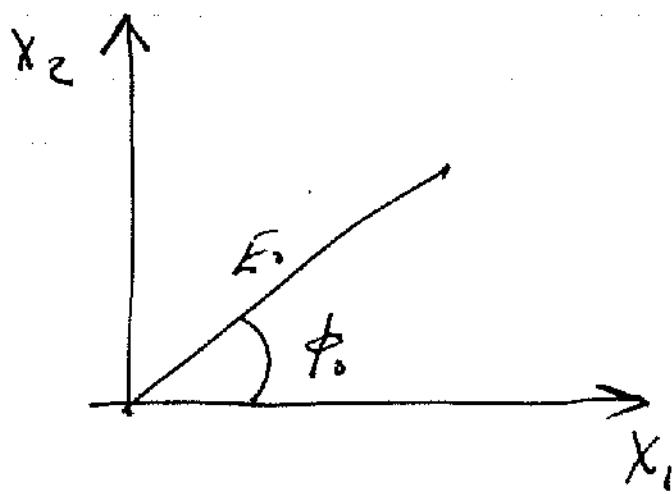
(one polarization)

$$E(\vec{r}, t) = E_0(\vec{r}, t) e^{i(\omega t - \vec{k} \cdot \vec{z})} e^{i\phi(\vec{r}, t)}$$

↑ ↑

carrier. shape of the wavefront
and absolute phase).

$$E(\vec{r}, t) = E_0(\vec{r}, t) \underbrace{\left(X_1 \cos \omega t + X_2 \sin \omega t \right)}_{\text{temporal part}} e^{i(kz + \phi)}$$



$$\phi_0 = \tan^{-1} \frac{x_1}{x_2}$$

it is a very useful representation, \$x_1\$ and \$x_2\$ will turn out to be conjugate variables

5 X 8

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Field strength, Intensity and amplitude

far away from the source (to avoid ~~comes to~~)

B is proportional to the time derivative of E .

$$\nabla \times B = \mu_0 \frac{\partial}{\partial t} E$$

$$\vec{k} \times \vec{B} = \frac{1}{\mu_0} \frac{\partial E}{\partial t}$$

We said that the quadratures had only the time dependence, so $X_1 \equiv E$

$$X_2 \equiv B$$

Remember $\epsilon_0 |E|^2$ is an energy density

$$\epsilon_0 |E|^2 + \frac{1}{\mu_0} |H|^2$$

Intensity = Power
Area.

$$\text{Power} = C \int \epsilon_0 E_0^2 (x, y) dx dy = \int I(x, y) dx dy.$$

factor of 2 (time average).

So we have to talk about volumes whenever we talk about ~~power~~ energy.
Power · time EM

Intensity · Area · time

$\frac{\epsilon_0 E_0^2}{\text{Energy density}} \cdot \text{Area} \cdot \text{C time},$
Energy density · Volumes.

Here we see the need to actually define the "Volume" associated with the EH field. It is common to talk about it as the "Volume" Volume.

→ Interference Volume. The "length" or "time" over which the field is measured and so there can be interference.

We will have to learn how to define quantitatively such a volume by looking at the formal theory of coherence.

Mode

We understand the propagation of a Gaussian beam.

The mode is "spatial" ~~propagation~~ properties. The shape of the wave front, polarization, direction of propagation and: Intensity, and frequency, a suitable phase

Later we will get very sloppy when talking about a mode saying the spatial part.

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(SI units).

3.- Maxwell's equations.

(no sources)

$$\nabla \times \vec{E}(\vec{r}, t) = -\frac{\partial}{\partial t} \vec{B}(\vec{r}, t)$$

$$\nabla \times \vec{B}(\vec{r}, t) = \frac{1}{c^2} \frac{\partial}{\partial t} \vec{E}(\vec{r}, t)$$

$$\nabla \cdot \vec{E}(\vec{r}, t) = 0$$

$$\nabla \cdot \vec{B}(\vec{r}, t) = 0$$

Transverse vector potential $\vec{A}(\vec{r}, t)$ in the
Coulomb gauge. (also known as the
radiation gauge).

It satisfies the homogeneous wave equation.

$$\nabla^2 \vec{A}(\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{A}(\vec{r}, t) = 0$$

[\vec{A} is transverse by definition]

$$\nabla \cdot \vec{A}(\vec{r}, t) = 0$$

← [the electrostatic potential
is transmitted instantaneously]

In terms of $\vec{A}(\vec{r}, t)$ then

$$\vec{E}(\vec{r}, t) = -\frac{\partial}{\partial t} \vec{A}(\vec{r}, t)$$

$$\vec{B}(\vec{r}, t) = \nabla \times \vec{A}(\vec{r}, t)$$

Fourier decomposition of $\vec{A}(\vec{r}, t)$

$$\vec{A}(\vec{r}, t) = \frac{1}{\epsilon_0^{1/2} L^{3/2}} \sum_{\vec{k}} A_{\vec{k}}(t) e^{i\vec{k} \cdot \vec{r}}$$

Fourier series. ~~The~~ The EM field is in a very large cube of side L with periodic boundary conditions. We shall let $L \rightarrow \infty$ later on. Any physically meaningful results should not depend on the magnitude of L .

\rightarrow The sum is over three integers n_1, n_2, n_3 $\epsilon_0^{1/2} L^{3/2}$ is there for convenience.

\vec{k} has components.

$$k_1 = 2\pi n_1 / L \quad n_1 = 0, \pm 1, \pm 2, \dots$$

$$k_2 = 2\pi n_2 / L \quad n_2 = 0, \pm 1, \dots$$

$$k_3 = 2\pi n_3 / L \quad n_3 = 0, \pm 1, \dots$$

Note that if $L \rightarrow \infty$ the discrete sum over \vec{k} vectors $\sum_{\vec{k}}$ becomes an integral

$\sum_{\vec{k}} \rightarrow \left(\frac{L}{2\pi}\right)^3 \int f(k) d^3 k$ where $\left(\frac{L}{2\pi}\right)^3$ is the density of modes.

$$k = \frac{2\pi n}{L} \quad \text{so} \quad \delta k = \frac{2\pi}{L} \delta n$$

The number of modes corresponding to δk is δn

Then for the three dimensional interval

$$\delta k_1 \delta k_2 \delta k_3 \text{ it is } \delta n_1 \delta n_2 \delta n_3 = \left(\frac{L}{2\pi}\right)^3 \delta k_1 \delta k_2 \delta k_3.$$

Since $\nabla \cdot \vec{A}(\vec{r}, t) = 0$ transversality condition.

$$\frac{i}{\epsilon_0^{1/2} L^{3/2}} \sum_{\vec{k}} \vec{k} \cdot \vec{A}_k(t) e^{i\vec{k} \cdot \vec{r}} = 0$$

for all \vec{r} so

$$\vec{k} \cdot \vec{A}_k(t) = 0$$

$\vec{A}(\vec{r}, t)$ is real then

$$\vec{A}_{-\vec{k}}(t) = \vec{A}_k^*(t)$$

$\vec{A}(\vec{r}, t)$ satisfies the homogeneous wave equation

so

The mode density ρ (number per unit volume, \bar{n})

$$d\bar{n} = \frac{d^3 k}{2\pi^3}$$

but now in k space

$$d^3 k = dk_1 dk_2 dk_3$$

$$d^3 k = 4\pi k^2 dk$$

integrating over t and ϕ

$$d\bar{n} = \frac{k^2 dk}{2\pi^2}$$

$$\rho(k) dk = \frac{k^2}{2\pi^2} dk$$

careful with polarizations.

$$\rho(\omega) d\omega = \frac{\omega^2}{2\pi^2 c^3} d\omega$$

then

$$\frac{1}{\epsilon_0^{1/2} L^{3/2}} \sum_k \left(-k^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \hat{A}_k(t) e^{ikx} = 0$$

so for all \vec{k} and so that

$\hat{A}_k(t)$ satisfies:

$$\left(\frac{\partial^2}{\partial t^2} + \omega^2 \right) \hat{A}_k(t) = 0$$

$\omega_k = ck = \omega$ is the frequency.

$$\hat{A}_k(t) = \hat{C}_k e^{-i\omega t} + \hat{C}_{-k}^* e^{i\omega t}$$

\vec{C}_k can be resolved into two orthogonal components.
so that $\vec{k} \cdot \hat{A}_k = 0$ is satisfied automatically.

$$\vec{k} \cdot \vec{E}_{ks} = 0 \quad s = 1, 2 \quad \text{transversality}$$

$$\vec{E}_{ks} \cdot \vec{E}_{ks'} = \delta_{ss'} \quad s, s' = 1, 2 \quad \text{orthogonality}$$

$$\vec{E}_k \cdot \vec{E}_{k_2} = \frac{\vec{k}}{1/k_1} \quad \text{in MM} \quad \text{right-handness}$$

$$\vec{C}_k = \sum_{s=1}^2 \vec{C}_{ks} \vec{E}_k$$

These conditions leave undetermined a lot about the wave vector \vec{k} .

These basis set is the polarization set. It can be linear or elliptical in general. The particular basis for a problem can be best determined w/ the end or the most convenient moment to simplify the calculation.

Then

$$\vec{A}(\vec{r}, t) = \frac{1}{\epsilon_0^{\nu_2} L^{3/2}} \sum_{k \times s} \left[C_{ks} \vec{E}_{ks} e^{-i\omega t} + C_{-ks}^* \vec{E}_{-ks}^* e^{i\omega t} \right] e^{ik \cdot \vec{r}}$$

[now changing the index]

$$= \frac{1}{\epsilon_0^{\nu_2} L^{3/2}} \sum_{k \times s} \left[C_{ks} \vec{E}_{ks} e^{i(k \cdot \vec{r} - \omega t)} + C_{ks}^* \vec{E}_{ks}^* e^{-i(k \cdot \vec{r} - \omega t)} \right]$$

$$= \frac{1}{\epsilon_0^{\nu_2} L^{3/2}} \sum_k \sum_s [u_{ks}(t) \vec{E}_{ks} e^{i k \cdot \vec{r}} + u_{ks}^*(t) \vec{E}_{ks}^* e^{-i k \cdot \vec{r}}]$$

where $u_{ks}(t) = C_{ks} e^{-i\omega t}$

$\vec{E}_{ks} e^{i k \cdot \vec{r}}$ are the fundamental mode functions with complex amplitude $u_{ks}(t)$

Each mode is labeled by a wave vector k and polarization index s

The mode function satisfies the Helmholtz equation

$$(\nabla^2 + k^2) E_{ks} e^{ik \cdot \vec{r}} = 0$$

The mode amplitude $u_{ks}(t)$ satisfies the H. O.

$$\left(\frac{\partial^2}{\partial t^2} + \omega^2 \right) u_{ks}(t) = 0$$

Then the mode expansions of the electric and magnetic field are: from $\vec{E} = -\frac{\partial}{\partial t} \vec{A}$ and $\vec{B} = \nabla \times \vec{A}$

$$\vec{E}(\vec{r}, t) = \frac{i}{\epsilon_0'' L^{3/2}} \sum_k \sum_s \omega [u_{ks}(t)] \vec{e}_{ks} e^{ik \cdot \vec{r}} - c.c.]$$

$$\vec{B}(\vec{r}, t) = \frac{i}{\epsilon_0'' L^{3/2}} \sum_k \sum_s [u_{ks}(t)] (\vec{k} \times \vec{e}_{ks}) e^{ik \cdot \vec{r}} - c.c.]$$

→ Sept. 10 1996

The energy of the EM field is:

$$H = \frac{1}{2} \int_V [E_0 \vec{E}^2(\vec{r}, t) + \frac{1}{\mu_0} \vec{B}^2(\vec{r}, t)] d^3 r$$

or in terms of the mode amplitudes

$$H = \frac{1}{2} \sum_k \sum_s [\omega^2 / u_{ks}(t)]^2$$

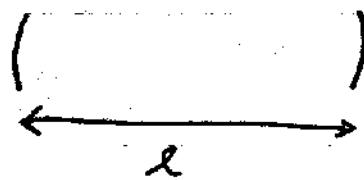
remember

$$\int_V e^{i(k-k') \cdot \vec{r}} d^3 r = V^3 \delta_{kk'}$$

and $(\vec{k} \times \vec{e}_{ks}^*) \cdot (\vec{k} \times \vec{e}_{ks'}) = k^2 \vec{e}_{ks}^* \cdot \vec{e}_{ks'} = k^2 \delta_{ss'}$

September 16 1999 13

Modes of a cavity (Longitudinal).



how many wavelengths fit here.

$$\frac{n\lambda}{2} = l$$

How much should I change λ to fit one more wavelength.

$$\frac{n\lambda}{2} = n+1 \frac{(\lambda + \Delta\lambda)}{2}$$

$$\text{but } \frac{\Delta\lambda}{\lambda} = -\frac{\Delta\nu}{\nu}$$

Assume $n \gg 1$

and

$$\frac{n}{n+1} = \frac{\lambda + \Delta\lambda}{\lambda}$$

$$n\lambda = 2l$$

$$n = \frac{2l}{\lambda}$$

$$\frac{1}{1+\frac{1}{n}} = 1 + \frac{\Delta\lambda}{\lambda}$$

$$1 - \frac{1}{n} = 1 + \frac{\Delta\lambda}{\lambda}$$

Change in frequency



$$\frac{1}{n} = \frac{\Delta\nu}{\nu}$$

$$\frac{\lambda}{2l} = \frac{\Delta\nu}{\nu}$$

$$\Delta\nu = \frac{\lambda\nu}{2l} = \frac{c}{2l}$$

Free spectral range