Physics 711, Symmetry Problems in Physics Fall 2005

Homework

Solutions for assignment 2

Georgi 1.D. From $D_2(g) = SD_1(g)S^{-1}$ and $D_2(g)A = AD_1(g)$, both $\forall g$, $D_2(g)A = AS^{-1}D_2(g)S$, and $D_2(g)AS^{-1} = AS^{-1}D_2(g)$, so by Schur's lemma $AS^{-1} = \lambda\mathbf{1}$, or $A = \lambda S$, where S is a nonsingular $n \times n$ matrix and $n = dimD_{1,2}$.

Group of integers $1, 2, ..., n-1$, n prime, mod n is a group of order $n-1$. The product is $(k_2, k_1) \to k_2k_1$, mod n. The powers of any element g in a finite group must recur. Let $g^s = g$ be the first recurrance for any element g. Then $g^{s-1} = e$. Taking cosets with respect to the subgroup generated by these powers, $s-1$ must divide $n-1$; i.e. $(n-1)/(s-1) = t$, where t is an integer. Then $g^{(s-1)t} = g^n = e$. In our case, $e = 1$, $g = k$, and $k^{n-1} = 1$, mod n.

Character table of S_3

$Rep\|Class$	111	3	12
(3)	1	1	1
(111)	1	1	-1
(21)	2	-1	0

Normalized character table of S_3

$Rep\|Class$	111	3	12
(3)	$1/\sqrt{6}$	$1/\sqrt{3}$	$-1/\sqrt{2}$
(111)	$1/\sqrt{6}$	$1/\sqrt{3}$	$1/\sqrt{2}$
(21)	$\sqrt{2/3}$	$-1/\sqrt{3}$	0

$\chi_{a,\alpha} = \sqrt{k_\alpha/N}$, $N = 6$, $k_\alpha = 1, 111; 2, 3; 3, 12.$