Physics 711, Symmetry Problems in Physics Fall 2005

Homework assignment 5

Due 11/22/05

1. Redo the analysis that gives the irreducible representations of the Poincare' group for particles of positive mass and energy using $SO(1,3)$ instead of $SL(2,C)$ for the Lorentz group. Give explicit formulas for the Lorentz transformations that correspond to the $SL(2,C)$ matrices in $A = A'_L A_R A_L$.

2. Use the Weyl representation for the gamma matrices to describe left and right chirality spinors. The Weyl gammas are

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \tag{1}$$

where $\sigma^\mu = (\mathbf{1}, \sigma)$, $\overline{\sigma}^\mu = (\mathbf{1}, -\sigma) = \sigma_\mu$, $\sigma = (\sigma^1, \sigma^2, \sigma^3)$,

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^1 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2}$$

3. Find the charge conjugation matrix C that takes Dirac spinors that obey $(i\partial\!\!\!/ - eA\!\!\!/ - m)\psi = 0$ to $(i\partial\!\!\!/ + eA\!\!\!/ - m)\psi^c = 0$, where $\psi^c = C\overline{\psi}^T$ and write the Majorana spinor that obeys $\psi^c_M = \psi_M$ in terms of a Weyl spinor and its complex conjugate. $\overline{\psi}$ is the Pauli adjoint.