Physics 711, Symmetry Problems in Physics, Fall 2005

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Textbooks: *Lie Algebras in Particle Physics*, by Howard Georgi, 2nd ed., Reading, Mass.: Perseus Books, Advanced Book Program, c1999.
Frontiers in physics ; v. 54, ISBN 0-7382-0233-9.
Group Theory in Physics, by Wu-Ki Tung, World Scientific, c1985, ISBN 9971-966-56-5; ISBN 9971-966-57-3 (pbk).
The class meets TuTh 2:00-3:15pm in Physics 4102.

Prerequisite: Knowledge of linear algebra.

Preamble: Symmetries and their mathematical structure in terms of group theor have contributed to many discoveries in physics. In classical physics the foremost example is the Lorentz group, the symmetry group of Maxwell's equations, and the special theory of relativity. With the advent of quantum mechanics and the superposition principle, symmetries played an increasing role. The symmetric group leads to Bose and Fermi statistics. The rotation group leads to the analysis of atomic spectra. Spin and the Lorentz group are connected with the Dirac equation. SU(2) is connected with isospin symmetry. SU(3) is connected with the eightfold way and, in another context, with color. SU(5) and SO(10) are connected with grand unification. Graded Lie algebras are connected with supersymmetry. One can expect this connection of symmetries and discoveries in physics to continue.

The plan of this course is first to present practical techniques that lead to physical results with a minimum of mathematical superstructure. The latter part of the course will contain more mathematical developments. Topics may be rearranged as the course progresses.

Syllabus

I. Definition of a group and elementary examples

A. Transformation groups

1. Discrete example–the symmetric group

2. Continuous example–the rotation group in three dimensions

B. Abstract groups

1. Subgroups, cosets

2. Conjugate elements, classes

3. Invariant subgroup, factor group

4. Isomorphism, homomorphism

II. Representation of groups

A. Matrix representations of discrete groups

B. Representations of continuous compact groups

C. Reducible and irreducible representations

D. Unitary representations, equivalence of representations

E. Characters

III. Compact Lie groups and their Lie algebras

A. Generators, structure constants

B. Fundamental and adjoint representations

C. su(2) as the simplest nonabelian Lie algebra

D. Tensor operators, Clebsch-Gordan series, and Wigner-Eckart theorem

E. Application to isospin

F. Cartan and Dynkin analysis of compact Lie algebras

G. su(3) and application to the eightfold way and the quark model

H. Interlude on the symmetric group

I. Young tableaux for S_n and SU(n)

J. Irreducible representations of SU(n) groups

K. Haar measure

L. Harmonic oscillator

M. SU(6) and the quark model

N. Color SU(3)
O. SU(5), SO(10) and unified theories
P. SO(N) groups
Q. Sp(2n) groups

IV. Noncompact groups
A. The Lorentz and Poincaré groups in 1+3 dimensions
B. Irreducible representations of the Lorentz and Poincar'e groups
C. van der Waerden's dotted and undotted spinors and the Dirac equation
D. Adjoining the discrete transformations, P, C, T and CPT- corepresentations
E. Graded Lie algebras, supersymmetry and supergroups
F. Clifford algebras and spinors in general spacetime dimensions.

V. Quantum groups
A. Algebra and coalgebra
B. Product and coproduct
C. Unit and counit
D. Bialgebras
E. Hopf algebras
F. Quantum groups
G. Twisted products and star products
H. Applications

Further applications will be presented depending on the interests of the class. The syllabus given above emphasises particle physics; however other areas will be covered according to the interests of the class.

There will be weekly homework, a midterm and a final project to be presented as a talk in class.