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Physics 704

HOMEWORK ASSIGNMENT #4

Spring 2006

Deadline Thursday, April 13

1. Random numbers

a) Consider a linear congruent algorithm $I_{j+1} = aI_j + c \pmod{m}$ for a machine with $m = 2^4 - 1 = 15$. Taking $a = 7$ and $c = 4$, take a seed $I_0 = 4$ and generate (and list) the first 16 pseudorandom numbers, and find the period.

b) Show that the period depends on the choice of the seed! (E.g. what happens with $I_0 = 11$?)

c) How many repeats do you need to establish that you are in the second period?

2. Use the applet at <http://www.math.utah.edu/%7Eepa/Random/Random.html> to compare the best and the worst random number generators. Here is an easy way to capture graphics: with the Random Graphics window open, enter Alt-PrintScreen, then paste the image into a Word document. The best submission of this problem will be reproduced in the solutions.

3. For a finite-size lattice of linear dimension L , the critical temperature should differ noticeably from the known value for the infinite system.

a) Locate the estimate for the critical temperature for at least 2 different L 's, using different applets (unless you want to write your own code, which I promised you would not need to do, or you find an applet with an adjustable L). You might use the peak of the susceptibility or specific heat or the point of inflection of $m^2(T)$.

b) According to finite-size scaling theory, how are these estimates of T_c related to T_c for an infinite lattice for the 2D Ising model?

4. We did not discuss the boundary conditions when doing the Ising model. Generally one uses periodic boundary conditions but others can also be used.

a) If you use free boundary conditions (so no bond to the non-site beyond the boundary) for a lattice of modest size, should the approximate critical temperature be higher or lower or the same as with periodic boundary conditions? Why?

b) Show that antiperiodic boundary conditions in one direction can be used to set up an interface for a 2D Ising model.

c) Find/compare the energy to flip a spin in i) an ordered region, ii) adjacent to a straight interface, iii) at a kink of the interface (i.e. at the interior corner of a right-angle bend of the interface).

d) What boundary conditions would you use to simulate the model treated in the infamous problem 3 of Set 3?

For those interested, LMB problem 7.9.1 has many worthwhile, if time-consuming, exercises. Observing the collapse of data using the correct scaled variables can be a thrilling experience!