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Physics 704 **HOMEWORK ASSIGNMENT #3** Spring 2006

Deadline Tuesday, March 28

1&2. LMB, problem 4.6.1. (Counts as 2 problems.)

Hint for part 2: You should find $\nu(4) = N$, $\nu(6) = 2N$, $\nu(8) = N(N-5)/2$. (The solution provided goes up to b = 12!)

3. Consider the solid-on-solid (SOS) model of an interface. In each column of a 1D lattice i, the interface lies at a position $n_i = 0, 1, 2, 3, \ldots$, which is single-valued (so no overhangs or voids):

$$\mathcal{H} = \varepsilon \Sigma_i |n_i - n_{i+1}| - K \Sigma_i \delta_{n_i,0} \qquad n_i = 0, 1, 2, 3, \ldots$$

This model corresponds to LMB Fig. 3.22.

a) Write down the transfer matrix of this model in terms of $x = \exp(-\varepsilon / k_B T)$ and $\kappa = \exp(K/k_B T)$.

b) By considering an eigenvector of the form

$(\varphi_0, \cos(q+\theta), \cos(2q+\theta), \ldots)$

show that there is a continuous spectrum of eigenvalues

$(1-x)/(1+x) \le \lambda \le (1+x)/(1-x)$.

c) Show that for $\kappa > (1-x)^{-1}$, there is also a bound state eigenvector of the form

$(\varphi_0, e^{-\mu}, e^{-2\mu}, \ldots)$ corresponding to the eigenvalue $\lambda_0 = [\kappa(1-x^2)(\kappa-1)]/[\kappa(1-x^2)-1]$.

One can also show that this λ_0, when it exists, is the largest eigenvalue; thus, it dominates the thermodynamics. Hence the critical value of K is given via $\kappa_c = (1-x)^{-1}$. (Adapted from Yeomans)

4. For T=Tc and h small one expects that the correlation length x will have the scaling form

$$\xi(h,0) \sim |h|^{-\nu_H}$$

and that the pair correlation function will have the approximate form

$$g(r,h,0) \sim r^{-(d-2+\eta_H)} \exp(-r/\xi) \quad .$$

a) Use Landau-Ginzburg theory to derive that the classical values of the critical exponents are $\nu_H = 1/3$ and $\eta_H = 0$. Start with the equivalent of eq. 4.107 (but with the straightforward last term from the ordinary Landau expansion rather than the complicated expression—cf. PB eq. 3.87). Let $h(r) = h_0 + h_1\, \delta\, (r)$ and $m(r) = m_0(T_c,h) + \varphi(r)$ and keep only the leading power of m_0 and terms linear in φ.

b) The susceptibility is expected to diverge as |h| -> 0 on the critical isotherm with an exponent γ_H. Express γ_H in terms of ν_H and η_H.