

**Department of Physics  
University of Maryland  
College Park, MD 20742-4111**

**Physics 704**

**HOMEWORK ASSIGNMENT #2**

Spring 2006

Due: Thursday, Feb. 23; deadline Tuesday, Feb. 28

1. LMB, problem 4.6.6, parts 4-6, (Part 1 is trivial. Using what we learned about functional derivatives, you should be able to derive the two "equations of motion"—at least the first—but you do not need to turn this in.) Note that we did most of 4.6.5 in class.

2. Consider an Ising model (eq. 3.30) in 1D which also includes a 3-spin term  $-J_t \sum_{\langle ij k \rangle} S_i S_j S_k$  where  $i, j$ , and  $k$  are consecutive sites along the chain. Convert this Hamiltonian to the lattice gas model  $H = E_t \sum_{\langle ij k \rangle} n_i n_j n_k + E \sum_{\langle ij \rangle} n_i n_j + E_H \sum_i n_i + \text{const.}$ , and find the relationship between  $J_t/J$  and  $E_t/E$ , showing that they are not simply proportional to each other. (As noted in class, this recognition has eluded some prominent researchers in the field!)

3. Consider Landau theory for a first-order phase transition.

a) Derive the relationship between  $T_c$  and  $T_0$ : Show  $T_c - T_0 \propto g_4^2/(ag_6)$  and determine the numerical proportionality factor.

b) Determine the latent heat of the first-order transition in terms of the Landau parameters.

4. a) Show that a system described by a free energy

$$G(T, m) = g_0 + (1/2)a(T-T_0)m^2 + (1/3)g_3 m^3 + (1/4)g_4 m^4$$

(with  $m$  real) has a first-order transition in Landau theory, and determine  $T_c$ .

b) Let us define  $\psi = m + g_3/(3g_4)$ .

i). Given  $\mathcal{H}(T, m) = (1/2)(\nabla m)^2 + g_0 + (1/2)a(T-T_0)m^2 + (1/3)g_3 m^3 + (1/4)g_4 m^4$ , work out  $\mathcal{H}(T, \psi)$ .

ii) Show that there is a term that corresponds to a magnetic field.

iii) In order to have a phase transition, this field must vanish. That happens trivially if  $g_3 = 0$ . Find the non-trivial relationship between the coefficients for which it also vanishes.

iv) Show that there is a critical point if ...