10S-1 Consider the field equation $R_{ab} - \frac{1}{4}Rg_{ab} = 0$. The trace (contraction with g^{ab}) of the left hand side vanishes identically, so this does not imply $R = 0$, and it is really only 9 independent equations. (a) Show using the contracted Bianchi identity (22.50) that this equation does however imply that R is a constant. Thus this equation is equivalent to the vacuum Einstein equation with an undetermined cosmological constant term. (b) Now consider the field equation $R_{ab} - \frac{1}{4}Rg_{ab} = 8\pi(T_{ab} - \frac{1}{4}Tg_{ab})$, where T_{ab} is the matter stress tensor and T is its trace. Show using the Bianchi indentity together with the local conservation of stress energy (22.40) that this equation implies the Einstein equation (22.51) with an additional undetermined cosmological constant term.

10S-2

(a) Compute the Christoffel symbols and Riemann tensor BY HAND (it's good for the soul to do this at least once in your life) for the line element

$$ds^2 = -dt^2 + a^2(t)q_{ij}dx^i dx^j,$$

where q_{ij} is a t-independent n-dimensional metric on the space labeled by coordinates x^i, $i = 1, ..., n$. Express your result in terms of a and $\dot{a} = da/dt$, δ^i_j, q_{ij}, and the Christoffel symbols and curvature tensor for the metric q_{ij}. (You should find there are Γ^t_{ij}, $\Gamma^k_{tj} = \Gamma^k_{jt}$, and Γ^k_{ij} components of the Christoffel symbol Γ, and all others vanish, and that the nonzero curvature components will have either no t indices or two t indices.)

(b) Specialize your result to the case $n = 1$, and characterize all the cases in which the 1+1-dimensional curvature vanishes. Explain how the time-dependent case is flat, by explaining how the (t, x) coordinates sit in the (1+1) Minkowski spacetime.

(c) In the case $n > 1$, characterize all the cases in which the $n + 1$ dimensional curvature vanishes. Explain how the time-dependent case is flat, by explaining how the (t, x^i) coordinates sit in the $(n + 1)$ Minkowski spacetime.

(d) (i) Now assume $n = 3$ and compute the Ricci tensor, Ricci scalar, and Einstein tensor. (ii) Assume q_{ij} is either R^3 (k=0), S^3 (k=+1) and H^3 (k=-1) and derive the Friedman equations from G_{tt} and G_{ij}.