Gravitational Waves: an Overview

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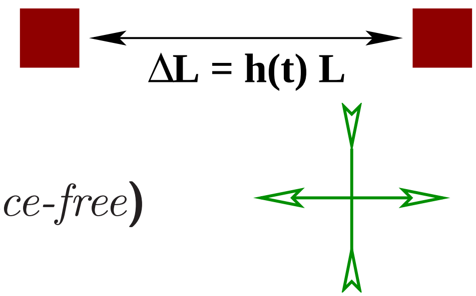
Phys675: Introduction to Relativity, Gravitation and Cosmology

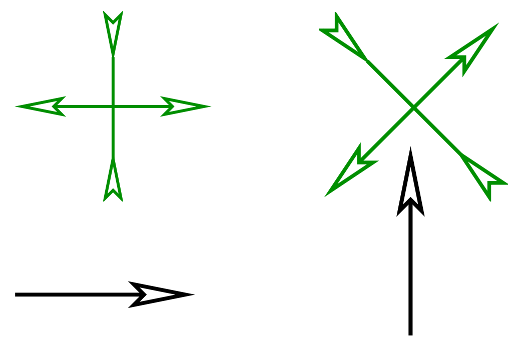
Properties of gravitational waves



- **Stretch and squeeze are**
  - *transverse* **to direction of propagation**
  - **equal and opposite along orthogonal axis (*trace-free*)**

- **Gravitons are spin-2 particles**

- **Two polarizations: h_+ and $h_\times$**



– **GW theory: polarizations rotated by $45^{\rm o}$**

– **EM theory: polarizations rotated by $90^{\rm o}$**

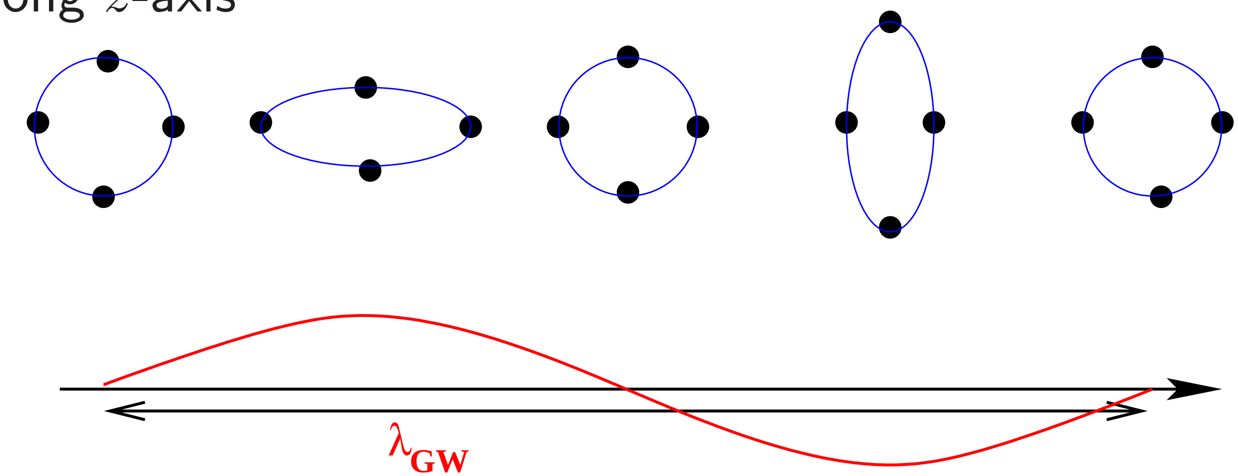
- **h_+ and $h_\times$ are double time integral of Riemann tensor**

$$\ddot{h}_{ij} \sim R_{i0j0} \sim \partial^2_{ij}\Phi \qquad \Phi \Rightarrow \text{non-static tidal potential}$$

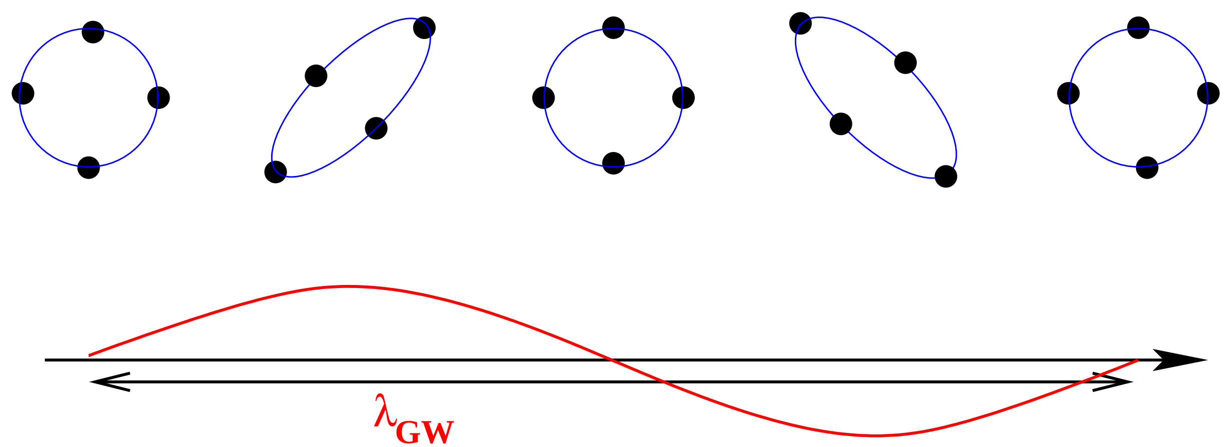
Interaction between GW and ring of free-falling particles

GW propagating along z-axis

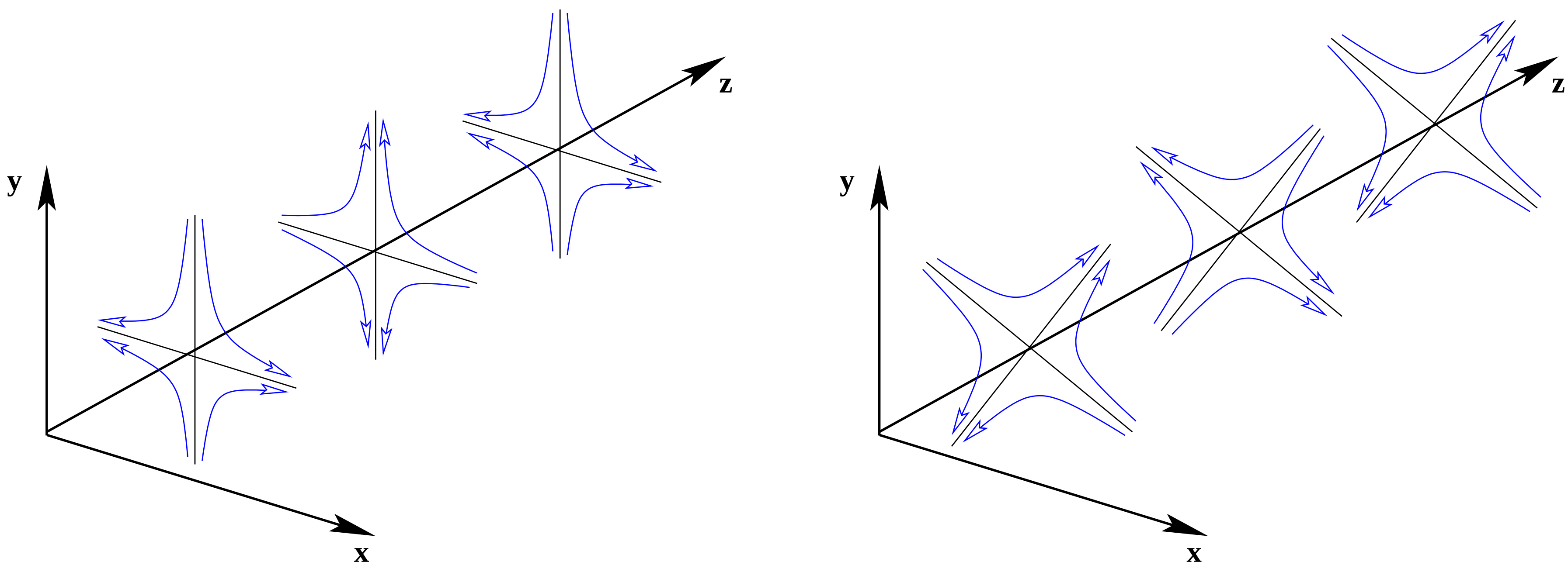
- Case: $h_+ \neq 0$
  $h_\times = 0$



- Case: $h_\times \neq 0$
  $h_+ = 0$



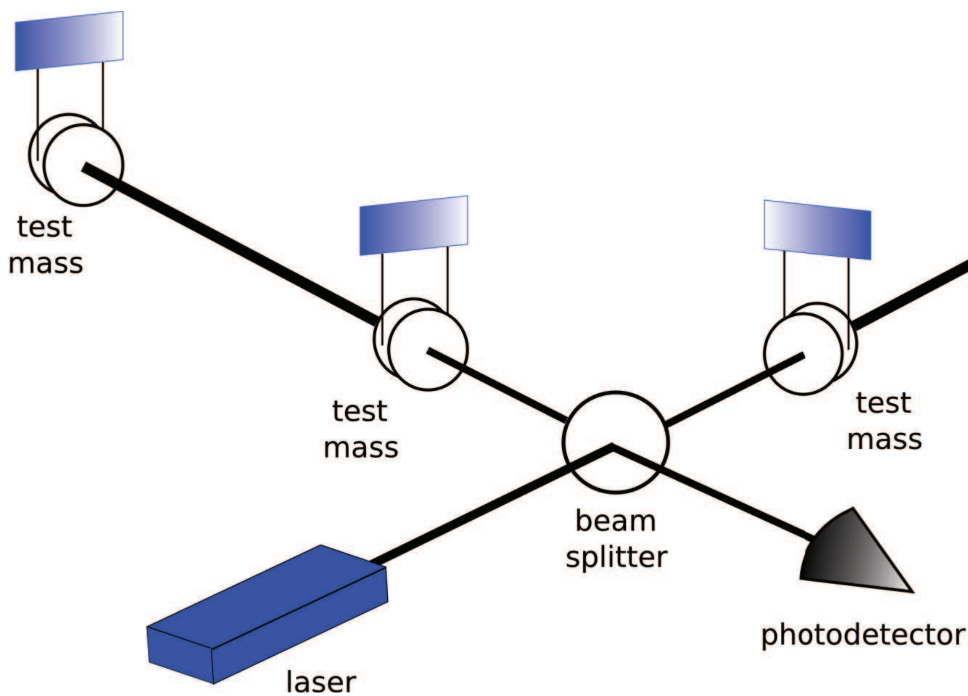
Force pattern for $h_\times$ and h_+



Force pattern of GWs are invariant under 180^o degree, by contrast force patterns of EM waves are invariant under 360^o degree

How to measure gravitational waves

Use light beams to measure the stretching and squeezing induced by GWs



$\Delta L = L\,h \sim 10^{-16}$ cm

being $L = 4$km and $h \sim 10^{-21}$

$\Delta\phi \sim 10^{-8}$ rad

Multipolar decomposition of waves in linear gravity

- Multipole expansion in terms of mass moments (I_L) and mass-current moments (J_L) of the source

$$h \sim \overbrace{\frac{G}{c^2}\frac{I_0}{r}}^{\text{can't oscillate}} + \overbrace{\frac{G}{c^3}\frac{\dot{I}_1}{r}}^{\text{can't oscillate}} + \overbrace{\frac{G}{c^4}\frac{\ddot{I}_2}{r}}^{\text{mass quadrupole}} + \cdots$$

$$\cdots + \underbrace{\frac{G}{c^4}\frac{\dot{J}_1}{r}}_{\text{can't oscillate}} + \underbrace{\frac{G}{c^5}\frac{\ddot{J}_2}{r}}_{\text{current quadrupole}} + \cdots$$

- Typical strength: $h \sim \frac{G}{c^4}\frac{M\,L^2}{P^2}\frac{1}{r} \sim \frac{G(E_{\rm kin}/c^2)}{c^2\,r}$

If $E_{\rm kin}/c^2 \sim 1 M_\odot$**, depending on** r $\quad \Rightarrow \quad h \sim 10^{-23}$–$10^{-17}$

Quadrupole nature of GW emission in linear gravity

EM theory: Luminosity $\propto \ddot{\mathbf{d}}^2$ $\mathbf{d} = e\,\mathbf{x} \Rightarrow$ electric dipole moment

- GW theory: electric dipole moment $\Rightarrow$ mass dipole moment

$$\mathbf{d} = \sum_i m_i\,\mathbf{x}_i \Rightarrow \dot{\mathbf{d}} = \sum_i m_i\,\dot{\mathbf{x}}_i = \mathbf{P}$$

Conservation of momentum $\Rightarrow$ no mass dipole radiation exists in GR

- GW theory: magnetic dipole moment $\Rightarrow$ current dipole moment

$$\mu = \sum_i m_i\,\mathbf{x}_i \times \dot{\mathbf{x}}_i = \mathbf{J}$$

Conservation of angular momentum $\Rightarrow$ no current dipole radiation exists in GR

Comparison between GW and EM luminosity

$$\mathcal{L}_{\rm GW} = \frac{G}{5c^5}\,(\dddot{I}_2)^2 \qquad I_2 \sim \epsilon\, M\, r^2$$

$r \longrightarrow$ **typical source's dimension,** $M \longrightarrow$ **source's mass,** $\epsilon \longrightarrow$ **deviation from sphericity**

$$\dddot{I}_2 \sim \omega^3\, \epsilon\, M\, r^2 \text{ **with** } \omega \sim 1/P \quad \Rightarrow \quad \mathcal{L}_{\rm GW} \sim \frac{G}{c^5}\, \epsilon^2\, \omega^6\, M^2\, r^4$$

$$\mathcal{L}_{\rm GW} \sim \frac{c^5}{G}\, \epsilon^2\, \left(\frac{G\, M\, \omega}{c^3}\right)^6 \left(\frac{r\, c^2}{G\, M}\right)^4 \Rightarrow \quad \frac{c^5}{G} = 3.6 \times 10^{59} \text{ erg/sec (huge!)}$$

- **For a steel rod of** $M = 490$ **tons,** $r = 20$ **m and** $\omega \sim 28$ **rad/sec:**
  $GM\omega/c^3 \sim 10^{-32}, rc^2/GM \sim 10^{25} \rightarrow \mathcal{L}_{\rm GW} \sim 10^{-27}$ **erg/sec** $\sim 10^{-60} \mathcal{L}^{\rm EM}_{\rm sun}$**!**

- **As Weber noticed in 1972, if we introduce** $R_S = 2GM/c^2$ **and** $\omega = (v/c)\,(c/r)$

$$\mathcal{L}_{\rm GW} = \frac{c^5}{G}\, \epsilon^2\, \left(\frac{v}{c}\right)^6 \left(\frac{r_S}{r}\right) \quad \underbrace{\Rightarrow}_{v\sim c,\, r\sim r_S} \quad \mathcal{L}_{\rm GW} \sim \epsilon^2\, \frac{c^5}{G} \sim 10^{26}\, \mathcal{L}^{\rm EM}_{\rm sun}!$$

GWs on the Earth: comparison with other kind of radiation

Supernova at 20 kpc:

- **From GWs:** $\sim 400 \frac{\rm erg}{\rm cm^2\, sec} \left(\frac{f_{\rm GW}}{1 \rm kHz}\right)^2 \left(\frac{h}{10^{-21}}\right)^2$ **during few msecs**

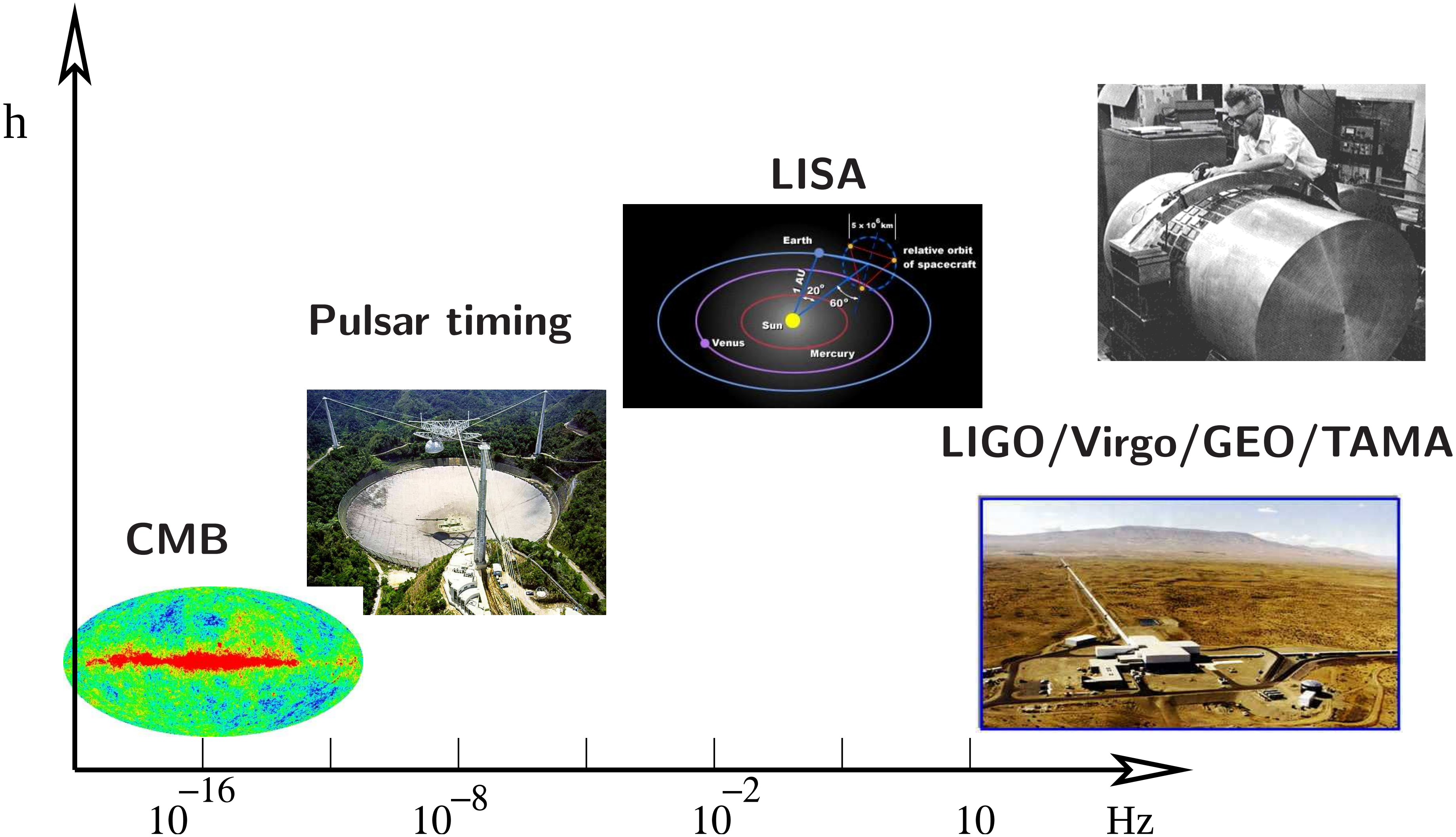
- **From neutrino:** $\sim 10^5 \frac{\rm erg}{\rm cm^2\, sec}$ **during 10 secs**

- **From optical radiation:** $\sim 10^{-4} \frac{\rm erg}{\rm cm^2\, sec}$ **during one week**

Electromagnetic astronomy versus gravitational-wave astronomy

EM astronomy	GW astronomy
– accelerating charges; time changing dipole	– accelerating masses; time changing quadr.
– incoherent superposition of emissions from electrons, atoms and molecules	– coherent superposition of radiation from bulk dynamics of dense source
– direct information about thermodynamic state	– direct information of system's dynamics
– wavelength small compared to source	– wavelength large compared to source
– absorbed, scattered, dispersed by matter	– very small interaction with matter
– frequency range: 10 MHz and up	– frequency range: 10 kHz and down

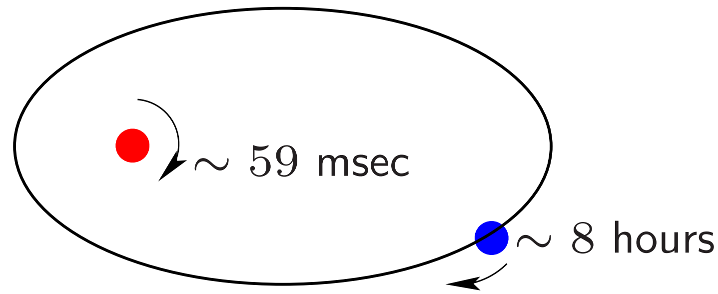
GW frequency spectrum extends over many decades



Indirect observation of gravitational wa

Neutron Binary System: PSR 1913 +16 - Timing Pulsars

Hulse & Taylor discovery (1974)



Separated by $\sim 10^6$ Km, $m_1 = 1.4 M_\odot$, $m_2 = 1.36 M_\odot$, eccentricity $= 0.617$

- **Prediction from GR: rate of change of orbital period**
- **Emission of gravitational waves:**
  - **due to loss of orbital energy**
  - **orbital decay in agreement with GR at the level of 0.5%**

Hulse-Taylor binary: cumulative shift of periastron time

To show agreement with GR, they compared the *observed* **orbital phase with a theoretical template phase**

If f_b **varies slowly with time, then to first order in a Taylor expansion**

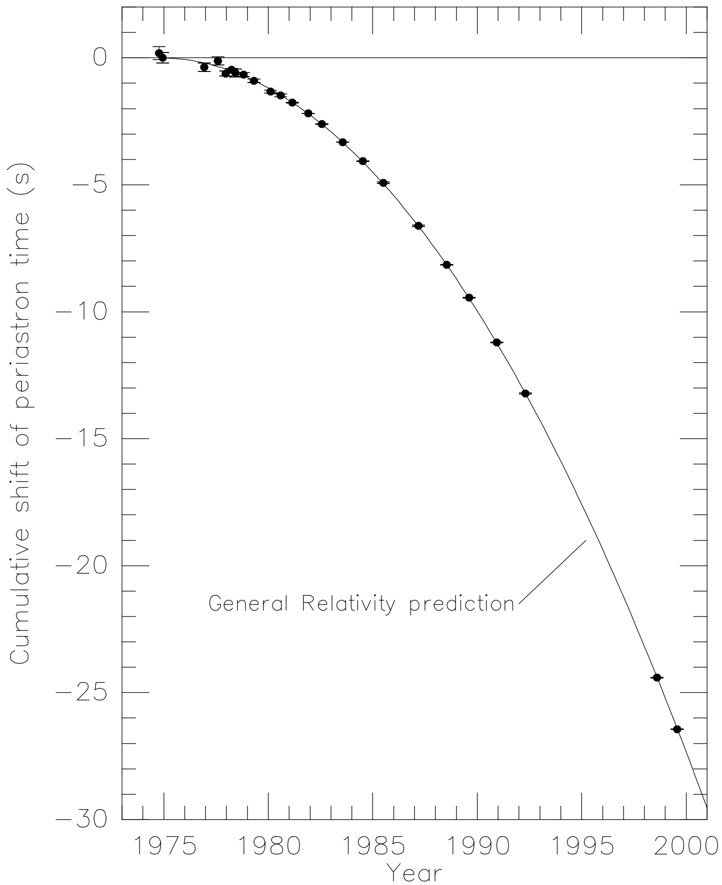
$$\Phi_b(t) = 2\pi\, f_b\, t + \pi\, \dot{f}_b\, t^2$$

Assuming that t_P **is the periastron passage time defined as**

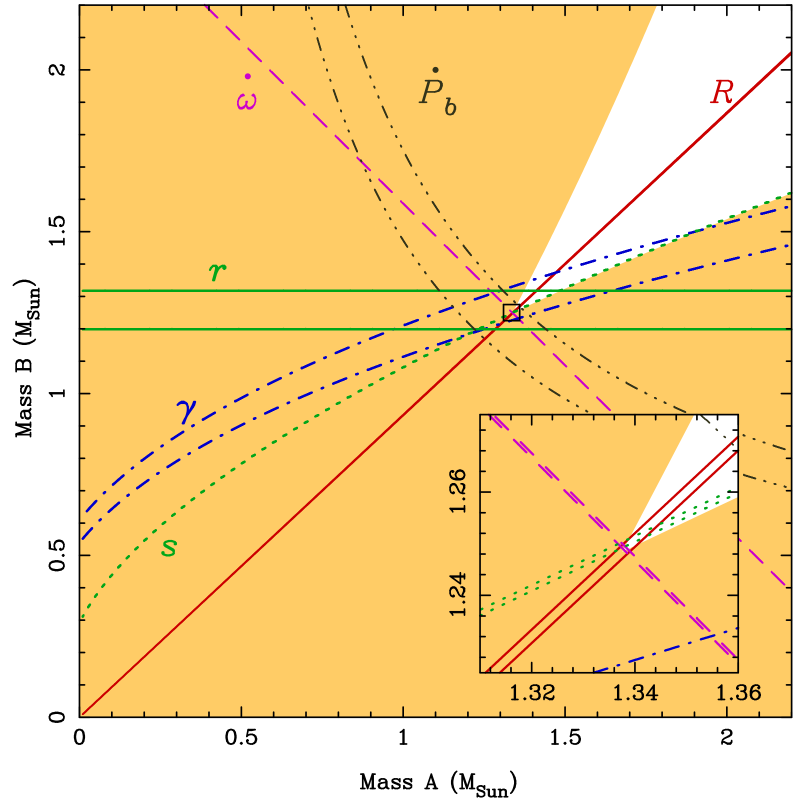
$\Phi(t_p) = 2\pi N$ $\qquad$ N **being an integer**

$$2\pi N = 2\pi f_b\, t_p + \pi\, \dot{f}_b\, t_p^2 \quad \Rightarrow \quad t_p - N/f_b = -\frac{1}{2}\dot{f}_b/f_b\, t^2$$

Hulse-Taylor binary: cumulative shift of periastron time



[from Taylor & Weisberg 2000]



[from Kramer et al. 2005]

Known double pulsar binaries

PSR J0737-3039 [Burgay et al. 03; Lyne et al. 2004]

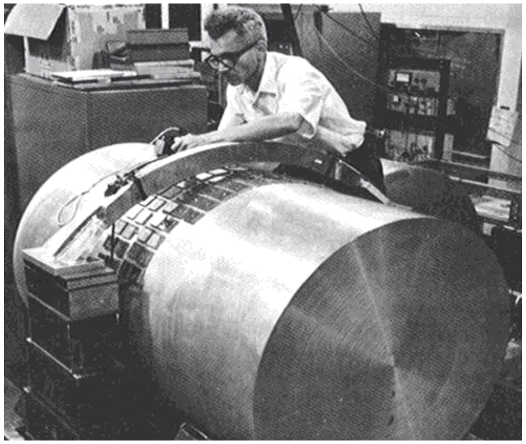
rot. period A 22.7 ms
rot. period B 2773.6 ms

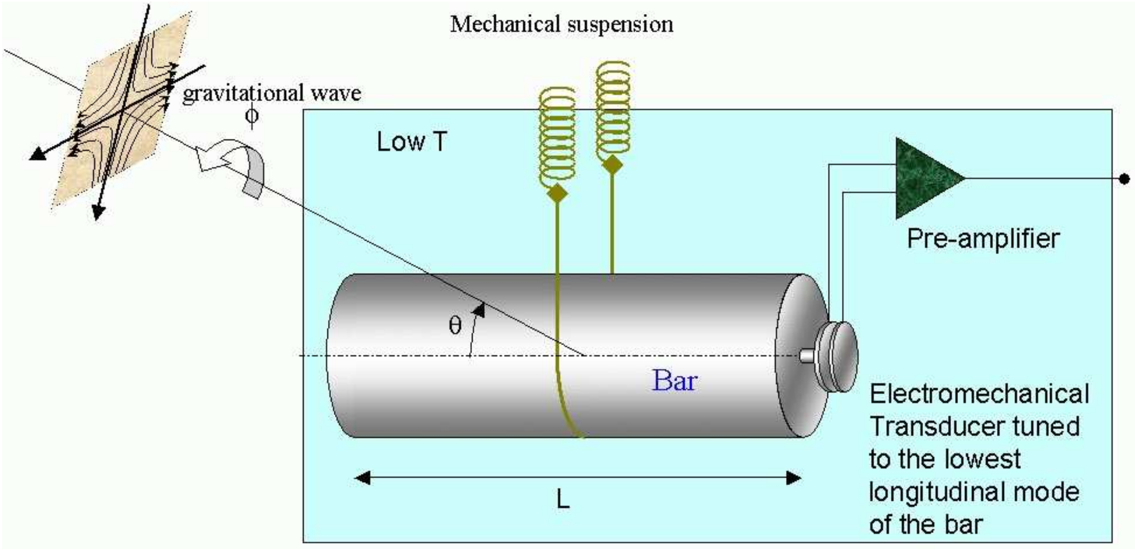
orb. period 2 h and 45 min (will merge in 85 Myr!)
$e = 0.088$
distance $\sim 0.6kpc$ **(close!)**

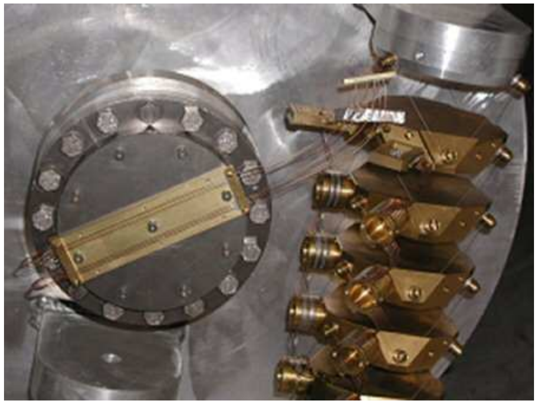
$\Delta\phi = 16.900(2)\mathrm{deg/yr}$ **(large!)** $\dot{P} = -1.20(8)10^{-12}$

Direct observation with resonant-mass detectors

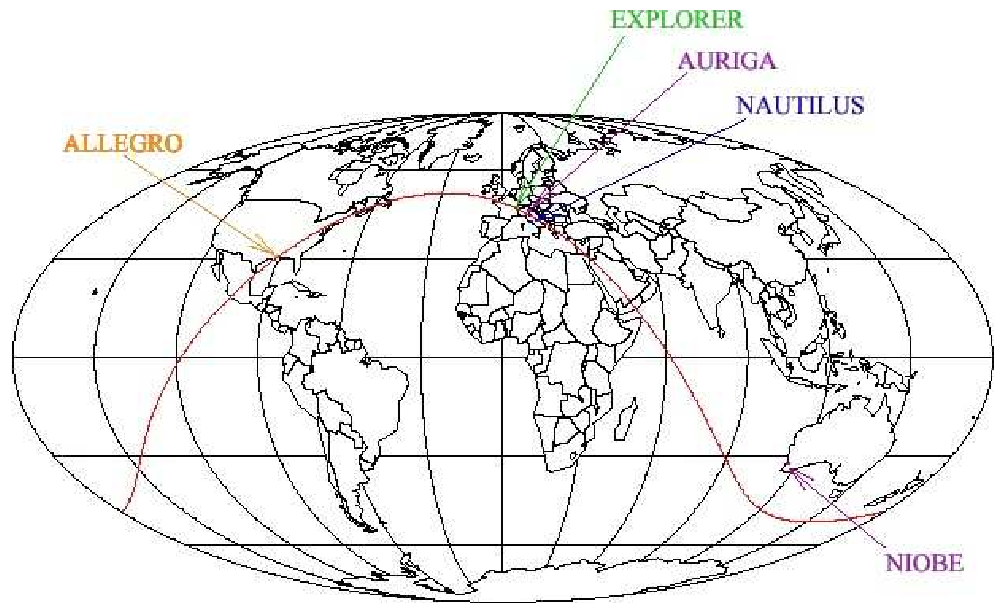
- **Pioneering work by** *Joe Weber* **at Maryland**







Resonant-mass detectors in the world



Resonant bar or sphere detectors (GW frequency $\sim$ 1 kHz)

Nautilus (Rome) Explorer (CERN) Schenberg (Brasil) MiniGRAIL (Belgium)

Allegro (Louisiana) Niobe (Perth) Auriga (Padova)

International network of GW interferometers (frequency band $\sim 10\text{–}10^3$ Hz)

LIGO at Livingston (Lousiana) ⇒

⇐ LIGO at Hanford (Washington State)

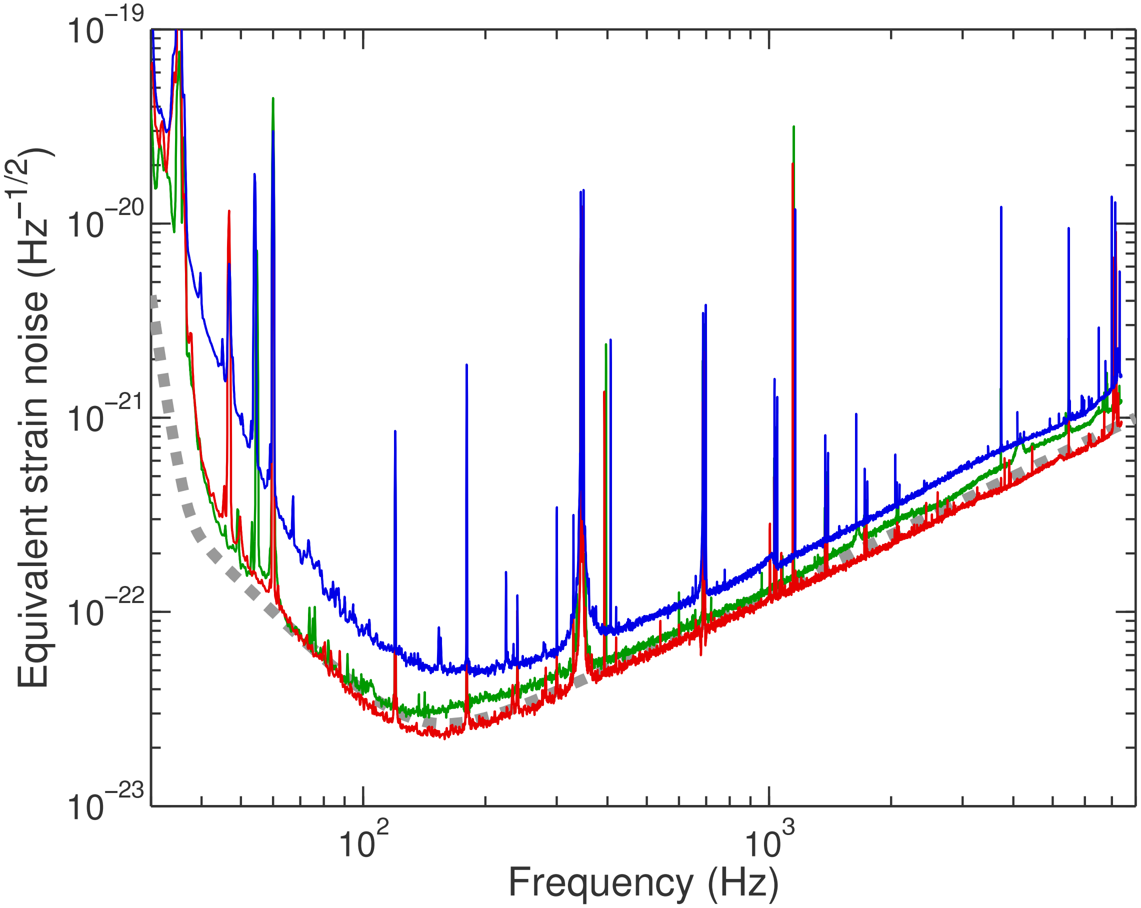
VIRGO (France-Italy) ⇒

⇐ GEO 600 (UK-Germany)

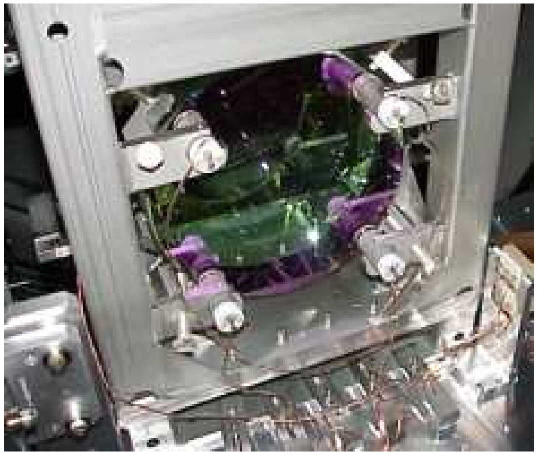
TAMA 300 (Japan)

Sensitivity of LIGO during run S5

$$\sqrt{S_h\,\Delta f} \sim h$$
$$h \sim \Delta L/L$$
$$\Delta f \sim f$$



A few LIGO/VIRGO/.... specifications and technical challenges

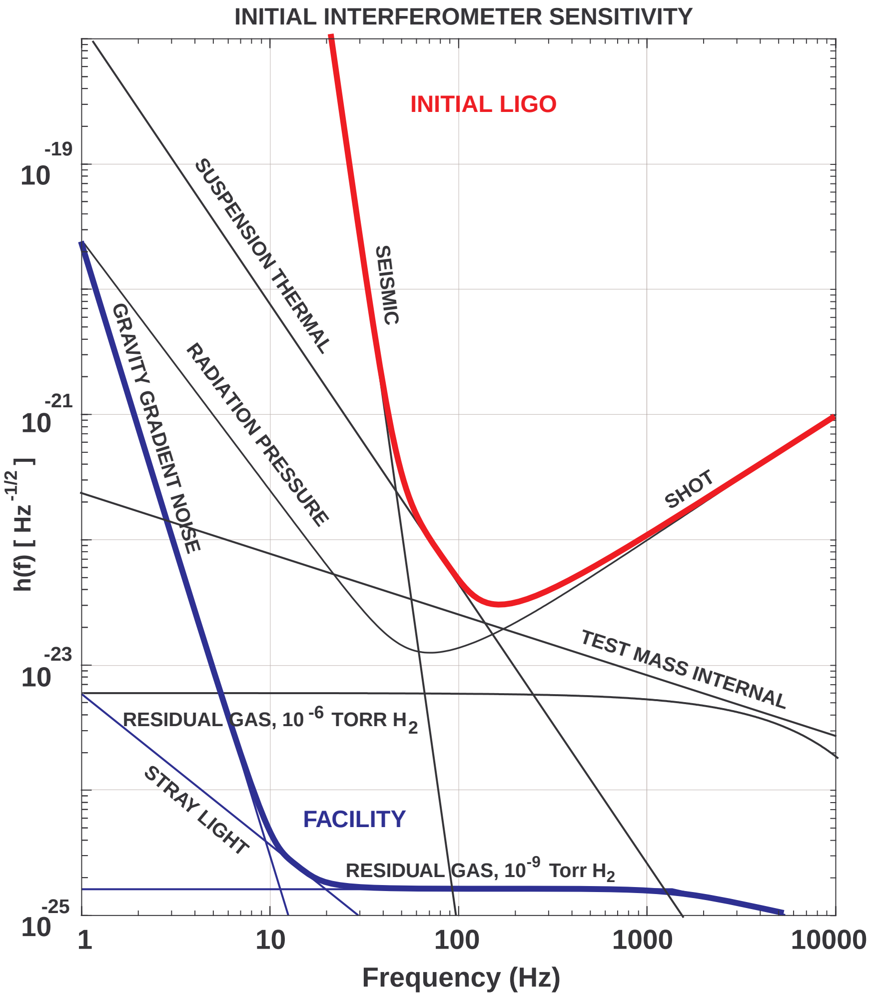


- **Monitor test masses (~ 11 kg) with precision of $\sim 10^{-16}$ cm with laser's wavelength of 10^{-4} cm**

- **Remove/subtract all non-gravitational forces such as thermal noise, seismic noise, suspension noise, etc.**

- **Beam tube vacuum level $\sim 10^{-9}$ torr**

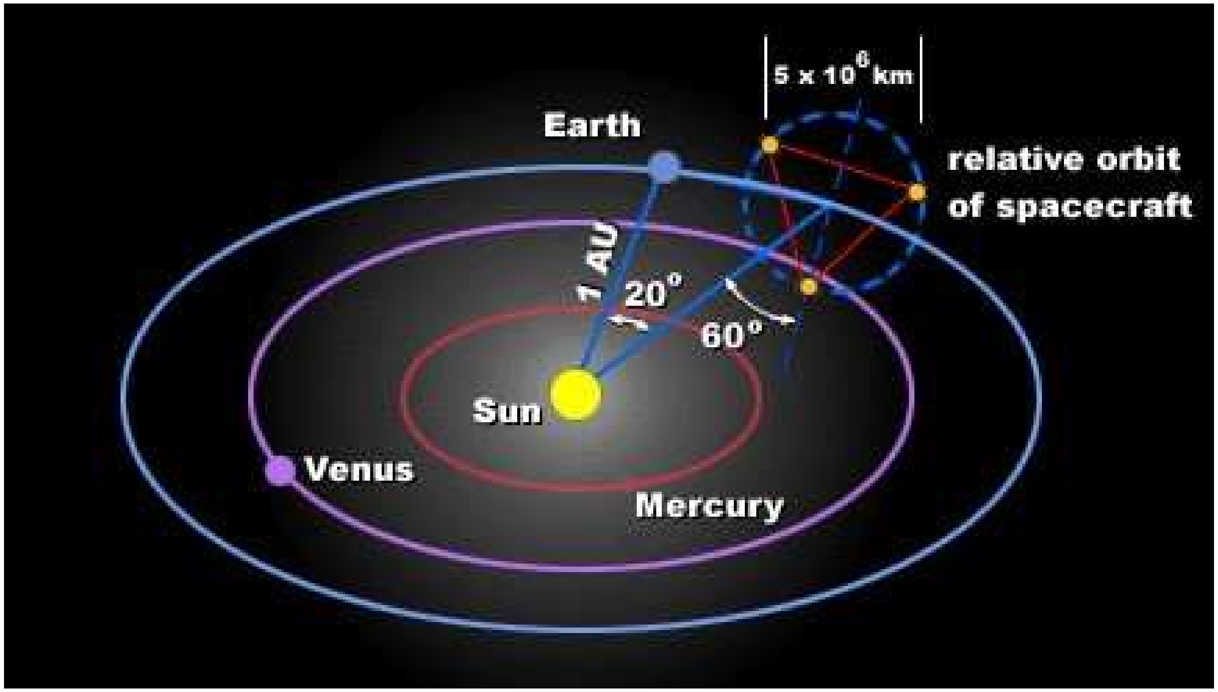
Typical noises in ground-based detectors



LISA: Laser Interferometer Space Antenna (frequency band: $10^{-4} - 0.1$ Hz)

LISA science goals complementary to ground-based interferometer ones

ESA/NASA mission in 2016?



A few LISA specifications and technical challenges

- Distance between spacecraft ~ 5 millions of km

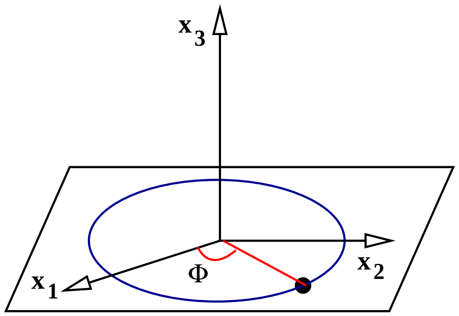
- Monitor test masses inside spacecrafts with a precision of $\sim 10^{-9}$ cm ($h \sim 10^{-21}$)

- Nd:YAG laser with wavelength $\sim 10^{-4}$ cm and power ~ 1 Watt

- Drag-free system to guarantee that only gravitational forces outside the spacecraft act on the proof masses

- Drag-free performances $\sim 10^{-15}\,\mathrm{m/sec}^2$ [LISA Pathfinder in 2010]

Binaries of black holes and/or neutron stars on circular orbit



Treating the two compact bodies as point particles with relative distance $\mathbf{x}$ and reduced mass μ:

$x_1 = r\,\cos\Phi$ and $x_2 = r\,\sin\Phi$

$$Q_{ij} = \mu\,(x_i\,x_j - \tfrac{1}{3}\delta_{ij}\,x^2)$$

$$Q_{11} = \mu r^2\,(\cos^2\Phi - 1/3), \qquad Q_{22} = \mu r^2\,(\sin^2\Phi - 1/3)$$

$$Q_{33} = -\mu r^2/3, \qquad Q_{12} = \mu r^2\,\sin\Phi\,\cos\Phi$$

$\dot{\Phi} = \sqrt{\frac{GM}{r^3}} = \omega$ [Newton law: $\omega^2\,r = \frac{GM}{r^2}$]

taking 3 time derivatives of $Q \Rightarrow \mathcal{L}_{\rm GW} = -\frac{dE}{dt} = \frac{32}{5}\frac{G^4}{c^5}\frac{\mu^2\,M^3}{r^5}$

Binary coalescence time

$$E = \frac{1}{2}\mu v^2 - \frac{G\mu M}{r} = -\frac{G\mu M}{2r} \quad \Rightarrow \quad r = -\frac{G\mu M}{2E}$$

$$\dot{r} = \frac{dr}{dE}\frac{dE}{dt} = -\frac{64}{5}\frac{G\mu M^2}{r^3} \quad \text{integrating} \quad \Rightarrow \quad r(t) = \left(r_0^4 - \frac{256}{5}\, G\mu\, M^2\, \Delta\tau_{\rm coal}\right)^{1/4}$$

If $r(t_f) \ll r_0 \quad \Rightarrow \quad \Delta\tau_{\rm coal} = \frac{5}{256}\frac{r_0^4}{G\mu\, M^2}$

Examples:

- **LIGO/VIRGO/GEO/TAMA source:** $M = (10 + 10)M_\odot$ **at** $r_0 \sim 500$ **km,**

  $f_{\rm GW} \sim 40$**Hz,** $T_0 \sim 0.05$**sec** $\Rightarrow$ $\Delta\tau_{\rm coal} \sim 1\,{\rm sec}$

- **LISA source:** $M = (10^6 + 10^6)M_\odot$ **at** $r_0 \sim 200 \times 10^6\,{\rm km}$**,**

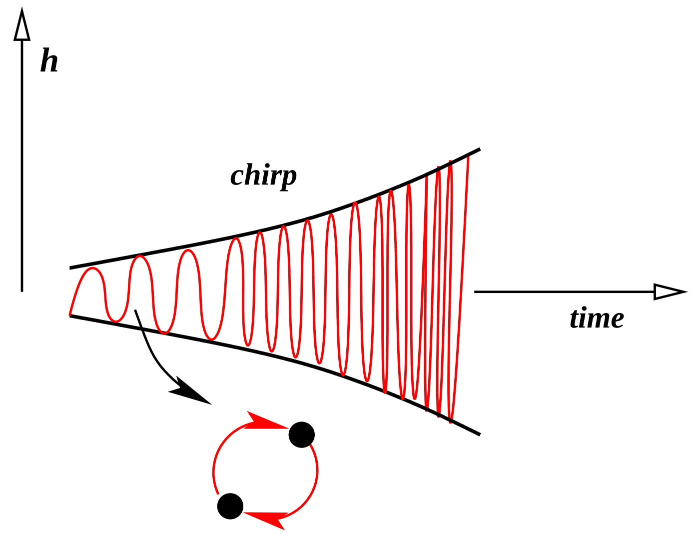
  $f_{\rm GW} \sim 4.5 \times 10^{-5}$ **Hz,** $T_0 \sim 11$ **hours** $\Rightarrow$ $\Delta\tau_{\rm coal} \sim 1\,{\rm year}$

Gravitational waves from compact binaries

- Mass-quadrupole approximation: $h_{ij} \sim \frac{G}{Rc^4} \ddot{Q}_{ij}$ $\quad Q_{ij} = \mu \, (x_i \, x_j - r^2 \, \delta_{ij})$

$$h \propto \frac{M^{5/3} \, \omega^{2/3}}{R} \cos 2\Phi$$

for quasi-circular orbits: $\omega^2 \sim \dot{\Phi}^2 = \frac{GM}{r^3}$



Chirp: The signal continuously changes its frequency and the power emitted at any frequency is very small!

$h \sim \frac{M^{5/3} \, f^{2/3}}{R}$ for $f \sim 100$ Hz, $M = 20 M_\odot$

R at 20 Mpc $\Rightarrow$ $h \sim 10^{-21}$

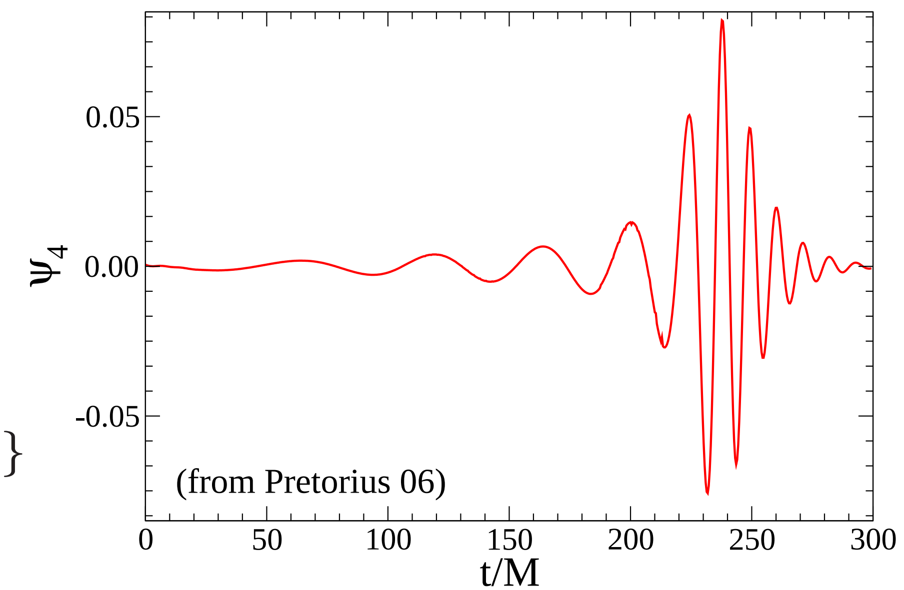
Typical features of coalescing black-hole binaries

• Inspiral: quasi-circular orbits

Throughout the inspiral $T_{\rm RR} \gg T_{\rm orb} \Rightarrow$ **natural** *adiabatic parameter* $\frac{\dot{\omega}}{\omega^2} = \mathcal{O}\left(\frac{v^5}{c^5}\right)$

For compact bodies $\frac{v^2}{c^2} \sim \frac{GM}{c^2 r} \Rightarrow$ **PN approximation: slow motion and weak field**

"Chirping" if $T_{\rm obs} \gtrsim \omega/\dot{\omega}$



• Inspiral: spin-precessing orbits

$T_{\rm RR} \gg T_{\rm prec} \gg T_{\rm orb}$; $\omega_{\rm GW} = \{\omega_{\rm prec}, 2\omega\}$

• Inspiral: eccentric orbits

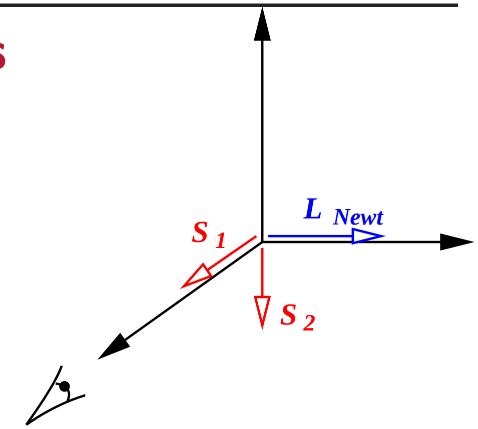
$T_{\rm RR} \gg T_{\rm peri\,prec} \gg T_{\rm orb}$; $\omega_{\rm GW} = \{N\,\omega, \cdots\}$

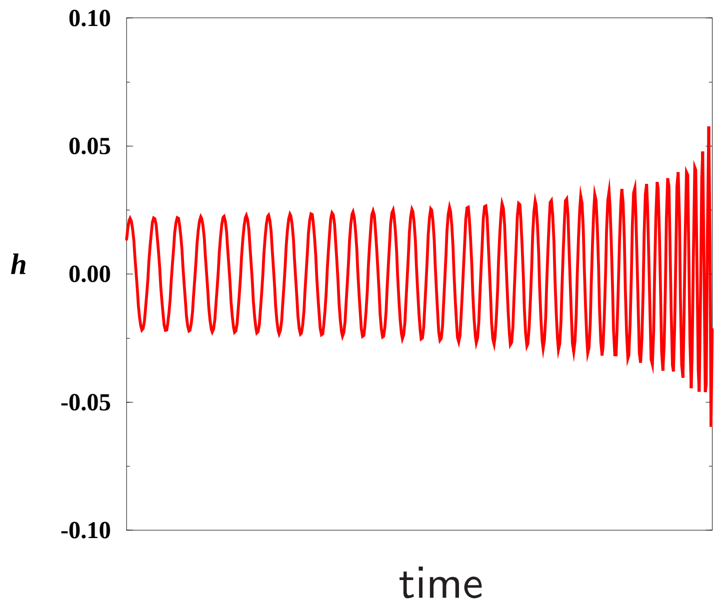
• Plunge-merger-ringdown

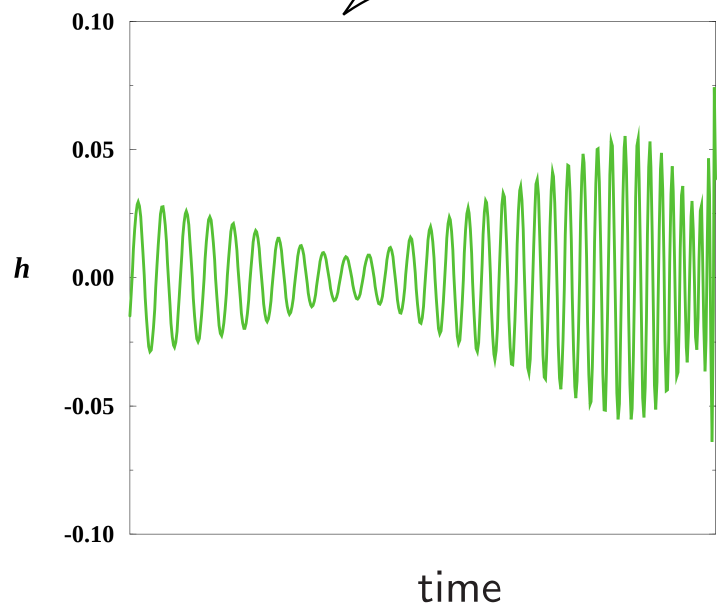
Numerical relativity; close-limit approximation; post-Newtonian resummation techniques

Waveforms including spin effects

Maximal spins $M = 10 M_{\odot} + 10 M_{\odot}$







Massive BH binaries

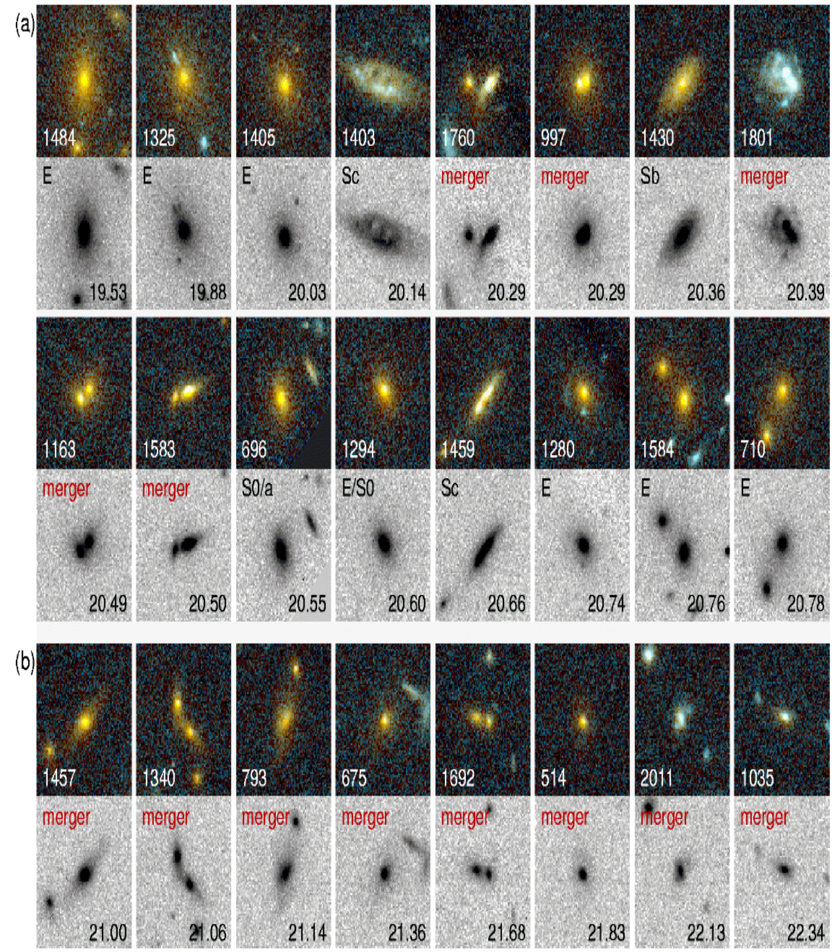
Massive BH binaries form following the mergers of galaxies and pregalactic structures

MS 1054-03 (cluster of galaxies) at $z = 0.83$: about 20% are merging!

(a) $\longrightarrow$ 16 most luminous galaxies in the cluster

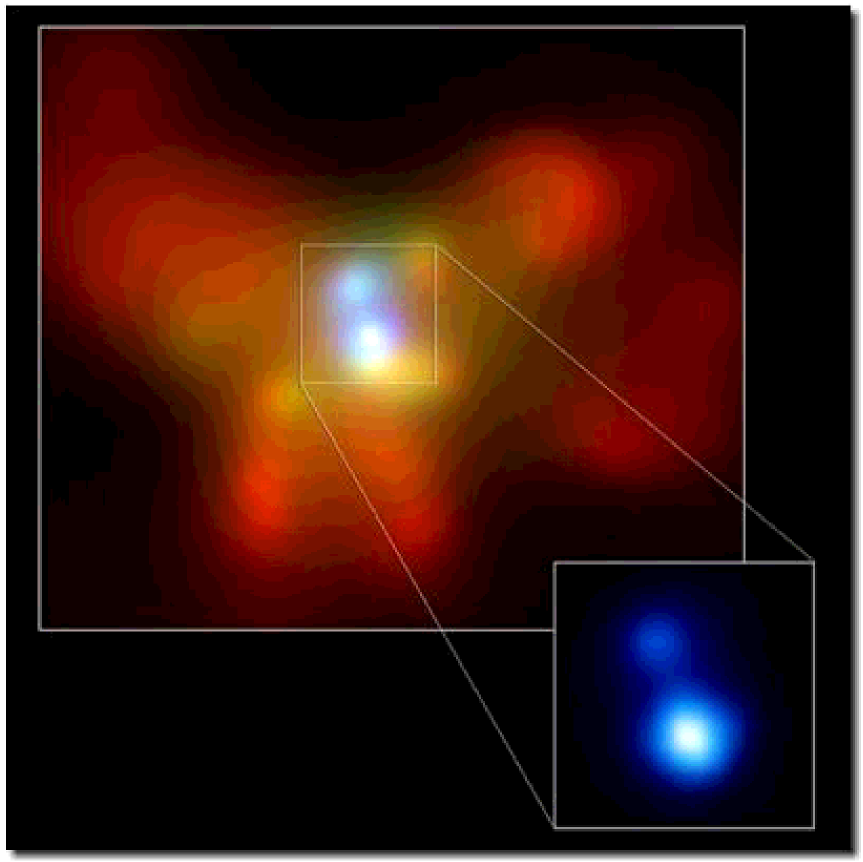
(b) $\longrightarrow$ 8 fainter galaxies

Masses and merger rates as function of redshift



van Dokkum et al., ApJ Letters, in press (astro-ph/9905394)

Image of NGC6240 taken by Chandra showing a butterfly shaped galaxy product of two smaller galaxies (two active giant BHs)



Massive BH binaries

Relativity:

- **From inspiraling post-Newtonian waveforms $\rightarrow$ precision tests of GR**
- **From merger waveforms (num. relativity) $\rightarrow$ tests of non-linear gravity**
- **Tests of cosmic censorship (is the final object a black hole?) and second law of BH mechanics (increase of horizon area)**

Astrophysics:

- **Cosmic history of massive BH formation from very high redshift to the present time**

Very high S/N (very large z); high accuracy in determining binary parameters, but event rates uncertain 0.1–$100\,\mathrm{yr}^{-1}$

Extreme mass-ratio inspiraling binaries

Small body spiraling into central body of $\sim 10^5$–$10^7\, M_\odot$ out to $\sim$ Gpc distance

Relativity:

- **Relativistic orbits (test of GR)**
- **Map of massive body's external spacetime geometry. Extract multiple moments. Test the BH no hair theorem (is it a black hole?)**

Astrophysics:

- **Probe astrophysics of dense clusters around BH's in galactic nuclei**
- **Existence and population of BH's in galactic nuclei**
- **Infer massive body's spin and mass with accuracy 10^{-3}–10^{-5}; sky position determined within $1^{\rm o}$; distance-to-source's accuracy of several 10%**