Syllabus for Phys.624
Introductory Quantum Field Theory
Fall, 2006

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Text Book: "Quantum Field Theory" by F. Mandl and G. Shaw (John Wiley)

" A First Book of Quantum Field Theory" by P. B. Pal and A. Lahiri (CRC Press)

Topics to be covered are Chapters 1-13 of Mandl and Shaw or Ch. 1 to 14 from Pal and Lahiri. After a discussion of preliminaries, I cover canonical quantization of scalar, spin half and vector fields, Noether's theorem and symmetries, Interaction picture and Feynman rules for perturbative calculation of physical processes, Renormalization, spontaneous symmetry breaking and introduction to gauge field theories.

I will mostly follow the Pal -Lahiri book but will heavily consult the Mandl-Shaw book as well as other books in the market. There will be two midterm exams: early Oct., mid-Nov. and a final exam on the date in the college schedule i.e. December 19, 8-10:AM.

There will be homework assignments every week. They will be collected the following week, same day it was assigned. The final grades will be based on all homeworks and the tests.

My office hours are wednesday and Friday 2-3; at other times, (except Tuesday and Thursdays) I will be available with appointment. Do take advantage of the office hours. Quantum Field Theory is a completely new language, although the basic rules are that of Quantum mechanics; knowledge of QFT is essential in all fields of theoretical physics. So you will need help in thoroughly assimilating the rules and techniques of this important topic.

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Final grade will be based on 40% from midterms, 40% from final and 20% homework.

Phys. 624 Homework 1
Fall, 2005
Due September 6, wednesday

1. Radiation field inside a cubic enclosure, which contains no charges, is specified by the quantum state:

$$|c\rangle = exp(-\frac{1}{2}|c|^2)\sum_{n=0}^{\infty}\frac{c^n}{\sqrt{n!}}|n\rangle$$

where $c = |c|e^{i\delta}$ is any complex number and $|n\rangle$ is the state in which there are n photons. Derive the following properties of $|c\rangle$: (i) $|c\rangle$ is a normalized state; (ii) $a|c\rangle = c|c\rangle$ where a is the annihilation operator as in the case of a simple harmonic oscillator; (iii) $\langle c|\mathbf{N}|c\rangle = |c|^2$, where $\mathbf{N}$ is the number operator.

2. Consider a one dimensional linear chain described by the Lagrangian

$$L \; = \; T - V = \textstyle\sum_{n=1}^{N} \frac{m}{2}\dot{q_n}^2 - \sum_{n=1}^{N} \frac{\kappa}{2}(q_{n+1} - q_n)^2.$$

Write down the field equation and the Hamiltonian for this system using your knowledge of classical mechanics. Using normal coordinates to expand the coordinates $q_n(t)$ i.e. $q_n(t) = \sum_k c_k(t)e^{ikan}/\sqrt{N}$, rewrite the equations for the normal coordinates. (a is the lattice spacing.) Solve for the time evolution of the normal coordinates and show that the Hamiltonian is time independent. Use periodic boundary conditions i.e. $q_{N+1}(t) = q_1(t)$ which endows the linear chain with the topology of a closed ring. Show that in the limit of $N \to \infty$ and $a \to 0$, this Lagrangian can be rewritten as a free field theory for a scalar field.

3. (a) Use the canonical commutation relations for a real scalar field ϕ and its conjuate momentum π,

$$[\pi(t,\vec{x}), \phi(t,\vec{y})] = -i\delta(\vec{x} - \vec{y})$$

to show that the momentum operator $\mathbf{P} = -\int d^3x\pi(x)\nabla\phi(x)$ satisfies

$$[\mathbf{P}, \phi(x)] = i\nabla\phi(x), [\mathbf{P}, \pi(x)] = i\nabla\pi(x)$$

. Hence show that any operator which can be constructed as a simultaneous power series in $\phi(x), \pi(x)$ i.e. $F(x) = \sum_{m,n} c_{mn}\phi^n(x)\pi(x)^m$ satisfies $[\mathbf{P}, F(x)] = i\nabla F(x)$.

(b) Under spatial transformation $\mathbf{x} \to \mathbf{x} + \mathbf{c}$ $\phi(x) \to \phi(t, \mathbf{x} - \mathbf{c})$; show that one can write $\phi(t, \mathbf{x} - \mathbf{c}) = U\phi(x)U^\dagger$ where $U \; = \; e^{i\mathbf{c}\cdot\mathbf{P}}$.

4. Consider two pairs of creation and annihilation operators: $(a, a^\dagger)$ and $(b, b^\dagger)$ which satisfy the commutation and anti-commutation relations respectively. Suppose the Hamiltonian of the system is given by $H = \omega_a a^\dagger a + \omega_b b^\dagger b$.

(a) Consider an operator $Q = b^\dagger a$ and its hermitean conjugate. Calculate the commutation relation between Q and $a^\dagger$ and anti-commutation relation between Q and $b^\dagger$.

(b) When does Q commute with H ? Calculate the anti-commutator of Q and $Q^\dagger$ in this case.