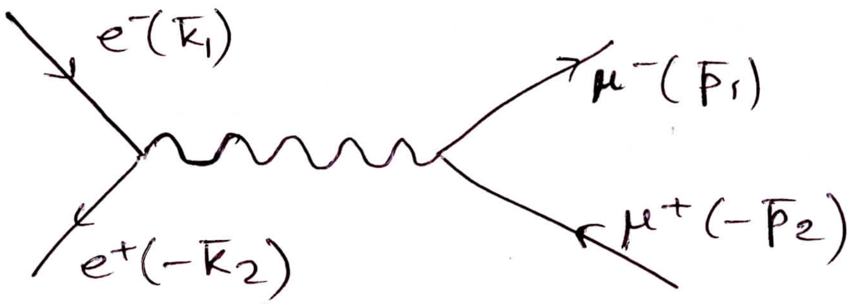
$\boxed{e^+e^- \to \mu^+\mu^-}$ → heavier particle with same charge as electron

(<u>in</u>elastic process, assuming enough energy in e^+e^-)

- $\mathcal{L}_{int} = -e A_\mu \left[\bar{\Psi}_{(e)} \gamma^\mu \psi_{(e)} + \bar{\Psi}_{(\mu)} \gamma^\mu \psi_{(\mu)} \right]$

i.e., another "copy" of 1st term with <u>same</u> feynman rule

- Feynman diagram (and notation for momenta):



with $i\mathcal{M} = (ie)^2 \, i D_{\lambda\nu}(k_1+k_2) \times$ ← photon propagator

$$\left[\bar{v}(k_2)\gamma^\lambda u(k_1)\right]\left[\bar{u}(p_1)\gamma^\nu v(p_2)\right]$$

↳ <u>drop</u> "muon" and spin label

where

$$D_{\lambda\nu}(k) = -\frac{1}{k^2+i\varepsilon}\left[g_{\lambda\nu} - (1-\xi)\, k_\lambda k_\nu / k^2\right]$$

→ drops out in 't Hooft-Feynman gauge

However,

$(k_1+k_2)_\lambda \left[\bar{v}(k_2)\gamma^\lambda u(k_1)\right]$

↑ from $(1-\xi)$ term of photon propagator

$$= \left[\bar{v}(k_2)\left(\underbrace{\not{k}_2}_{\ominus m_e} + \underbrace{\not{k}_1}_{+m_e}\right)u(k_1)\right]$$

$\Rightarrow$ $(1-\xi)$ - part of propagator doesn't contribute!

So, propagator and hence $\mathcal{M}$ simplifies to

$$\mathcal{M} = \frac{e^2}{s}\left[\bar{v}(k_2)\gamma^\lambda u(k_1)\right]\left[\bar{u}(p_1)\gamma_\lambda v(p_2)\right]$$

actually $(k_1+k_2)^2$ (from photon propagator) ... but this expression is Lorentz invariant $\Rightarrow$ evaluate it in CM frame, where it is total energy^2 (since $\vec{k}_1 = -\vec{k}_2$ in that frame)

denoted by s earlier (in $\nu_\mu e \to \mu \nu_e$)

For unpolarized cross-section sum over muon spins and average over electron spins to give

$$\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2$$

from $[\bar{v}(k_2)\ldots][\bar{v}(k_2)\ldots]^*$

$$= \frac{e^4}{4s}\,\mathrm{Tr}\left[(\not{k}_2 - m_e)\gamma^\lambda(\not{k}_1 + m_e)\gamma^\rho\right] \times$$

$$\mathrm{Tr}\left[(\not{p}_1 + m_\mu)\gamma_\lambda(\not{p}_2 - m_\mu)\gamma_\rho\right]$$

from $[\bar{u}(p_1)\ldots][\bar{u}(p_1)\ldots]^*$

[Use Eq. 7.18 of Lahiri and Pal to convert sum over spins to traces, with $F = \gamma_\lambda$ ... or do it from scratch like in notes on $\nu_\mu e \to \mu \nu_e$.

Recall that "λ" on γ_λ in $\mathcal{M}$ is dummy index so that we get another such index ("ρ") from $\mathcal{M}^*$]

— Neglect m_e since we must have $\sqrt{s} \geq 2m_\mu$ (for process to be allowed kinematically) and m_μ ($\approx$ 100 MeV) $\gg$ m_e ($\approx$ 0.5 MeV)

— Each trace gives $4 \times [g^{\cdot\cdot} g^{\cdot\cdot} + \ldots]$ as usual:

$$|\mathcal{M}|^2 = \frac{4e^4}{s}\left[k_2^\lambda k_1^\rho + k_2^\rho k_1^\lambda - (k_1 \cdot k_2)\, g^{\lambda\rho}\right] \times$$
$$\left[p_{1\lambda} p_{2\rho} + p_{1\rho} p_{2\lambda} - g_{\lambda\rho}\left(p_1 \cdot p_2 + m_\mu^2\right)\right]$$

m_μ^1 vanishes due to $\mathrm{Tr}[\gamma^3]$

(Next, use the fact each term in $[\ldots]$ is symmetric under $\lambda \Leftrightarrow \rho$)

$$= \frac{8e^4}{s}\Big[(k_2 \cdot p_1)(k_1 \cdot p_2) + (k_2 \cdot p_2)(k_1 \cdot p_1) - (k_2 \cdot k_1)(p_1 \cdot p_2 + m_\mu^2)$$
$$- (k_1 \cdot k_2)(p_1 \cdot p_2) + 2(k_1 \cdot k_2)(p_1 \cdot p_2 + m_\mu^2)\Big]$$

[$(k_1 \cdot k_2)(p_1 \cdot p_2)$ terms cancel; use $k_2 \cdot p_1 = k_1 \cdot p_2$ in 1st term (since $(k_1 - p_2)^2 = (p_1 - k_2)^2$) and $k_2 \cdot p_2 = k_1 \cdot p_1$ in 2nd term (since $(k_1 - p_1)^2 = (p_2 - k_2)^2$]

$$= \frac{8e^4}{s}\left[(k_2 \cdot k_1)\, m_\mu^2 + (k_1 \cdot p_2)^2 + (k_1 \cdot p_1)^2\right]$$

— Go to CM frame where θ is angle between e^- and μ^- so that

$k_1 \cdot p_1 = E\left(E - |\vec{p}|\cos\theta\right)$; $k_1 \cdot p_2 = E\left(E + |\vec{p}|\cos\theta\right)$

and $k_1 \cdot k_2 = 2E^2$

with $|\vec{p}_1| = |\vec{p}_2| = |\vec{p}| = \frac{1}{2}\sqrt{s - 4m_\mu^2}$ and $s = 4E^2$

(E being energy of e^- or e^+)

$$\Rightarrow \overline{|\mathcal{M}|^2} \text{ (in CM frame)} = e^4\left[1 + \frac{1}{E^2}\left(|\vec{p}|^2\cos^2\theta + m_\mu^2\right)\right]$$

$$= e^4\left[\left(1 + \cos^2\theta\right) + \frac{m_\mu^2}{E^2}\left(1 - \cos^2\theta\right)\right]$$

so that $\alpha = e^2/(4\pi)$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\underbrace{\sqrt{1 - \frac{4m_\mu^2}{s}}}_{\text{from } \int \text{LIPS}}\left[\left(1 + \cos^2\theta\right) + \frac{4m_\mu^2}{s}\left(1 - \cos^2\theta\right)\right]$$

drop it for $E \gg m_\mu$

($\sigma \to 0$ as $s \to 2m_\mu$ irrespective of $\mathcal{M}$)

$$\text{and } \sigma = \frac{4\pi\alpha^2}{3s}\sqrt{1 - \frac{4m_\mu^2}{s}}\left(1 + \frac{2m_\mu^2}{s}\right)$$

Cross-section into specific helicities in massless limit: as usual, insert L, R projection operators, but keep doing spin-sum: e.g.

$$\sum_{\text{spin}}\left[\bar{v}(k_2)\gamma^\lambda L u(k_1)\right]\left[\bar{v}(k_2)\gamma^\rho L u(k_1)\right]^* \ldots \text{will}$$

pick out only LH e^- (figure out chirality/helicity of e^+)
(again, this way is easier to evaluate using traces)