Physics 623, Spring 2010, O.W. Greenberg
Problem set on stability of matter

1. Give a qualitative overview of the issues concerning the stability of matter. Include a. the stability of a single atom, b. the stability of bulk matter, and c. the existence of a thermodynamic limit for matter. For each case state the qualitative issue that is crucial for the demonstation of stability.

2. Construct a trial wave function that obeys the Heisenberg-Weyl inequality

$$T_\psi < |x|^2 >_\psi \geq 9/4, \tag{1}$$

where $T_\psi = \int [\nabla \psi(x)]^2 d^3 x$, $< |x|^2 >_\psi = \int [x]^2 [\psi(x)]^2 d^3 x$, subject to $\int [\psi(x)]^2 d^3 x = 1$, and yet allows the energy of a single-electron atom to be unbounded below.

3. Use the Sobolev inequality

$$T_\psi \geq K_s [\int [\psi(x)]^6 d^3 x]^{1/3} \tag{2}$$

to find a finite lower bound for the energy of a single-electron atom.

K_s is a positive number whose value is not needed. The x's are three dimensional vectors.

Reference: E.H. Lieb, Rev. Mod. Phys. 48, 553 (1976).