Solutions to homework due May 8, 2009 for Physics 623, Spring 2009, O.W. Greenberg

Your name must be written in large legible letters.

1. Consider the Klein-Gordon equation with an attractive square well potential of depth $-|V_0|$ and range R. Find the weakest potential that produces an S-wave bound state
(a) if the potential is introduced as the time component of the vector potential;
(b) if the potential is introduced as a scalar potential, i.e. as a mass term.
2. Find a 5th anticommuting matrix in addition to α_i and β.
Represent 4x4 matrices as

$$A \otimes B = \begin{pmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{pmatrix} \tag{1}$$

where

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \tag{2}$$

Then $\alpha_i = \sigma_1 \otimes \sigma_i$, $\beta = \sigma_3 \times 1$. To anticommute with the three α's we need σ_2 or $\sigma_3 \otimes 1$.To anticommute with β need $\sigma_2 \otimes 1$ or any multiple of it.
3. Derive a formula for $\gamma^\mu \gamma^\nu$ in terms of $g^{\mu\nu}$ and $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\mu]_-$.
Trivial. $\gamma^\mu \gamma^\nu = (1/2)[\gamma^\mu, \gamma^\nu]_+ + (1/2)[\gamma^\mu, \gamma^\nu]_- = g^{\mu\nu} 1 + (1/i)\sigma^{\mu\nu}$.
(I left out the 1/2 in the definition of $\sigma^{\mu\nu}$ in the problem on the web.)

In 4. and 5. express your work in terms of the 2x2 identity matrix and the Pauli matrices.
4. Derive a set of gamma matrices in which $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ is diagonal.
Pick $\gamma_5 = \sigma_3 \otimes 1$. Then $\sigma_1 \times \sigma_i$ are three that anticommute with γ_5. $\sigma_2 \otimes 1$ anticommutes with the other four.
5. Derive a set of gamma matrices that are pure imaginary.
There are 16 combinations of $A \otimes B$ with A arbitrarily chosen real, so $A = i, \sigma_1, i\sigma_2, i\sigma_3$, and B arbitrarily chose imaginary, so $B = i, i\sigma_1, \sigma_2, i\sigma_3$. Choose

the γ_0 hermitian, the others antihermitian. Pick one of the hermitian ones, say $\sigma_1 \otimes \sigma_2 = \gamma_0$. By inspection you can find that $\gamma_1 = 1 \otimes i\sigma_1$, $\gamma_2 = i\sigma_2 \otimes \sigma_2$, $\gamma_3 = \sigma_1 \otimes \sigma_3$ satisfy the conditions. I think there are lots of other ways to do this using the above technique.