Homework 7: Due October 24

1. The current operator we derived in class $\hat{\vec{J}}(\vec{x}) = \frac{\hat{\vec{p}}\hat{\rho}(\vec{x}) + \hat{\rho}(\vec{x})\hat{\vec{p}}}{2m}$ was valid for Hamiltonians of the form $\hat{H} = \frac{\hat{\vec{p}}^2}{2m} + V(\hat{\vec{x}})$. In this problem I want you to consider the case of a particle in an electro-magnetic field $\hat{H} = \frac{\left(\hat{\vec{p}} - q\vec{A}(\hat{\vec{x}})\right)^2}{2m} + q\phi(\vec{x})$. Find the appropriate form for $\hat{\vec{J}}(\vec{x})$ for this problem and show that current is conserved in the sense that $\frac{d\hat{\rho}^H(x)}{dt} = -\vec{\nabla}\cdot\hat{\vec{J}}^H(\vec{x})$. Hint: recall that for the classical version of the original problem the velocity is $\frac{\vec{p}}{m}$. What is the classical velocity in the new problem?

2. Consider a particle of charge *q* and mass *m* moving in a constant magnetic field in of strength *B* and oriented in the z direction. The purpose of this problem is find the eigenstates of energy for this situation. These turn out to be the famous Landau levels. To proceed we need to pick a gauge. In this problem we will take gauge $\vec{A}(\vec{x}) = -B\,\hat{x}\,y$ where $\hat{x}$ is a unit vector in the x direction (and not an operator)
    a. Show that this vector potential does in fact correspond the situation of interest.
    b. Show that solutions to the time-independent Schrodinger equation can be written as $\psi(x,y,z) = \exp\left(i\left(k_z z + k_x x\right)\right)\psi_n^{HO}\left(y + \frac{\hbar k_x}{qB}\right)$ where ψ_n^{HO} is the eigenfunction for the nth state of a one dimension harmonic oscillator for a system of mass *m* and $\omega = \left|\frac{qB}{m}\right|$ with associated energies given by
    $E_{k_z k_x n} = \frac{\hbar^2 k_z^2}{2m} + \hbar\omega\left(n + \tfrac{1}{2}\right)$.
    Note that the eigenstates are specified by two continuous parameters k_z, k_x and one discrete parameter *n*. Note also that the energies are completely independent of k_x and hence all level are infinite degenerate.