Homework 11: Due December 3

1. Consider a Hamiltonian which is periodic in time $\hat{H}(t) = \hat{H}(t+\tau)$.
   a. Show that $\hat{U}(t+n\tau, t_0) = \hat{U}(t,t_0)\hat{U}(t_0+\tau, t_0)^n$
   b. Show that $\hat{U}(t,t_0)$ can be written in the form $\hat{U}(t,t_0) = \hat{U}_0(t,t_0)e^{-i\hat{\bar{H}}(t-t_0)}$ where $\hat{U}_0(t+\tau,t_0) = \hat{U}_0(t,t_0)$ and $\hat{\bar{H}}$ is a Hemitian operator given by $\hat{\bar{H}} = i\dfrac{\log\left(\hat{U}(\tau+t_0,t_0)\right)}{\tau}$.
   The Log of the operator as defined as the inverse of the exponential (*i.e.* $\hat{B} = \log(\hat{A})$ if and only if $\hat{A} = \exp(\hat{B})$) .
   c. Using the result in a. show that there exist solutions to the time-dependent Schrodinger equation of the form $\left|\psi(t)\right\rangle = \exp(-i\varepsilon_n t)\left|\psi_n(t)\right\rangle$ where $\left|\psi_n(t)\right\rangle$ is periodic, $\left|\psi_n(t+\tau)\right\rangle = \left|\psi_n(t)\right\rangle$, and ε_n is a constant (sometimes called a quasi-energy. This is the time analog of Bloch's theorem.

Sakurai **chapter 4 : 1, 2 , 5, 6**