ELECTRODYNAMICS
PROBLEM SET 9
due April 13th, before class

I. REFLECTION WITH DISPERSION

A plane wave of frequency ω is incident normally from vacuum on a semi-infinite slab of material with a complex $n(\omega)$.

a) Show that the ratio of reflected power to incident power is

$$R = \left| \frac{1 - n(\omega)}{1 + n(\omega)} \right|^2 . \tag{1}$$

II. ANISOTROPIC MEDIUM

Plane waves propagate in a homogeneous, nonpermeable ($\mu = 1$) but anisotropic dielectric medium. The dielectric is characterized by a tensor ϵ_{ij} such that $D_i = \epsilon_{ij} E_j$. ϵ_{ij} is symmetric so it can be diagonalized by choosing an appropriate orthogonal coordinate system..

a) Show that plane waves with frequency ω and wave vector $\mathbf{k}$ must satisfy

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) + \frac{\omega^2}{c^2} \mathbf{D} = 0. \tag{2}$$

b) Show that for a given wave vector $\mathbf{k} = k\mathbf{n}$ there are two distinct modes of propagation with different phase velocities $v = \omega/k$ which satisfy the *Fresnel equation*:

$$\sum_{i=1}^{3} \frac{n_i^2}{v^2 - v_i^2} = 0, \tag{3}$$

where $v_i = c/\sqrt{\epsilon_i}$ and n_i is the ith component of $\mathbf{k}/|k|$.

III. ORDERS OF MAGNITUDE

Without looking at books/internet, try to guess the (order of magnitude of) the wavelengths of
a) visible light
b) radio waves
c) microwave
d) X-Ray
Add a very short (one line?) rationale of how you arrive at this estimate.
Find from some reliable source a *rough* estimate for the numbers above and compare them with your guesses.

IV. OPTICALLY ACTIVE MEDIUM

A dextrose solution is optically active and is characterized by a polarization vector satisfying $\mathbf{P} = \gamma \nabla \times \mathbf{E}$ for a real constant γ which depends on the dextrose concentration. The solution is non-condctung and non-magnetic (that is, the magnetization vanishes). Consider a plane wave with frequency ω propagating in this solution. For definiteness, assume the propagation is in the z direction. Also assume $\gamma\omega << c$ so that square rots can be approximated as $\sqrt{1 + A} \approx 1 + A/2$.

a) Find the two possible indices of refraction for such a wave.

b) Suppose linearly polarized light s incident on the dextrose solution. After traveling a distance L through the solution, the light is still linearly polarized but the direction of polarization has been rotated by an angle ϕ (this is called Faraday rotation). Find ϕ in terms of L, γ and ω.